TD #1

Advanced Mathematical Programming

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Software

Modelling

Implementation

Section 1

Software

Structured and flat formulations

- Mathematical Programs (MP) describing *problems* involve sets and parameters
  e.g. $\min\{c^\top x \mid Ax \geq b\}$
- For each set of values assigned to the parameters, MP describes a different *instance*
  e.g. $\min\{x_1 + 2x_2 \mid x_1 + x_2 >= 1\}$

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- Humans reason in terms of problems (*structured formulations*)
- Solvers provide solutions for instances (*flat formulations*)
- Need a translation from problems to instances: modelling languages
  (e.g. AMPL, Python+PyOMO, Matlab+YALMIP, Julia+JuMP, …)

AMPL vs. Python

- AMPL
  - wonderful syntax close to mathematics
  - interfaces with lots of solvers, including MINLP (but little SDP)
  - imperative sub-language: poor (no function calls, no libraries)
  - good for rapid prototyping or "just use the solver"
- Python
  - mixture of declarative (PyOMO) and imperative (Python)
  - interfaces with many solvers, including SDP (but little MINLP)
  - excellent imperative sub-language (Python itself)
  - good for "doing further stuff with the solution"

Installing AMPL

- Windows (64bit)
  1. make directory `C:\ampl`
  2. copy `ampl_mswin64.zip` inside `C:\ampl` and unzip it
  3. insert `C:\ampl` in the `PATH` environment variable
     System Properties dialog/*Advanced* tab/*Environment Variables* button/*Path* field/*Edit* button/add `C:\ampl` to the string, separated by semicolons
- MacOS X: open terminal, and type

```
cd ; mkdir ampl ; cd ampl
unzip ~/Downloads/ampl_macosx64.zip
cd ; echo "export PATH=$PATH:~/ampl" >> ~/.bash_profile
source ~/.bash_profile
```

- Linux (64bit): as for MacOS X
  but replace `ampl_macosx64.zip` *by* `ampl_linux-intel64.zip`

Testing AMPL

1. open a command prompt / terminal window
2. Save the following to `test.run`

```
set M := 1..50;
set N := 1..10;
param c{N} default Uniform01();
param A{M,N} default Uniform(0,1);
param b{M} default Uniform(1,2);
var x{N} >= 0;
minimize f: sum{j in N} c[j]*x[j];
subject to C{i in M}:
  sum{j in N} A[i,j]*x[j] >= b[i];
option solver cplex;
solve;
display x,f,solve_result;
```

3. type `ampl < test.run`
4. optimal objective function value is `f = 1.34199`

Section 2

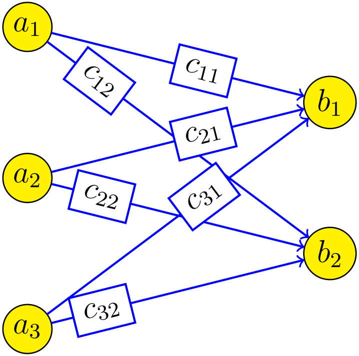
Modelling

The transportation problem

Given a set P of production facilities with production capacities a_i for $i \in P$, a set Q of customer sites with demands b_j for $j \in Q$, and knowing that the unit transportation cost from facility $i \in P$ to customer $j \in Q$ is c_{ij}, find the optimal transportation plan

The art of modelling!

- *Use drawings — they help to think*



First fundamental question

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   1. what's given?
   2. costs — unit, refers to quantities
   3. capacities and demand based on quantities
   4. ⇒ *let's decide quantities*
   5. (pitfall: the question "quantity *of what*?" is irrelevant — and you don't know in advance which questions are irrelevant)

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- *As you go on with the model, you might find your initial choices were poor — you might have to go back and change them*

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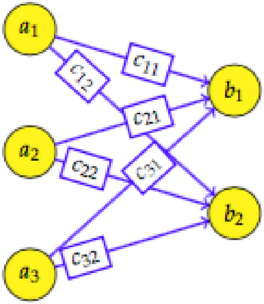

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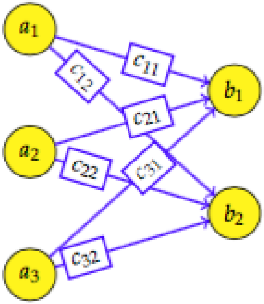

- How about:
  z_i = qty. produced at i
  y_j = qty. demanded at j

Let's try this choice

1. *Sets and indices*
   a. $i \in P \subset \mathbb{N}$
   b. $j \in Q \subset \mathbb{N}$
2. *Parameters*
   a. $\forall i \in P \quad a_i \in \mathbb{R}_+$
   b. $\forall j \in Q \quad b_j \in \mathbb{R}_+$
   c. $\forall i \in P, j \in Q \quad c_{ij} \in \mathbb{R}_+$
3. *Decision variables*
   a. $\forall i \in P \quad z_i \in [0, a_i]$
   b. $\forall j \in Q \quad y_j \in [b_j, \infty]$
4. *Constraints*
   a. All that is produced must be delivered: $\sum_{i \in P} z_i = \sum_{j \in Q} y_j$

      necessary condition, but is it sufficient?
5. *Objective function*: ???

   no way of knowing what fraction of the production out of i went to j, so how do we consider transportation costs?

Bummer! Let's go back

- Failure to express "*fraction of i going to j*" must inspire us!
  Let's try x_{ij} = qty. transported from i to j

1. *Sets*: as before
2. *Parameters*: as before
3. *Decision variables*
   a. $\forall i \in P, j \in Q \quad x_{ij} \in \mathbb{R}_+$
4. *Objective function*

$$\min \sum_{i \in P} \sum_{j \in Q} c_{ij} x_{ij}$$

5. *Constraints*
   a. No facility can produce more than the maximum:
   $\forall i \in P \quad \sum_{j \in Q} x_{ij} \leq a_i$
   b. No customer must receive less than its demand:
   $\forall j \in Q \quad \sum_{i \in P} x_{ij} \geq b_j$

Much better!

Section 3

Implementation

The AMPL encoding

- Three files:
  - `file.mod`: the *model file*
    containing the description of the structured formulation
  - `file.dat`: the *data file*
    containing the description of the instance
  - `file.run`: the *run file*
    the "imperative part": choice of solver, run, analyze solution…
  - Run "`ampl < file.run`" and get results on file or screen

The transportation problem in AMPL: `.mod`

```
# transportation.mod
param Pmax integer;
param Qmax integer;
set P := 1..Pmax;
set Q := 1..Qmax;
param a{P};
param b{Q};
param c{P,Q};
var x{P,Q} >= 0;
minimize cost: sum{i in P, j in Q} c[i,j]*x[i,j];
subject to production{i in P}:
  sum{j in Q} x[i,j] <= a[i];
subject to demand{j in Q}:
  sum{i in P} x[i,j] >= b[j];
```

The transportation problem in AMPL: .dat

```
# transportation.dat
param Pmax := 2;
param Qmax := 1;
param a :=
 1  2.0
 2  2.0
;
param b :=
 1  1.0
;
param c :=
 1 1  1.0
 2 1  2.0
;
```

The transportation problem in AMPL: `.run`

```
# transportation.run
model transportation.mod;
data transportation.dat;
option solver cplex;
solve;
display x, cost;
```