TD #2 (easy version)

Large-scale Mathematical Programming

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INF580

Monitoring an electrical grid
(for those who have not had time to do it in TD1)

Other easy problems

Section 1

Monitoring an electrical grid

(for those who have not had time to do it in TD1)

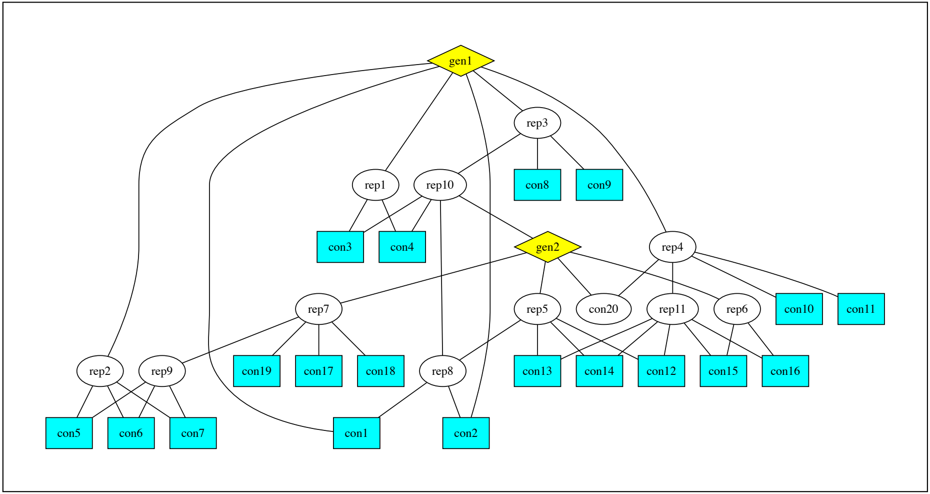
The problem

An electricity distribution company wants to monitor certain quantities at the lines of its grid by placing measuring devices at the buses. There are three types of buses: consumer, generator, and repeater. There are five types of devices:

- A: installed at any bus, and monitors all incident lines (cost: 0.9MEUR)
- B: installed at consumer and repeater buses, and monitors two incident lines (cost: 0.5MEUR)
- C: installed at generator buses only, and monitors one incident line (cost: 0.3MEUR)
- D: installed at repeater buses only, and monitors one incident line (cost: 0.2MEUR)
- E: installed at consumer buses only, and monitors one incident line (cost: 0.3MEUR).

Provide a least-cost installation plan for the devices at the buses, so that all lines are monitored by at least one device.

The electrical grid



Formulation

- Index sets:
  - V: set of buses v
  - E: set of lines $\{u, v\}$
  - A: set of *directed* lines (u, v)
  - $\forall u \in V$ let N_u = buses adjacent to u
  - D: set of device types
  - D_M: device types covering > 1 line
  - $D_1 = D \smallsetminus D_M$
- Parameters:
  - $\forall v \in V \quad b_v$ = bus type
  - $\forall d \in D \quad c_d$ = device cost

Formulation

- Decision variables
  - $\forall d \in D, v \in V \quad x_{dv} = 1$
    iff device type d installed at bus v
  - $\forall d \in D, (u, v) \in A \quad y_{duv} = 1$
    iff device type d installed at bus u measures line $\{u, v\}$
  - all variables are binary
- Objective function

$$\min_{x,y} \sum_{d \in D} c_d \sum_{v \in V} x_{dv}$$

Formulation

- Constraints
  - device types:

$$\begin{aligned} \forall v \in V \quad b_v = \mathsf{gen} \quad &\rightarrow \quad x_{\mathsf{B}v} = 0 \\ \forall v \in V \quad b_v \in \{\mathsf{con}, \mathsf{rep}\} \quad &\rightarrow \quad x_{\mathsf{C}v} = 0 \\ \forall v \in V \quad b_v \in \{\mathsf{gen}, \mathsf{con}\} \quad &\rightarrow \quad x_{\mathsf{D}v} = 0 \\ \forall v \in V \quad b_v \in \{\mathsf{gen}, \mathsf{rep}\} \quad &\rightarrow \quad x_{\mathsf{E}v} = 0 \end{aligned}$$

  - at most one device of any type at each bus

$$\forall v \in V \quad \sum_{d \in D} x_{dv} \leq 1$$

Formulation

- Constraints
  - A: every line incident to installed device is monitored

$$\forall u \in V, v \in N_u \quad y_{\mathsf{A}uv} = x_{\mathsf{A}u}$$

  - B: two monitored lines incident to installed device

$$\forall u \in V \quad \sum_{v \in N_u} y_{\mathsf{B}uv} = \min(2, |N_u|) x_{\mathsf{B}u}$$

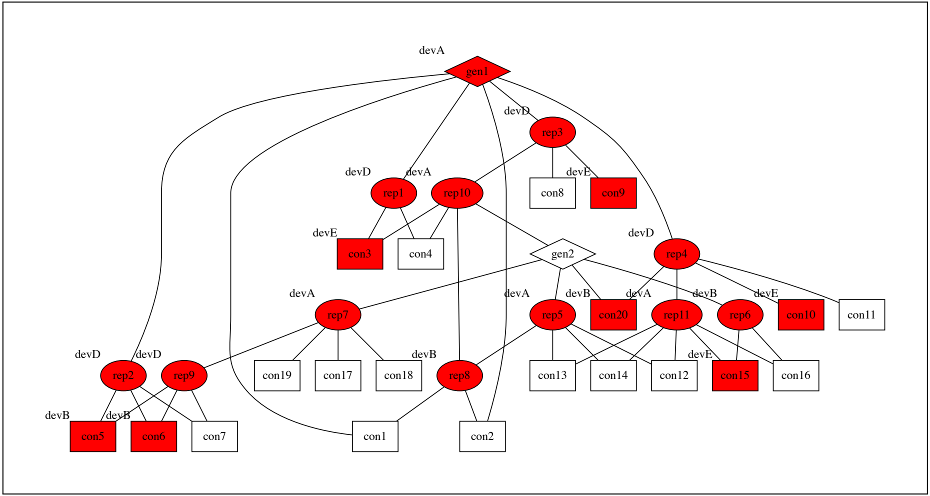
  - C,D,E: one monitored line incident to installed device

$$\forall d \in D_1, u \in V \quad \sum_{v \in N_u} y_{duv} = x_{du}$$

  - line is monitored

$$\forall \{u, v\} \in E \quad \sum_{d \in D} y_{duv} + \sum_{e \in D} y_{evu} \geq 1$$

Solution



all lines monitored, no redundancy, cost 9.2MEUR

Section 2

Other easy problems

Blending

A refinery produces two types of fuel by blending three types of crude. The first type of fuel requires at most 30% of crude 1 and at least 40% of crude 2, and retails at 5.5EUR per unit. The second type requires at most 50% of crude 1 and at least 10% of crude 2, and retails at 4.5EUR. The availability of crude 1 is 3000 units, at a unit cost of 3EUR; for crude 2 we have 2000 units and a unit cost of 6EUR; for crude 3, 4000 and 4EUR. How do we choose the amounts of crude to blend in the two fuels so as to maximize net profit?

Assignment

There are n jobs to be dispatched to m identical machines. The j-th job takes time p_j to complete. Jobs cannot be interrupted and resumed. Each machine can only process one job at a time. Assign jobs to machines so the whole set of jobs is completed in the shortest possible time. Also write a random instance generator so you can actually solve this problem using AMPL.

Demands

A small firm needs to obtain a certain number of computational servers on loan. Their needs change every month: 9 in January, 5 in February, 7 in March, 9 in April. The loan cost depends on the length: 200EUR for one month, 350 for two, and 450 for three. Plan the needed loans in the cheapest possible way.

Demands, again

A computer service firm estimates the need for service hours over the next five months as follows: 6000, 7000, 8000, 9500, 11000. Currently, the firm employs 50 consultants: each works at most 160 hours/month, and is paid 2000EUR/month. To satisfy demand peaks, the firm must recruit and train new consultants: training takes one month, and 50 hours of supervision work of an existing consultant. Trainees are paid 1000EUR/month. It was observed that 5% of the trainees leave the firm for the competition at the end of training. Plan the activites at minimum cost.

Multi-period production

A manufacturing firm needs to plan its activities on a 3-month horizon. It can produce 110 units at a cost of 300$ each; moreover, if it produces at all in a given month, it must produce at least 15 units per month. It can also subcontract production of 60 supplementary units at a cost of 330% each. Storage costs amount to 10$ per unit per month. Sales forecasts for the next three months are 100, 130, and 150 units. Satisfy the demand at minimum cost.

Capacities

A total of n data flows must be routed on one of two possible links between a source and a target node. The j-th data flow requires c_j Mbps to be routed. The capacity of the first link is 1Mbps; the capacity of the second is 2Mbps. Routing through the second link, however, is 30% more expensive than routing through the first. Minimize the routing cost while respecting link capacities. Write a random instance generator and solve instances with AMPL.

Covering, set-up costs and transportation

A distribution firm has identified n candidate sites to build depots. The i-th candidate depot, having given capacity b_i, costs f_i to build (for $i \leq n$). There are m stores to be supplied, each having a minimum demand d_j (for $j \leq n$). The cost of transporting one unit of goods between depot i and store j is c_{ij}. Plan openings and transportation so as to minimize costs. Write a random instance generator and solve instances with AMPL.

Circle packing

Maximize the number of cylindrical crates of beer (each having 20cm radius) which can be packed in the carrying area (6m long and 2.5m wide) of a pick-up truck.