Optimally running a biomass-based energy production process

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Abstract

A multi-plant biomass-based energy production process is able to extract the chemical energy from various agricultural products. Such a process consists of several plants that are able to deal with biomasses of different types. Each type of plant has distinct mass-to-energy yields for each particular product type. Since the scale of the process may be geographically wide, transportation costs also have an impact on the overall profitability. Biomasses have different unit costs, and end-products (electrical energy, refined bioethanol, but also several other cross-products of the biomasses that are not necessarily energy-related) have different selling prices; hence, deciding the amount of each different biomass to process in order to maximize revenues and minimize costs is a nontrivial task. In this paper we propose a mathematical programming formulation of this problem and discuss its application to a real-world example.

Keywords: renewable energy, biomass exploitation, mathematical programming, linear programming.

1 Introduction

This paper is concerned with a mathematical programming formulation of the problem of optimally running an energy production process based on biomasses. This model was developed for practical reasons arising in the establishment of a bioenergy production process in central Italy. Specifically, the involved chemical, agricultural and engineering enterprises needed to justify the profitability of the process to banks and funding agencies. This was carried out by employing sensitivity analysis around the optimum of the mathematical program describing the process.

The production of energy from biomasses is proving more and more popular what with the energy from fossil carbon-based fuels being costly to both the environment and society [12]. Mathematical programming is one of the main planning tool in this area. [10] examines the competitiveness of biomass-based fuel for electrical energy opposed to carbon-based fuel. In [4], a mathematical programming approach is proposed to localize both energy conversion plants and biomass catchments basins in provincial area. Among the advantages of this type of energy production, there is the potential for employing wasted materials of biological origin, like used alimentary fats and oils, agricultural waste and so on. A factory producing energy with such materials would benefit from both the sales of the energy and the gains obtained by servicing waste [1]. Other mathematical models for specific biomass discrete facility location problems are developed in [6] and [3].

The biomass-based energy production process considered in this paper (see Fig. 1) involves several processing plants of different types (for example, a solid biomass plant, a squeeze plant and a fermentation-distillation plant). Some of these plants (e.g. solid biomass plant) produce energy. Others (e.g. the fermentation-distillation plant) produce intermediate products which will then be routed to other plants for further processing. There are several possible input products (e.g. agricultural products, biological waste), obtained from different sources (e.g. direct farming or acquisition on the markets) at different unit costs. Apart from the energetic output, there may be other output products which are sold in different markets (e.g. molasses obtained from the fermentation-distillation plant and sold in the feed market). The optimization problem stemming from the process is that of modelling the production process as a net gain maximization supposing the type of plants involved and the end product demands are known.

Section 2 presents the mathematical programming formulation. In Section 3 we discuss a real-world application of our model. Section 4 concerns some realistic improvements to the model. Section 5 concludes the paper.

2 Modelling the production process

Modelling a flowsheet as that presented in Fig. 1 presents many difficulties. Notice that the products can be inputs, intermediate, outputs, or both (like alcohol, which is both an output product and an intermediate product). Likewise, processes can be intermediate or final or a combination (like the fermentation-distillation plant). Consider also that the decision maker may choose to buy an intermediate product from a different source to cover demand needs, thus making the product a combination of intermediate and input. Of course the input products may be acquired or produced at different locations and at different prices. Moreover, each flow arrow has an associated transportation cost. The time horizon for the optimization process is one year.

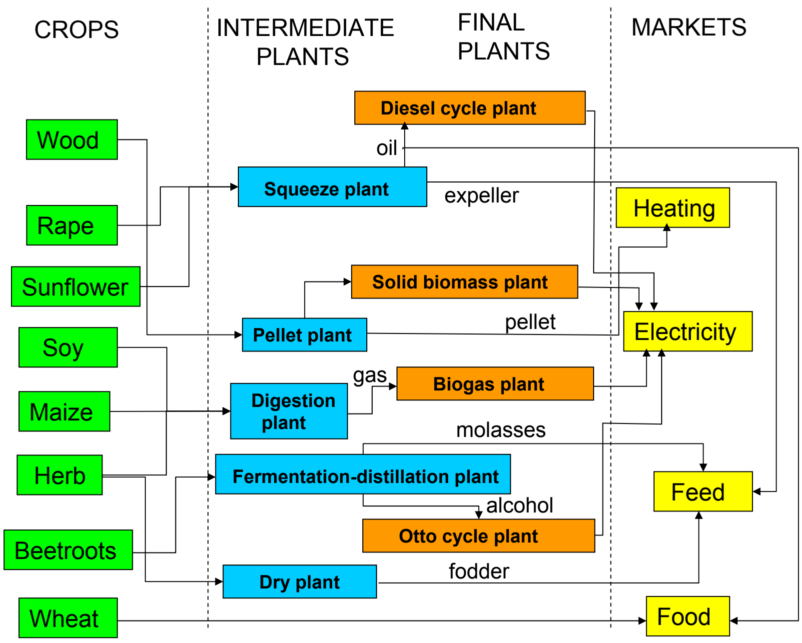

Figure 1: A typical process flowsheet.

Because essentially this problem is connected with the transportation and processing of various materials through a network, we employ a model based on multicommodity flow, which is a standard and well-understood modelling technique in Operations Research **citations: Maurizio**. The main concept in our model is the *process site*. A process site is a geographical location with at most one processing plant and/or various storage spaces for different types of goods (commodities). A place where production of a given commodity occurs is represented by a process site with a storage space. Thus, for example, a geographical location with two fields producing rapes and sunflowers is a process site with two storage spaces and no processing plant. The fermentation-distillation plant is a process site with no storage spaces and one processing plant. Each output in Fig. 1 is represented by a process site with just one storage space for each output good. In this interpretation the concepts of input, output and intermediate products, and those of intermediate and final process, lose importance: this is appropriate because, as we have emphasized earlier, these distinctions are not always well-defined. Instead, we focus the attention on the material balance and on the transformation process in each process site. Furthermore, we are able to deal with the occurrence that a given commodity may be obtained at different costs depending on whether it is bought or produced directly.

We represent the process sites by a set V of vertices of a graph $G = (V, A)$ where the set of arcs A is given by the logistical connections among the locations. To each vertex $v \in V$ we associate a set of commodities $H^-(v)$ which may enter the process site, and a set of commodities $H^+(v)$ which may leave it. Thus, for example, the squeeze plant is a process site vertex where $H^-(\text{squeeze plant}) = \{\text{rape}, \text{sunflower}\}$ and $H^+(\text{squeeze plant}) = \{\text{oil, expeller}\}$. Furthermore, we let $H = \bigcup_{v\in V}(H^-(v) \cup H^+(v))$ be the set of all commodities involved in the production process, and we partition $V = V_0 \cup V_1$ into V_0, the set of process sites with an associated processing plant, and $V_1 = V \setminus V_0$. Fig. 2 is the graph derived from the example in Fig. 1.

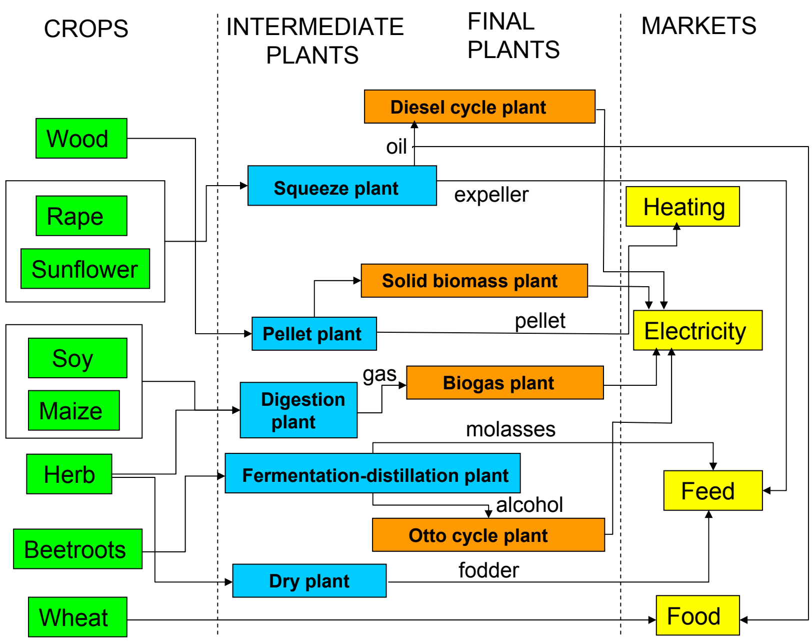

Figure 2: The graph derived from the example in Fig. 1. **maurizio, change a bit**

We assume the following to be known parameters:

- c_{vk}: cost of supplying vertex v with a unit of commodity k (negative costs are associated with output nodes, as these represent selling prices; a negative cost may also be associated to the input node "waste", since waste disposal is a service commodity);
- C_{vk}: maximum quantity of commodity k in vertex v;
- τ_{uvk}: transportation cost for a unit of commodity k on the arc (u, v);
- T_{uvk}: transportation capacity for commodity k on arc (u, v);

- λ_{vkh}: cost of processing a unit of commodity k into commodity h in vertex v;
- π_{vkh}: yield of commodity h expressed as unit percentage of commodity k in vertex v;
- d_{vk}: demand of commodity k in vertex v.

It is clear that certain parameters make sense only when associated to a particular subset of vertices, like e.g. the demands may only be applied to the vertices representing the outputs. In this case, the corresponding parameter should be set to 0 in all vertices for which it is not applicable.

The decision variables are:

- x_{vk}: quantity of commodity k in vertex v;
- y_{uvk}: quantity of commodity k on arc (u, v);
- z_{vkh}: quantity of commodity k processed into commodity h in vertex v.

Since the output demands are known *a priori*, we would like to minimize the total operation costs subject to demand satisfaction. There are three types of costs:

- cost of supplying vertices with commodities:
$$\gamma_1 = \sum_{k \in H} \sum_{v \in V} c_{vk} x_{vk};$$
- transportation costs:
$$\gamma_2 = \sum_{k \in H} \sum_{(u,v) \in A} \tau_{uvk} y_{uvk};$$
- processing costs:
$$\gamma_3 = \sum_{v \in V} \sum_{k \in H^-(v)} \sum_{h \in H^+(v)} \lambda_{vkh} z_{vkh},$$

so the objective function is

$$\min \sum_{i=1}^{3} \gamma_i(x, y, z). \tag{1}$$

We need to make sure that some material conservation equations are enforced in each process site where a plant is installed:

$$\sum_{k \in H^-(v)} \pi_{vkh} z_{vkh} = x_{vh}, \;\; \forall v \in V_0, h \in H^+(v). \tag{2}$$

Notice that these constraints do not actually enforce a conservation of mass, for in most processing plants a percentage of the input quantities goes to waste; but it is nonetheless a conservation law subject to the yield properties of the particular transformation process of the plant.

Secondly, the quantity of processed commodity must not exceed the quantity of input commodity in each vertex:

$$\sum_{h \in H^+(v)} z_{vkh} \leq x_{vk}, \;\; \forall v \in V_0, k \in H^-(v). \tag{3}$$

Furthermore, we need the quantity of input commodity in each vertex to be consistent with the quantity of commodity in the vertex itself, and similarly for output commodities:

$$\sum_{u \in V:(u,v) \in A} y_{uvk} = x_{vk}, \ \forall v \in V, k \in H^-(v) \tag{4}$$

$$\sum_{u \in V:(v,u) \in A} y_{vuh} = x_{vh}, \ \forall v \in V, h \in H^+(v). \tag{5}$$

Finally, we have the bounds on the variables:

$$d_{vk} \leq x_{vk} \leq C_{vk}, \ \forall v \in V, k \in H \tag{6}$$

$$0 \leq y_{uvk} \leq T_{uvk}, \ \forall (u,v) \in A, k \in H \tag{7}$$

$$z_{vkh} \geq 0, \ \forall v \in V, k \in H^-(v), h \in H^+(v) \tag{8}$$

and some fixed variables for irrelevant vertices:

$$x_{vk} = 0, \ \forall v \in V_1, k \in H \backslash (H^-(v) \cup H^+(v)) \tag{9}$$

$$y_{uvk} = 0, \ \forall (u,v) \in A, k \in H \backslash H^-(v), \tag{10}$$

$$y_{uvk} = 0, \ \forall (u,v) \in A, k \in H \backslash H^+(u). \tag{11}$$

The main advantage to this model is that it can be easily extended to deal with more commodities and plants in a natural way, by adding appropriate vertices or changing the relevant $H^-(v), H^+(v)$ and related parameters.

3 A real-world application

The model described in Section 2 is a Linear Programming (LP) problem, which can be solved by using one of several LP solvers. Using our model we solved an instance derived from a real world application within the "Marche Bioenergia" project (the administrative Italian region where this project took place is called "Marche"). This project consists in the study of replacement/integration of the traditional crops (beetroots, wheat) with new crops exploitable by biomass-based energy production plants, as represented in Fig. 1. The target territory consists of some 40,000 ha of land around San Severino Marche, a small village in the center of Italy. One of the aims of the "Marche Bioenergia" project was that of estimating the real value of the national economical incentive to produce electric energy from agricoltural products (so-called *green certificates*). With our model, we can do this by looking at the reduced cost attained at the optimum by the non-basic variables z_{vkh} corresponding to unused power plants.

The processing costs λ_{vkh} and the transformation yields π_{vkh} take the values summarized in Tables 1, 2, 3 and 4. In particular Table 1 lists the yields and agricultural costs of the crops: such data have been obtained in collaboration with the regional farmers association "Coldiretti" of Ancona (the main city of the Marche region). Table 2 lists the yields and transformation costs of the intermediate plants (also supplied by "Coldiretti"), whereas Table 3 lists the yields and transformation costs of the power plants supplied directly by the "Marche Bioenergia" firm: large-scale solid biomass and Otto cycle plants (10 MW each) and small-scale Diesel cycle and biogas plants (1 MW each). Finally, Table 4 lists the prices $-c_{vk}$ of the final products obtained from "Sole 24 Ore" (1st June 2006 issue), the most important financial journal in Italy. The transportation cost τ_{uvk} have been set equal to 10 euro/ton for all products and 10 euro/MWh for electric power since the territory considered is relatively small. All capacities, C_{vk} and T_{uvk}, and all demands d_{vk} are considered unbounded: the problem is bounded anyway by the total available land (40,000 ha).

We remark that most data in our model is financial and physical process related, and is thus subject to errors. However, as was mentioned in the introduction, the main application purpose of our study was

Crop	**Cost** (euro/ha)	**Yield** (ton/ha)
wood	1,000	130.00
rape	445	2.27
sunflower	697	2.25
soy	470	3.40
maize	704	6.00
herb	600	7.08
beetroots	1,360	33.70
wheat	473	4.00

Table 1: The agricultural costs and yields of considered crops.

Plant	**Cost** (euro/ton)	**Output**	**Yield**
squeeze	18.00	oil	35%
		expeller	65%
pellet	70.00	pellet	95%
digestion	10.00	biogas	0.38%
fermentation-distillation	5	alcohol	20%
		molasses	80%
dry	7.30	fodder	75%

Table 2: The processing costs and the yields of the intermediate plants.

Plant	**Cost** (euro/ton)	**Yield** (MWh/ton)
Diesel cycle	23.00	4.25%
Solid biomass	10.00	1.07%
Biogas	50.00	1.00%
Otto cycle	8.70	2.87%

Table 3: The processing costs and the yields of the final power plants.

Product	**Market**	**Price** (euro/ton)
pellet	heating	150
electrical power	electricity	150 (euro/MWh)
molasses	feed	100
fodder	feed	115
rape oil	food	550
rape expeller	food	150
sunflower oil	food	650
sunflower expeller	food	125
wheat	food	135

Table 4: The prices of the final products sold in different markets.

to justify profitability of the enterprise to banks and funding agencies. It turned out that in practice an approximated cost estimate was enough to attain this purpose, even without considering randomness of data. On the other hand, obtaining robust solutions of LPs subject to uncertain data reduces to solving another LP [2], so it is computationally as tractable as solving the original one.

We solved (in a few seconds **Maurizio: find seconds**) the LP model described in Section 2 to optimality on the instance presented here using the AMPL [5] modelling language and the CPLEX [7] solver. The obtained solution is shown graphically in Fig. 3.

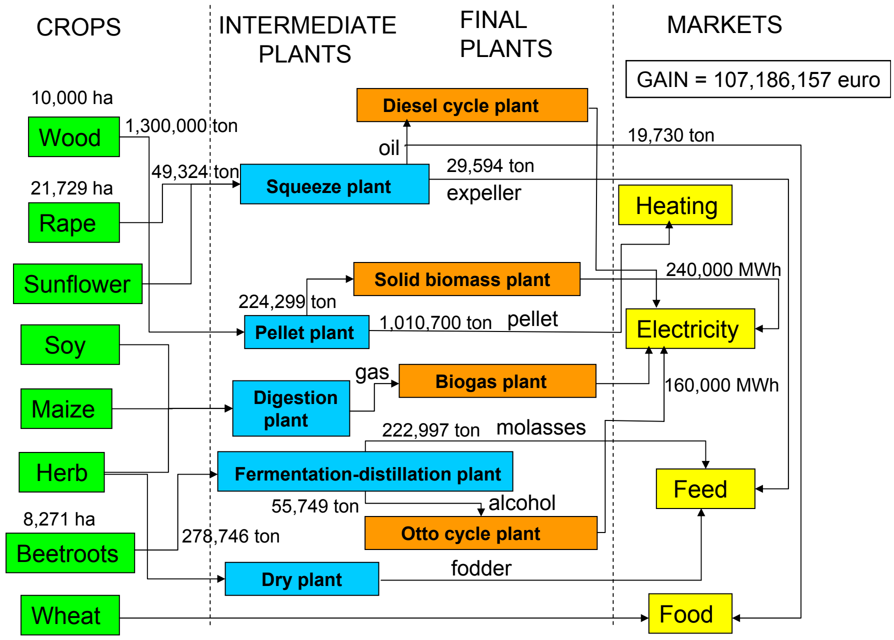

Figure 3: The optimal solution obtained with the LP model.

In the proposed optimal solution, about half of the agricultural resources (21.728,60 ha of land) is devoted to traditional market (rapes for food and feed markets). Slightly less than half of the total land is used to grow wood and beetroots for supplying the solid biomass and the Otto cycle plants, respectively. No other biomass-based energy plant is profitable: from post-optimality sensitivity analysis we infer that in order to produce electricity with a biogas plant, production costs should decrease by 317 euro/MWh (reduced cost of variable z_{vkh}, where v =biogas production plant, k =gas, h =electricity); and in order to produce electricity with a Diesel cycle plant, production costs should decrease by 153 euro/MWh. Finally, comparing the our optimal solution and the solution associated to current traditional agricultural production (represented in Fig. 4), we notice that exploitation of crops providing biomass for power production more than doubles the total gain (from about 48 million of euro to about 107 million of euro).

4 Model improvements

The model of Sect. 2 relies on some simplifications of real-world conditions. A more realistic model can be obtained as follows.

- Some of the plants considered in this paper produce electricity. These have very specific properties and behaviours [8, 9], among which:
  1. in a true market situation (i.e. no subsidization), electricity prices vary during the course of a single day, as demand rises and subsides;
  2. some electricity production plants are often designed to produce electricity *and* heat (which is either stored or conveyed directly into buildings in the area) — such plants are called Combined Heat Power (CHP);

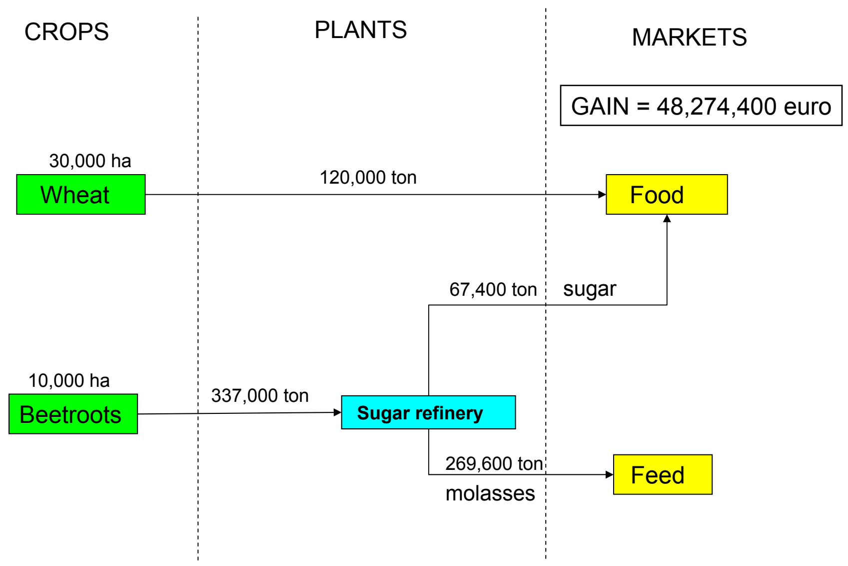

Figure 4: The traditional agricultural production.

3. CHPs generate heat and electricity at the same hour and same location.

- Transportation costs do not depend linearly on the distances due to the different means of transportation used [11]. For very short transportation distances, tractors may be used, which have higher transportation cost than lorries, used for medium to long distances; for very long transportation distances, trains or ships are used. More generally, the geography of the production process region deeply influences the costs of the single process activities; [11] suggests a methodology that combines Geographical Information Systems (GIS) software with process analysis to estimate these costs.

It turns out that the model in Sect. 2 can be extended to accommodate all of the above features. As regards the variability of energy prices during the course of a day and of the year, this can be dealt with in two ways: by employing storage space, or by explicitly adding a time dependency in the model. The former involves adding biomass storage space near the electricity plants (the energyPRO model [8] proposes an electricity plant planning methodology that locally optimizes each plant over a yearly time horizon with hourly time-steps). For the latter, consider the time set $T = \{1, \ldots, 8760\}$ of hours in a 365-day year. We reindex the cost parameters c_{vk} to c^t_{vk} for all $v \in V, k \in H, t \in T$, the decision variables x, y, z to x^t_{vk}, y^t_{uvk}, z^t_{vkh} for all appropriate $u, v \in V$, $k, h \in H$, $t \in T$. We rewrite $\gamma_1, \gamma_2, \gamma_3$ (terms of the objective function) as follows:

$$\begin{aligned}
\gamma_1 &= \sum_{t\in T}\sum_{k\in H}\sum_{v\in V} c^t_{vk} x^t_{vk} \\
\gamma_2 &= \sum_{t\in T}\sum_{k\in H}\sum_{(u,v)\in A} \tau_{uvk} y^t_{uvk} \\
\gamma_3 &= \sum_{t\in T}\sum_{v\in V}\sum_{k\in H^-(v)}\sum_{h\in H^+(v)} \lambda_{vkh} z^t_{vkh}.
\end{aligned}$$

All constraints (3)-(11) are changed accordingly (all occurrences of decision variables x, y, z and parameters c gain an index t and a quantifier $\forall t \in T$ is added to each constraint). This provides a model that can be exactly decomposed in $|T|$ separate LPs as that of Sect. 2, i.e. one for each hour $t \in T$. Although it may be possible to solve this problem every hour, it would not be reasonable to expect to change input or transported quantities every hour. Thus, we "connect" the decomposed LPs by means of equality constraints on input and transported quantities. We assume that for each vertex $v \in V$ and commodity $k \in K$ the input quantity x^t_{vk} can be changed every χ_{vk} hours, and that transporting commodity k on the arc (u,v) takes ξ_{uvk} hours (for simplicity we suppose that χ_{vk}, ξ_{uvk} are divisors of $|T|$). We then have:

$$\forall v \in V, k \in H, i \leq \frac{|T|}{\chi_{vk}}, t_1 < t_2 \in \left\{ \frac{|T|}{\chi_{vk}}(i-1), \ldots, \frac{|T|}{\chi_{vk}} i \right\} \quad x^{t_1}_{vk} = x^{t_2}_{vk} \tag{12}$$

$$\forall (u,v) \in A, k \in H, i \leq \frac{|T|}{\xi_{vk}}, t_1 < t_2 \in \left\{ \frac{|T|}{\xi_{vk}}(i-1), \ldots, \frac{|T|}{\xi_{vk}} i \right\} \quad y^{t_1}_{uvk} = y^{t_2}_{uvk}. \tag{13}$$

Constraints (12)-(13) simply state that input and transported quantities may only change at predetermined times. The solution of such a large-scale LP is not practically unreasonable using commercial-strength LP solvers such as CPLEX [7].

The combined production of electricity and heat (point 2 in the list above) can be dealt by our model by simply introducing an output process site representing heat, and adapting the λ and π parameters relative to the CHP, various inputs and heat output to reflect the situation. As a consequence of point 3 in the list above, this modelling is not wholly satisfactory, as generation of heat is time-dependent because it is linked to the generation of electricity: but again this time dependency can be dealt with by using process sites representing heat storage capacity or simply adding to the plant cost parameter.

Nonlinear transportation costs are already fully dealt with by our model, for with each arc we associate a transportation cost which is not unitary but rather depends on the vertices adjacent to the arc. Since the arc length is not used anywhere in the model, each arc can be assigned its proper cost.

5 Conclusion

In this paper we described a Linear Programming (LP) model for running a biomass-based energy production process, with a real-world application. Our model makes it possible to double the profit associated to traditional agricultural production. The financial benefit was so large that "Marche Bioenergia" was able to self-finance the project without having to seek economical incentives.

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