Optimal running and planning of a biomass-based energy production process

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Abstract

We propose mathematical programming models for solving problems arising from planning and running an energy production process based on burning biomasses. The models take into account different aspects of the problem: determination of the biomasses to produce and/or buy, transportation decisions to convey the materials to the respective plants, and plant site locations. Whereas the "running model" is linear, we propose two "planning models", both of which are Mixed-Integer Nonlinear Programming problems. We show that a spatial Branch-and-Bound type algorithm applied to them is guaranteed to converge to an exact optimum in a finite number of steps.

Keywords: renewable energy, biomass exploitation, mathematical programming, spatial Branch-and-Bound, finite convergence, global optimization.

1 Introduction

Producing energy derived from fossil carbon-based fuels is proving costly to both the environment (in terms of pollution) and society (in terms of monetary investment). As the prices of crude oil increase, governments and other institutions are researching the most cost-efficient ways to produce energy from alternative sources [Iny05]. One of the most popular contendents is energy produced by biomasses of several kinds [Pro98]. In [MAAC00] the competitiveness of biomass-based fuel for electrical energy opposed to carbon-based fuel is examined using a mathematical programming model. Among the advantages of this type of energy production, there is the potential for employing wasted materials of biological origin, like used alimentary fats and oils, agricultural waste and so on. A factory producing energy with such materials would benefit from both the sales of the energy and the gains obtained by servicing waste [ABMN04]. In [FMG05] a mathematical program is proposed to localize both energy conversion plants and biomass catchment basins in provincial areas. Other mathematical models for specific biomass discrete facility location problems are developed in [FMR$^+$04] and [DV01]. A model that combines detailed energy conversion plant optimization with energy/heat transportation cost is given in [SP06].

This paper describes an optimization problem arising from the deployment of such an energy production process in central Italy. This involves several processing plants of different types (for example, a liquid biomass plant, a squeeze plant and a fermentation-distillation plant). Some of these plants (e.g. liquid biomass plant) produce energy; others (e.g. the fermentation-distillation plant) produce intermediate products which will then be routed to other plants for further processing. There are several possible input products (e.g. agricultural products, biological waste), obtained from different sources (e.g. direct farming or acquisition on the markets) at different unit costs. Apart from the energetic output, there may be other output products which are sold in different markets (e.g. bioethanol obtained from the fermentation-distillation plant and sold in the bioethanol market). See Fig. 1 for a typical process flowsheet.

There are in fact three optimization problems relating to this description. The first (and simplest) is that of modelling the production process as a net gain maximization supposing the type of plants involved and the end product demands are known. The second is that of deciding the type of plants to involve in the process to maximize the net gain, subject to known end product demands. Although post-optimal sensitivity analysis may be used to gather hints on how to improve the process, an optimization model provides the ultimate process planning tool. The second model is in fact a simple variant of the first, in that we simply let some of the parameters of the first (linear) problem be decision variables in the second. The third problem is an evolution of the second, taking into account plant installation costs, some features of electricity production plants, and transportation issues [LA05, LM06b, MÖ5]. We remark that we only carried out computational experiments on the first and second model, since the practical needs of the industry that commissioned the research were limited. The third model is supplied to show that this modelling approach can be extended to a more complicated and realistic setup.

Section 2 describes the model relating to the production process when the plant types are known ("running model"). Section 3 describes the model relating to the process planning ("planning model"), an exact mixed-integer linear reformulation thereof, and shows that an application of a standard spatial Branch-and-Bound algorithm (e.g. [ST05, SP99]) yields a finitely convergent exact method. In Section 4 we discuss the application of the production process and planning models to a real-life case. Section 5 discusses the third model, and Section 6 concludes the paper.

2 Optimizing the production process

Modelling a flowsheet as that presented in Fig. 1 presents many difficulties. Notice that the products can be inputs, intermediate, outputs, or both (like alcohol, which is both an output product and an intermediate product). Likewise, processes can be intermediate or final or a combination (like the fermentation-distillation plant). Consider also that the decision maker may choose to buy an intermediate product from a different source to cover demand needs, thus making the product a combination of intermediate and input. Of course the input products may be acquired or produced at different locations and at different prices. Moreover, each flow arrow has an associated transportation cost. The time horizon for the optimization process is one year.

The central concept in our model is the process site. A *process site* is a geographical location with at most one processing plant and/or various storage spaces for different types of goods (commodities). A place where production of a given commodity occurs is represented by a process site with a storage space. Thus, for example, a geographical location with two fields producing maize and sunflowers is a process site with two storage spaces and no processing plant. The fermentation-distillation plant is a process site with no storage spaces and one processing plant. Each output in Fig. 1 is represented by a process site with just one storage space for each output good. In this interpretation the concepts of input, output and intermediate products, and those of intermediate and final process, lose importance: this is appropriate because, as we have emphasized earlier, these distinctions are not always well-defined. Instead, we focus the attention on the material balance and on the transformation process in each process site. Furthermore, we are able to deal with the occurrence that a given commodity may be obtained at different costs depending on whether it is bought or produced directly.

We represent the process sites by a set V of vertices of a graph $G = (V, A)$ where the set of arcs A is given by the logistic connections among the locations. To each vertex $v \in V$ we associate a set of commodities $H^-(v)$ which may enter the process site, and a set of commodities $H^+(v)$ which may leave it. Thus, for example, the fermentation-distillation plant is a process site vertex where $H^-(\text{fermentation-distillation plant}) = \{\text{cane}, \text{beetroots}\}$ and $H^+(\text{fermentation-distillation plant}) = \{\text{alcohol}\}$. Furthermore, we let $H = \bigcup_{v \in V}(H^-(v) \cup H^+(v))$ be the set of all commodities involved in

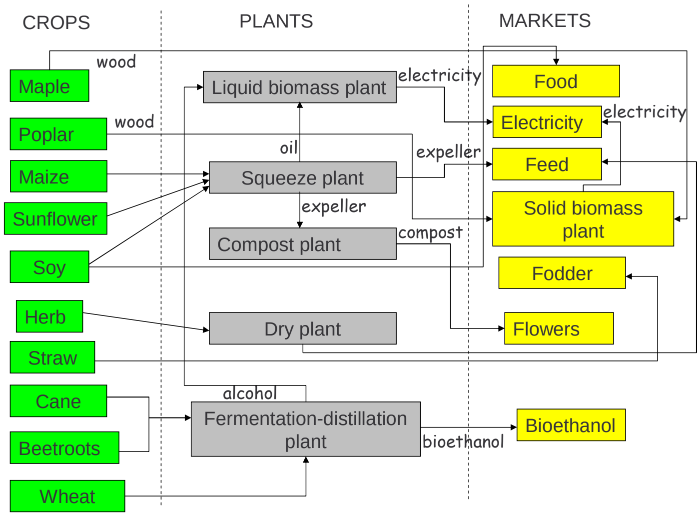

Figure 1: A typical process flowsheet.

the production process, and we partition $V = V_0 \cup V_1$ into V_0, the set of process sites with an associated processing plant, and $V_1 = V \backslash V_0$.

Fig. 2 is the graph derived from the example in Fig. 1.

The following parameters define the problem instance:

- c_{vk}: cost of supplying vertex v with a unit of commodity k (negative costs are associated with output nodes, as these represent selling prices; a negative cost may also be associated to the input node "waste", since waste disposal is a service commodity);
- C_{vk}: maximum quantity of commodity k in vertex v;
- τ_{uvk}: transportation cost for a unit of commodity k on the arc (u, v);
- T_{uvk}: transportation capacity for commodity k on arc (u, v);
- λ_{vkh}: cost of processing a unit of commodity k into commodity h in vertex v;
- π_{vkh}: yield of commodity h expressed as unit percentage of commodity k in vertex v;
- d_{vk}: demand of commodity k in vertex v.

It is clear that certain parameters make sense only when associated to a particular subset of vertices, like e.g. the demands may only be applied to the vertices representing the outputs. In this case, the corresponding parameter should be set to 0 in all vertices for which it is not applicable.

The decision variables are:

- x_{vk}: quantity of commodity k in vertex v;
- y_{uvk}: quantity of commodity k on arc (u, v);
- z_{vkh}: quantity of commodity k processed into commodity h in vertex v.

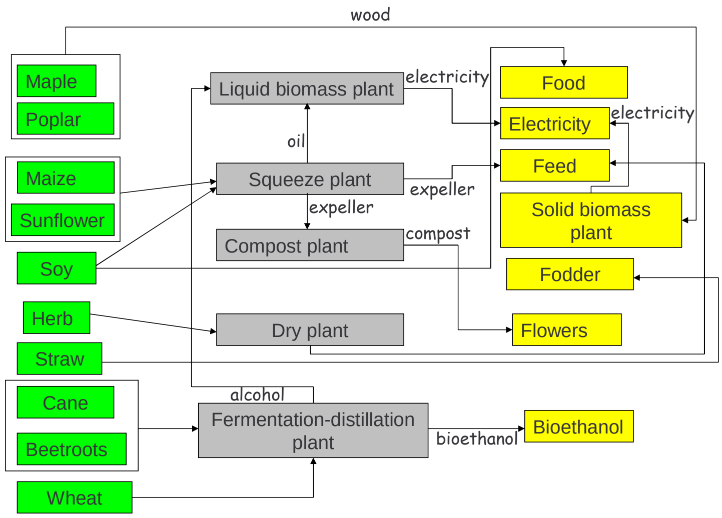

Figure 2: The graph derived from the example in Fig. 1.

Since the output demands are known *a priori*, we would like to minimize the total operation costs subject to demand satisfaction. There are three types of costs:

- cost of supplying vertices with commodities:
$$\gamma_1 = \sum_{k \in H} \sum_{v \in V} c_{vk} x_{vk};$$
- transportation costs:
$$\gamma_2 = \sum_{k \in H} \sum_{(u,v) \in A} \tau_{uvk} y_{uvk};$$
- processing costs:
$$\gamma_3 = \sum_{v \in V} \sum_{k \in H^-(v)} \sum_{h \in H^+(v)} \lambda_{vkh} z_{vkh},$$

so the objective function is

$$\min \sum_{i=1}^{3} \gamma_i(x, y, z). \tag{1}$$

We need to make sure that some material conservation equations are enforced in each process site where a plant is installed:

$$\sum_{k \in H^-(v)} \pi_{vkh} z_{vkh} = x_{vh}, \ \forall v \in V_0, h \in H^+(v). \tag{2}$$

Notice that these constraints do not actually enforce a conservation of mass, for in most processing plants a percentage of the input quantities goes to waste; but it is nonetheless a conservation law subject to the yield properties of the particular transformation process of the plant.

Secondly, the quantity of processed commodity must not exceed the quantity of input commodity in each vertex:

$$\sum_{h \in H^+(v)} z_{vkh} \leq x_{vk}, \ \forall v \in V_0, k \in H^-(v). \tag{3}$$

Furthermore, we need the quantity of input commodity in each vertex to be consistent with the quantity of commodity in the vertex itself, and similarly for output commodities:

$$\sum_{u \in V:(u,v) \in A} y_{uvk} = x_{vk}, \ \forall v \in V, k \in H^-(v) \tag{4}$$

$$\sum_{u \in V:(v,u) \in A} y_{vuh} = x_{vh}, \ \forall v \in V, h \in H^+(v). \tag{5}$$

Finally, we have the bounds on the variables:

$$d_{vk} \leq x_{vk} \leq C_{vk}, \ \forall v \in V, k \in H \tag{6}$$

$$0 \leq y_{uvk} \leq T_{uvk}, \ \forall (u,v) \in A, k \in H \tag{7}$$

$$z_{vkh} \geq 0, \ \forall v \in V, k \in H^-(v), h \in H^+(v) \tag{8}$$

and some fixed variables for irrelevant vertices:

$$x_{vk} = 0, \ \forall v \in V_1, k \in H \backslash (H^-(v) \cup H^+(v)) \tag{9}$$

$$y_{uvk} = 0, \ \forall (u,v) \in A, k \in H \backslash H^-(v), \tag{10}$$

$$y_{uvk} = 0, \ \forall (u,v) \in A, k \in H \backslash H^+(u). \tag{11}$$

The main advantage to this model is that it can be easily extended to deal with more commodities and plants in a natural way, by adding appropriate vertices or changing the relevant $H^-(v), H^+(v)$ and related parameters.

This problem is a Linear Program, and can be solved by using one of several LP solvers (e.g. CPLEX [ILO05]).

3 Planning the production process

In this section, we suppose no processing plants are yet present at the process sites. At each process site $v \in V_0$, we wish to install an appropriate processing plant chosen from a set $P(v)$ of possible plants (e.g. there may be different types of liquid biomass plants, each having different yield levels on the input commodities). We therefore wish to make decisions as regards the plant installation, feasible with the material balance constraints as in Section 2, which minimize the total operation costs.

We re-define the parameters λ, π to make them dependent on a processing plant p as follows:

- λ_{vkhp}: cost of using plant p to transform a unit of commodity k into commodity h in vertex v (this includes a per-unit estimate of the initial investment costs for building the plant);
- π_{vkhp}: yield of commodity h, using plant p, expressed as unit percentage of commodity k in vertex v.

We consider the following additional binary decision variables:

$$w_{vp} = \begin{cases} 1 & \text{if plant } p \text{ is installed in vertex } v \\ 0 & \text{otherwise} \end{cases}$$

Moreover, the node capacities C_{vk} and arc capacities T_{uvk}, which are considered as parameters in the previous model, are to be considered as decision variables instead, bounded above and below by relevant values.

The objective function (1) changes in the γ_3 term, which becomes:

$$\gamma_3' = \sum_{v \in V} \sum_{k \in H^-(v)} \sum_{h \in H^+(v)} \left(\sum_{p \in P(v)} \lambda_{vkhp} w_{vp} \right) z_{vkh}.$$

The material conservation constraints (2) become:

$$\sum_{k \in H^-(v)} \left(\sum_{p \in P(v)} \pi_{vkhp} w_{vp} \right) z_{vkh} = x_{vh}, \ \forall v \in V, h \in H^+(v). \tag{12}$$

The following constraints enforce consistency on the assignment variables (we remark that (13) allows a process site to host no plant at all):

$$\sum_{p \in P(v)} w_{vp} \leq 1, \ \forall v \in V_0, \tag{13}$$

$$\sum_{p \in P(v)} w_{vp} = 0, \ \forall v \in V_1. \tag{14}$$

Finally, constraints (3)-(8) are also part of the formulation.

3.1 Solution of the problem

The model described in Section 3 is a Mixed-Integer Nonlinear Programming problem (MINLP) with nonconvex terms in both the objective function and the constraints. Problems of this type are solved either by employing heuristic methods, like Multi Level Single Linkage (MLSL) [KS04, LK05] or Variable Neighbourhood Search (VNS) [HM01, LD05], or by using an ε-approximate method called spatial Branch-and-Bound (sBB) [SP99, ADFN98, TS04], which provides a proof of ε-optimality. sBB algorithms are Branch-and-Bound (BB) type algorithms (i.e. searches on trees where each node represents a restriction of the problem to a particular subdomain, the union of all the subdomains being the entire search space) where branching is possible on continuous variables appearing in nonlinear terms (branching on binary variables is carried out by fixing the variable at 0 in the left subnode and at 1 in the right subnode). Bounding is obtained by solving a suitable convex relaxation of the problem restricted at the current node's variable ranges. We assume that the branching scheme ensures that all binary variables are chosen for branching within a finite time limit (such a scheme is readily available by e.g. branching on each binary variable in turn). In general, when $\varepsilon = 0$, the sBB algorithm has no finite termination guarantee. Applied to this particular problem, however, the sBB yields a finitely terminating exact method, as shown in Thm. 3.2.

At each BB node we obtain a lower bound by solving a linear relaxation of the problem, built as follows.

1. Distribute products over sums, so that all the bilinear terms can be expressed as $w_{vp} z_{vkh}$ (also see [TS02]).

2. Replace each bilinar term $w_{vp} z_{vkh}$ by a new added variable ζ_{vkhp} called a *linearization variable*. More precisely, the bilinear terms in γ_3' in the objective and in constraints (12) should be replaced by the corresponding linearization variable, yielding:

$$\gamma_3'' = \sum_{v \in V} \sum_{k \in H^-(v)} \sum_{h \in H^+(v)} \sum_{p \in P(v)} \lambda_{vkhp} \zeta_{vkhp}$$

and

$$\sum_{k\in H^-(v)}\sum_{p\in P(v)} \pi_{vkhp}\zeta_{vkhp} = x_{vh},\ \forall v\in V, h\in H^+(v). \tag{15}$$

Naturally, to keep the reformulation exact, we must add the bilinear constraints

$$(\zeta_{vkhp} = w_{vp}z_{vkh}),\ \forall v\in V, k,h\in H, p\in P(v) \tag{16}$$

to the formulation. These are called *defining constraints*. The usefulness of this step is that it isolates the nonconvex terms in (16).

3. Replace (16) by their convex envelopes:

$$\begin{aligned}
(\zeta_{vkhp} &\geq 0),\ \forall v\in V, k,h\in H, p\in P(v) && (17)\\
(\zeta_{vkhp} &\geq z_{vkh} + z^U_{vkh}(w_{vp}-1)),\ \forall v\in V, k,h\in H, p\in P(v) && (18)\\
(\zeta_{vkhp} &\leq z^U_{vkh}w_{vp}),\ \forall v\in V, k,h\in H, p\in P(v) && (19)\\
(\zeta_{vkhp} &\leq z_{vkh}),\ \forall v\in V, k,h\in H, p\in P(v) && (20)
\end{aligned}$$

where z^U_{vkh} is a tight upper bound to z_{vkh} for each $v\in V,\ k,h\in H$. Constraints (17)-(20) are known as McCormick envelopes [McC76]. The relaxed problem is a mixed-integer linear relaxation of the original problem.

4. A linear (and hence convex) relaxation of the problem is readily obtained by relaxing the integrality constraints on the binary variables.

The linear relaxation thus derived is further tightened by adding Reformulation-Linearization Technique (RLT) cuts as in [SA86, SA92, SA99] by multiplying constraints by appropriate variables and then linearizing the resulting bilinear terms, as detailed below:

- constraints (3) by bound factors w_{vp} and $(1-w_{vp})$ for all $p\in P(v)$;
- constraints (13) and (14) by variables z_{vkh} for all k,h, to obtain:

$$\begin{aligned}
\sum_{p\in P(v)} \zeta_{vkhp} &\leq z_{vkh},\ \forall v\in V_0, k,h\in H && (21)\\
\sum_{p\in P(v)} \zeta_{vkhp} &= 0,\ \forall v\in V_1, k,h\in H. && (22)
\end{aligned}$$

Constraints (22) are a particular subclass of RLT constraints ([SA92, SA99]) called *reduction constraints* [Lib04b, Lib05, Lib04a], with very interesting properties. In particular, although the bilinear defining constraints (16) are not in the formulation, it can be shown that in consequence of (21) and (22), a certain subset of them still hold at the relaxed solution. Constraints (21) are normal level-1 RLT constraints.

To sum up, the linear relaxation at each sBB node consists in minimizing $\gamma_1+\gamma_2+\gamma_3''$ subject to (15) and (3) with the RLT cuts derived from them, (4), (5), (6)-(8), (9)-(11), (13) and (14) with the reduction constraints (21) and (22) derived from them, and the McCormick envelopes (17)-(20). We shall call this linear relaxation $\bar{L}$.

3.1 Proposition ([SA99], Prop. 8.11)
Let $W\subseteq\mathbb{R}^n$ and $Z\subseteq\mathbb{R}^m$ be the two nonempty polytopes (in variables w and z respectively) described in the planning model above, and consider the set $\Omega=\{(w,z,\zeta)\mid w\in W\wedge z\in Z\wedge$ (17)-(22)$\}$. Then, for any $(\bar{w},\bar{z},\bar{\zeta})\in\Omega$, if either $\bar{w}$ is a vertex of W or $\bar{z}$ is a vertex of Z, constraints (16) hold.

3.2 Theorem
As long as the branching scheme ensures that all binary variables are chosen for branching within a finite time limit, an sBB algorithm applied to the MINLP problem of Section 3 ((3)-(8),(12)-(14), excluding the linearization constraints of Sect. 3.1) converges to an exact solution or is shown to be infeasible in a finite amount of time.

Proof. Because of the assumption in the branching scheme, after a finite amount of running time in the sBB algorithm, a node N will be reached where all binary variables are fixed at either 0 or 1. Let $\bar{P}_N$ be the lower bounding LP at node N. $\bar{P}_N$ may be infeasible, unbounded or feasible with optimal solution x^N. If $\bar{P}_N$ is infeasible, the current Branch-and-Bound tree branch is pruned at the node N. If $\bar{P}_N$ is unbounded, the MINLP is also unbounded and the sBB terminates. Otherwise, if $\bar{P}_N$ is feasible, since the w variables have values 0 or 1 by virtue of the branching process, w will be at a vertex of its feasible polyhedron (which is a sub-polytope of $\{w_{vp} \in [0,1] \mid v \in V \wedge p \in P(v) \wedge \sum_{p \in P(v)} w_{vp} \leq 1$ if $v \in V_0$ and 0 othw.$\}$). By Prop. 3.1, this implies that x^N is feasible w.r.t. the bilinear defining constraints (16). Thus, x^N is also an upper bounding solution in the original problem and the node is fathomed, which implies that the current Branch-and-Bound tree branch is pruned at the node N. This also shows that the node is pruned even with $\varepsilon = 0$. Therefore, no branch of the Branch-and-Bound tree can be infinitely long, which proves finite convergence. To show exactness, we remark that since the algorithm was shown to converge even with $\varepsilon = 0$, the solution it provides is exact. Lastly, if no node yields a feasible solution x^N, the problem is infeasible. □

3.3 Proposition
Let L be the linear relaxation $\bar{L}$ subject to the integrality constraints on the w variables. L is an exact Mixed-Integer Linear Programming (MILP) reformulation of the planning model.

Proof. Let (w^*, z^*, ζ^*) be an optimal solution of L. Since w are either 0 or 1, again by Prop. 3.1 we have that $\forall v \in V, k, h \in H, p \in P(v)$ $(\zeta^*_{vkhp} = w^*_{vp} z^*_{vkh})$, which shows that the optimal solution of L is feasible in the original planning model. □

By Prop. 3.3, we can also solve the planning model with a MILP solver (e.g. CPLEX [ILO05]).

4 Computational experience

The main driving force for this paper was a real-life instance of the production model occurring in the Marche region of Italy. The owners of an agricultural ground currently producing beetroots and wheat wanted to switch to a more diversified scheme which could provide enough biomass to fuel an energy production plant. This instance gave rise to an LP model with fewer than 100 variables and constraints, the solution performance details are totally irrelevant. The instance was solved by CPLEX 10.1 [ILO05] to optimality. Most of the material used for this computational study can be found at `http://www.lix.polytechnique.fr/~liberti/bioenergy`.

As for the planning model, we considered two separate sets of instances. The first one is based on the real-life production instance mentioned above, modified to contain more production sites and potential plants at each site. The second one consists of randomly generated instances; see Table 1 for details.

We solved the instances in Table 1 to optimality using several solvers.

- BARON [ST05] (under the GAMS [BKM88] interface), a global optimization sBB solver acting on the original MINLP formulation.
- CPLEX [ILO05] (under the AMPL [FG02] interface), a BB solver acting on the MILP reformulation as per Corollary 3.3.
- MINLP_BB [Ley99] (under the AMPL interface), a BB solver for MINLPs which only branches on integer variables, acting on the original MINLP formulation — MINLP_BB only guarantees optimality if the NLP relaxation of the problem is convex (which is not the case here), but it proves to be an effective heuristic for nonconvex MINLPs.

Instance name	$\lvert V_0\rvert$	max$\lvert P(v)\rvert$	avg$\lvert P(v)\rvert$	Vars	(where 0-1)	Constrs	Noncvx terms
planning1	2	4	3.50	41	7	31	20+13
planning2	10	3	1.50	228	9	161	34+18
planning3	10	3	1.50	228	9	161	34+18
planning4	10	2	1.40	227	8	161	32+16
planning5	4	3	2.00	165	7	106	24+14
planning6	4	3	2.00	165	7	106	24+14
planning7	4	2	1.75	164	6	106	22+12
planning8	2	3	2.50	39	5	31	14+10
planning9	9	3	1.33	198	5	139	8+8
planning10	9	2	1.11	195	2	138	4+4
rnd10	2	6	3.50	179	6	131	72+42
rnd14	1	3	3.00	249	3	154	18+2
rnd16	1	8	8.00	158	8	90	16+16
rnd20	3	12	5.67	913	17	493	4+0
rnd23	4	9	6.50	690	26	406	60+50
rnd24	5	9	5.40	2379	26	1293	0+173
rnd26	7	9	6.14	177	42	101	28+7
rnd33	1	5	5.00	5361	5	2982	0+783
rnd37	3	3	2.00	5491	5	2876	0+384
rnd43	3	11	7.33	3289	22	1741	0+543
rnd47	1	5	5.00	4686	4	2726	23+15
rnd63	5	7	4.20	2681	21	1589	1957+923
rnd96	20	9	5.25	7908	80	4011	46+10

Table 1: The planning model instances. The nonconvex terms are expressed in the form $o + c$ where o is the number of nonconvex terms in the objective, and c is the number of nonconvex terms in the constraints.

Running times were generally very low for all solved instances: well within 2s CPU time for most instances, with a few exceptions which clocked at under a minute (on a PIV 1.2GHz with 640MB RAM running Linux). Running times were not deemed to be significant comparative indicators, particularly in view of the fact that such problems need usually not be solved in real time. The results are given in Table 2.

There are at least two important conclusions that can be drawn from the results in Table 2. First, all the considered methods scale well with the problem size. We feel it is particularly important to remark that the global optimization solvers perform on this application as well as the CPLEX MILP solver. Secondly, the experimental results are in line with the conclusion of Theorem 3.2 (we note that all global optimization solvers were run with a convergence tolerance of $\varepsilon = 0$).

5 A more realistic planning model

The models of Sect. 2 and Sect. 3 rely on several simplifications of real-life conditions. A truer picture would encompass other realistic features, as detailed below.

- Some of the plants considered in this paper produce electricity. These have very specific properties and behaviours [LA05, LM06b], among which:
    1. in a true market situation (i.e. no subsidization), electricity prices vary during the course of a single day, as demand rises and subsides;
    2. some electricity production plants are often designed to produce electricity *and* heat (which is

Instance name	BARON	CPLEX	MINLP_BB
planning1	-1993500.000006	-1993500.000000	-1993500.000000
planning2	-1487141.062815	-1487141.062815	-1487141.062815
planning3	-1293674.672558	-1293674.672558	-1293674.672558
planning4	-1298674.672558	-1298674.672558	-1298674.672558
planning5	-1592708.399909	-1592708.399909	-1592708.399909
planning6	-1282208.399909	-1282208.399909	-1282208.399909
planning7	-1287208.399909	-1287208.399909	-1287208.399909
planning8	-1688000.000000	-1688000.000000	-1688000.000000
planning9	-656478.709039	-656478.709039	-656478.709039
planning10	-482425.869355	-482425.869355	-482425.869355
rnd10	-10172670.9000	-10172670.900000	-10172670.900000
rnd14	-46793640.7549	-46793640.754903	-46793640.754903
rnd16	-2517826.01334	-2517826.013343	-2517826.013343
rnd20	-74172261.5403	-74172261.540278	-74172261.540278
rnd23	-225161225.148	-225161225.147790	-225161225.147790
rnd24	-24121864.8543	-24130018.218155	-23841483.937430*
rnd26	-2609446.88325	-2609446.883250	-2609446.883250
rnd33	-720041744.091*	-922056072.533006	-917574429.510109*
rnd37	-348612891.222*	-352566723.200840	-288592249.147212*
rnd43	-78666258.8413	-78666633.904820	-78666633.904820
rnd57	-28534845.5788	-28534845.578841	-28534845.578841
rnd63	-138429853.758	-143249498.228375	-138391489.573639
rnd96	-14015008.6266	-14015008.626632	-14015008.626632

Table 2: Objective function values found by BARON, CPLEX, MINLP_BB. Non-optimal values are marked *.

either stored or conveyed directly into buildings in the area) — such plants are called Combined Heat Power (CHP) [BS98];

3. CHPs generate heat and electricity at the same hour and same location.

- Transportation costs do not depend linearly on the distances due to the different means of transportation used [MÖ5]. For very short transportation distances, tractors may be used, which have higher transportation cost than lorries, used for medium to long distances; for very long transportation distances, trains or ships are used. More generally, the geography of the production process region deeply influences the costs of the single process activities; [MÖ5] suggests a methodology that combines Geographical Information Systems (GIS) software with process analysis to estimate these costs.

- All production plants incur a one time installation cost. This varies according to many factors, the most important of which are production capacity and physical location. A given production capacity can be obtained by building a single huge plant or many different small-sized plants. With respect to the latter choice, the former yields: (a) higher production efficiency but lower total efficiency (because usually it is difficult to convey auxiliary heat from one single place to many neighbouring urban areas); (b) lower specific building costs; (c) higher transportation costs due to distances [MÖ5].

It turns out that the planning model in Sect. 3 can be extended to accommodate most of the features above, excluding the electricity price variability over short time periods (point 1 in the list above). For this we would need an explicit time dependency in the model. Since the electricity price variability range is much shorter than the envisaged time horizon in the planning optimization (one day as opposed to one year), an hourly time discretization would multiply the number of variables by 8760 ($= 365 \times 24$), thus yielding a MINLP or MILP whose size is excessive with respect to the current solver state of the art. It is worth emphasizing, however, that since most of the available biomasses can be easily stored, the variability of electricity prices can be dealt with by adding biomass storage space near the electricity plants, which would either add more processing sites or simply reflect on the plant cost parameter. Storage space is not the only way to deal with the situation: the energyPRO model [LA05] proposes an electricity plant planning methodology that locally optimizes each plant over a yearly time horizon with hourly time-steps. The combined production of electricity and heat (point 2 in the list above) can be dealt by our model by simply introducing an output process site representing heat, and adapting the λ and π parameters relative to the CHP, various inputs and heat output to reflect the situation. As a consequence of point 3 in the list above, this modelling is not wholly satisfactory, as generation of heat is time-dependent because it is linked to the generation of electricity: but again this time dependency can be dealt with by using process sites representing heat storage capacity or simply adding to the plant cost parameter.

Nonlinear transportation costs are already fully dealt with by our model, for with each arc we associate a transportation cost which is not unitary but rather depends on the vertices adjacent to the arc. Since the arc length is not used anywhere in the model, each arc can be assigned its proper cost.

In order to cater for the last point, i.e. plant installation costs, we consider the parameters I_{vp} = cost of building plant $p \in P(v)$ in vertex $v \in V_0$. The dependency of the installation cost with the vertex v allows us to consider geography dependencies as outlined in [MÖ5]. We then add the following term λ_4 to the objective function:

$$\lambda_4 = \sum_{v \in V_0} \sum_{p \in P(v)} I_{vp} w_{vp}. \tag{23}$$

This does not change the convergence results given in Sect. 3.1.

We remark that although we treat plants as rather simple entities defined by their input, yields and outputs much like other nodes, this is a simplistic view. Some refined models that describe these these energy conversion units are given in [BS98, BSM98, BD02]; in particular, each of these nodes may sometimes consist of a set of different plants and utilization points where transportation of energy/heat has an impact on the overall efficiency. More recently, a full supply-chain model was provided that also cater for multi-company interactions [SOT$^+$06].

6 Conclusion

We describe a Linear Programming model for running a biomass-based energy production process and a Mixed-Integer Nonlinear Programming model for a simplified planning of the installation of processing plants used in the production process. Although the solution of the first model is readily obtained by any good quality LP solver, the second (nonlinear) model is nonconvex, exhibits multiple local minima and therefore needs to be solved using Global Optimization techniques. We show that a spatial Branch-and-Bound type algorithm converges exactly to the optimum and that the MINLP model can be reformulated exactly to a MILP; this result is also apparent in the computational results, ranging over a set of realistic instances and a set of randomly generated ones. Finally, we extend the planning model to deal with more realistic features.

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