

TD #1

Large-scale Mathematical Programming

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Software

Modelling

Implementation

Section I

Software

Structured and flat formulations

- ▶ Mathematical Programs (MP) describing *problems* involve sets and parameters
e.g. $\min\{c^\top x \mid Ax \geq b\}$
- ▶ For each set of values assigned to the parameters, MP describes a different *instance*
e.g. $\min\{x_1 + 2x_2 \mid x_1 + x_2 \geq 1\}$

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e.g. $\min\{x_1 + 2x_2 \mid x_1 + x_2 \geq 1\}$
- ▶ Humans reason in terms of problems (*structured formulations*)
- ▶ Solvers provide solutions for instances (*flat formulations*)
- ▶ Need a translation from problems to instances: **modelling languages**
(e.g. AMPL, Python+PyOMO, Matlab+YALMIP, Julia+JuMP, ...)

AMPL vs. Python

▶ AMPL

- ▶ wonderful syntax close to mathematics
- ▶ interfaces with lots of solvers, including MINLP (but little SDP)
- ▶ imperative sub-language: poor (no function calls, no libraries)
- ▶ good for rapid prototyping or “just use the solver”

▶ Python

- ▶ mixture of declarative (PyOMO) and imperative (Python)
- ▶ interfaces with many solvers, including SDP (but little MINLP)
- ▶ excellent imperative sub-language (Python itself)
- ▶ good for “doing further stuff with the solution”

Installing AMPL

▶ Windows (64bit)

1. make directory C:\ampl
2. copy ampl_mswin64.zip inside C:\ampl and unzip it
3. insert C:\ampl in the PATH environment variable
System Properties dialog/Advanced tab/Environment Variables button/Path field/Edit button/add C:\ampl to the string, separated by semicolons

▶ MacOS X: open terminal, and type

```
cd ; mkdir ampl ; cd ampl
unzip ~/Downloads/ampl_macosx64.zip
cd ; echo "export PATH=$PATH:~/ampl" >> ~/.bash_profile
source ~/.bash_profile
```

▶ Linux (64bit): as for MacOS X

but replace ampl_macosx64.zip by ampl_linux-intel64.zip

Testing AMPL

1. open a command prompt / terminal window
2. Save the following to `test.run`

```
set M := 1..50;
set N := 1..10;
param c{N} default Uniform01();
param A{M,N} default Uniform(0,1);
param b{M} default Uniform(1,2);
var x{N} >= 0;
minimize f: sum{j in N} c[j]*x[j];
subject to C{i in M}:
    sum{j in N} A[i,j]*x[j] >= b[i];
option solver cplex;
solve;
display x,f,solve_result;
```

3. type `ampl < test.run`
4. optimal objective function value is $f = 1.34199$

Section 2

Modelling

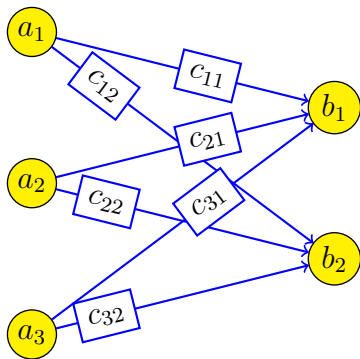
The transportation problem

Given a set P of production facilities with production capacities a_i for $i \in P$, a set Q of customer sites with demands b_j for $j \in Q$, and knowing that the unit transportation cost from facility $i \in P$ to customer $j \in Q$ is c_{ij} , find the optimal transportation plan



The art of modelling!

- *Use drawings — they help to think*



First fundamental question

- I. What decisions does the problem require?

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1. what's given?
2. costs — unit, refers to quantities
3. capacities and demand based on quantities
4. \Rightarrow *let's decide quantities*
5. (pitfall: the question “quantity *of what?*” is irrelevant — and you don't know in advance which questions are irrelevant)

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- *As you go on with the model, you might find your initial choices were poor — you might have to go back and change them*

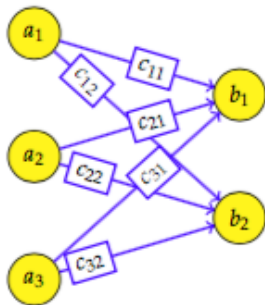
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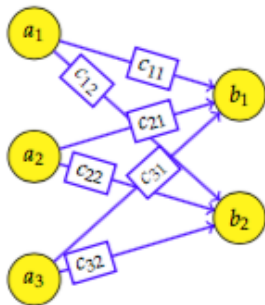
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► How about:

z_i = qty. produced at i

y_j = qty. demanded at j

Let's try this choice

1. *Sets and indices*

- a. $i \in P \subset \mathbb{N}$
- b. $j \in Q \subset \mathbb{N}$

2. *Parameters*

- a. $\forall i \in P \quad a_i \in \mathbb{R}_+$
- b. $\forall j \in Q \quad b_j \in \mathbb{R}_+$
- c. $\forall i \in P, j \in Q \quad c_{ij} \in \mathbb{R}_+$

3. *Decision variables*

- a. $\forall i \in P \quad z_i \in [0, a_i]$
- b. $\forall j \in Q \quad y_j \in [b_j, \infty]$

4. *Constraints*

- a. All that is produced must be delivered: $\sum_{i \in P} z_i = \sum_{j \in Q} y_j$

necessary condition, but is it sufficient?

5. *Objective function: ???*

no way of knowing what fraction of the production out of i went to j , so how do we consider transportation costs?

Bummer! Let's go back

- ▶ Failure to express “fraction of i going to j ” must inspire us!
Let's try $x_{ij} = \text{qty. transported from } i \text{ to } j$

1. *Sets*: as before
2. *Parameters*: as before
3. *Decision variables*

- a. $\forall i \in P, j \in Q \quad x_{ij} \in \mathbb{R}_+$

4. *Objective function*

$$\min \sum_{i \in P} \sum_{j \in Q} c_{ij} x_{ij}$$

5. *Constraints*

- a. No facility can produce more than the maximum:

$$\forall i \in P \quad \sum_{j \in Q} x_{ij} \leq a_i$$

- b. No customer must receive less than its demand:

$$\forall j \in Q \quad \sum_{i \in P} x_{ij} \geq b_j$$

Much better!

Section 3

Implementation

The AMPL encoding

- ▶ Three files:
 - ▶ `file.mod`: the *model file*
containing the description of the structured formulation
 - ▶ `file.dat`: the *data file*
containing the description of the instance
 - ▶ `file.run`: the *run file*
the “imperative part”: choice of solver, run, analyze solution...
 - ▶ Run “`ampl < file.run`” and get results on file or screen

The transportation problem in AMPL: .mod

```
# transportation.mod
param Pmax integer;
param Qmax integer;
set P := 1..Pmax;
set Q := 1..Qmax;
param a{P};
param b{Q};
param c{P,Q};
var x{P,Q} >= 0;
minimize cost: sum{i in P, j in Q} c[i,j]*x[i,j];
subject to production{i in P}:
    sum{j in Q} x[i,j] <= a[i];
subject to demand{j in Q}:
    sum{i in P} x[i,j] >= b[j];
```

The transportation problem in AMPL: .dat

```
# transportation.dat
param Pmax := 2;
param Qmax := 1;
param a :=
  1  2.0
  2  2.0
;
param b :=
  1  1.0
;
param c :=
  1 1  1.0
  2 1  2.0
;
```

The transportation problem in AMPL: .run

```
# transportation.run
model transportation.mod;
data transportation.dat;
option solver cplex;
solve;
display x, cost;
```