Brief announcement: On the impossibility of detecting concurrency

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Concurrent specifications

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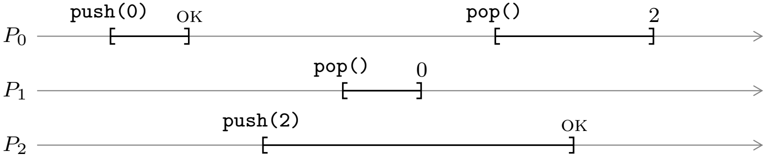

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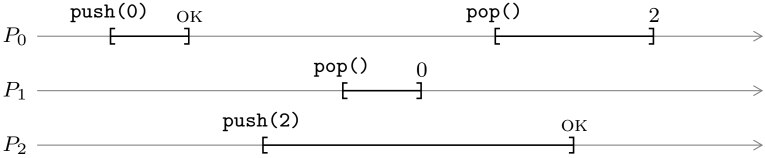

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- A *concurrent specification* is a subset $\sigma \subseteq \mathcal{T}$.

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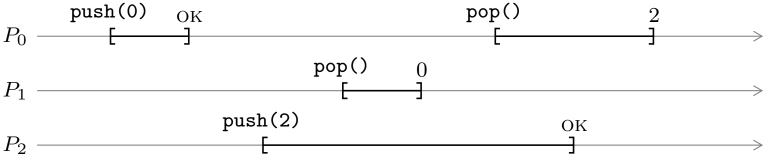

Write $\mathcal{T}$ for the set of all execution traces.

- A *concurrent specification* is a subset $\sigma \subseteq \mathcal{T}$.
- A program implements a specification σ if all the traces that it can produce belong to σ.

Linearizability (Herlihy & Wing, 1990)

$$\text{SeqSpec} \xrightarrow{\text{Lin}} \text{ConcSpec}$$

- **Input:** a sequential specification σ (e.g. list, queue, ...).
- **Output:** a concurrent specification $\text{Lin}(\sigma)$.

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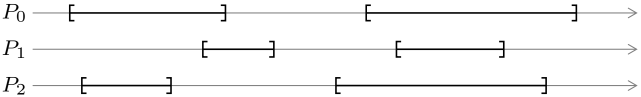

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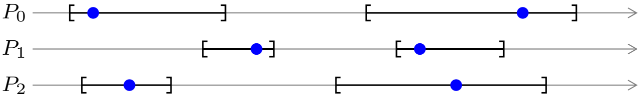

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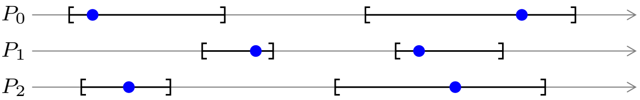

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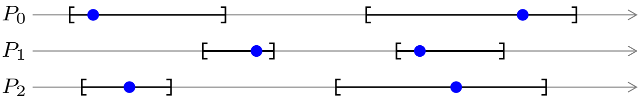

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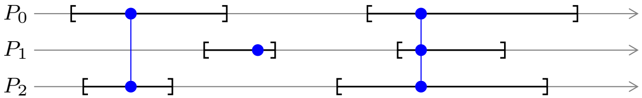
Some objects are not linearizable!
Their specification cannot be expressed as $\text{Lin}(\sigma)$, for any σ.

Concurrent variants of linearizability

Set-linearizability (Neiger)



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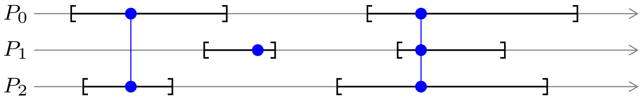

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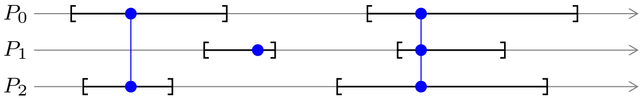

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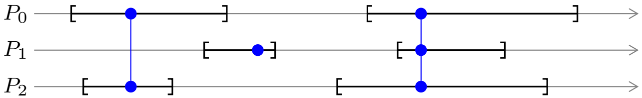

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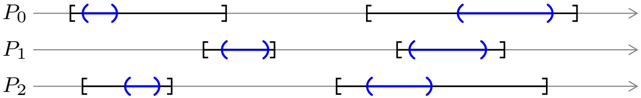

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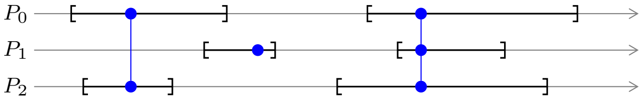

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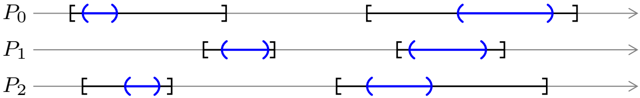

- Can specify every task!

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Concurrent specifications

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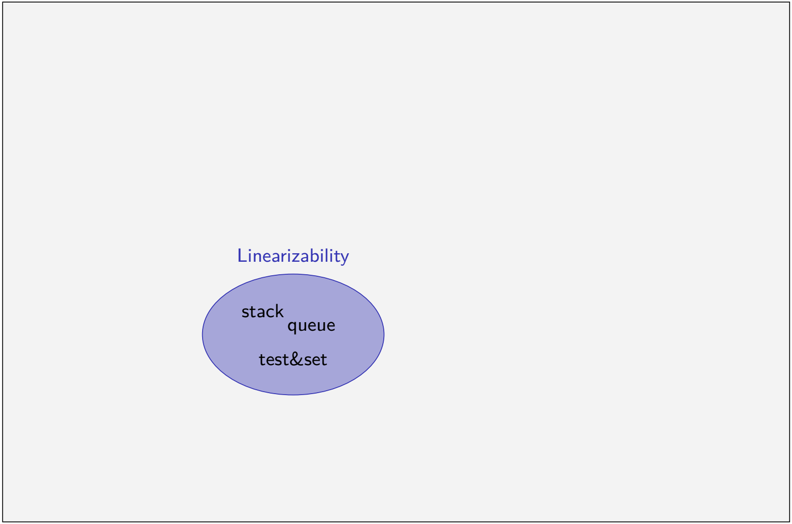

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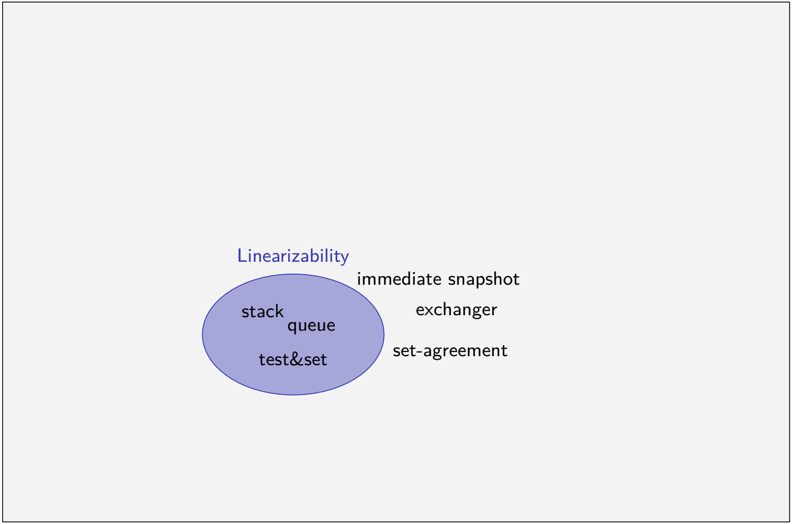

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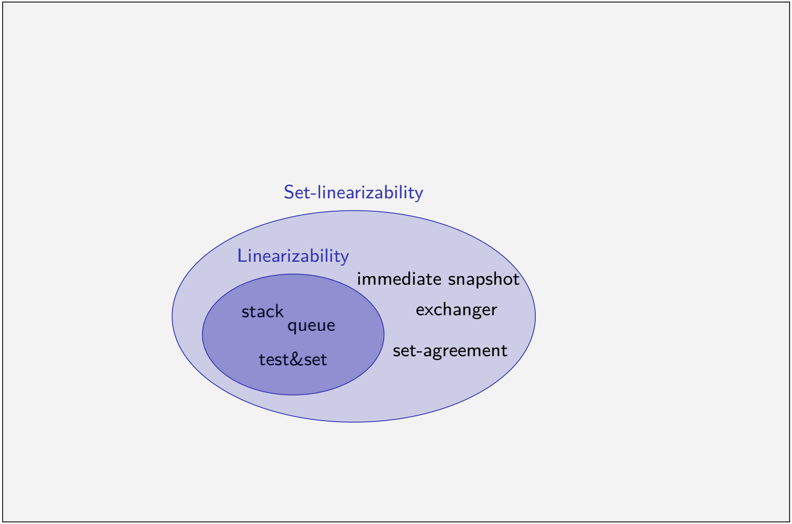

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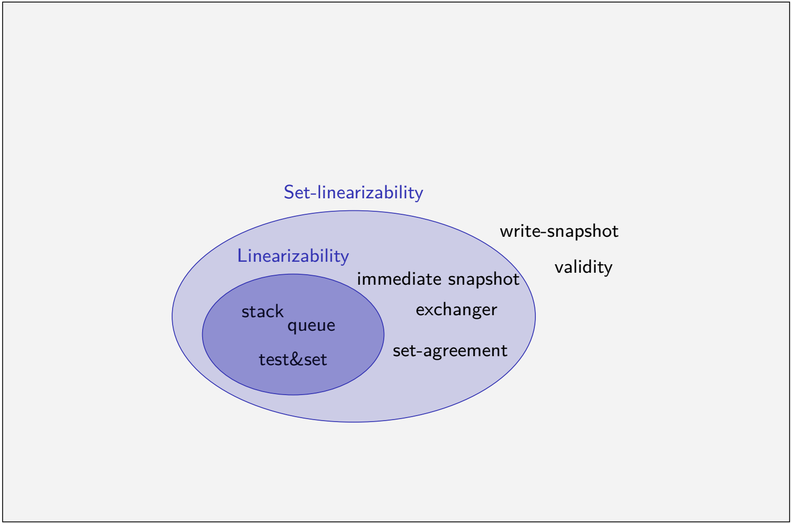

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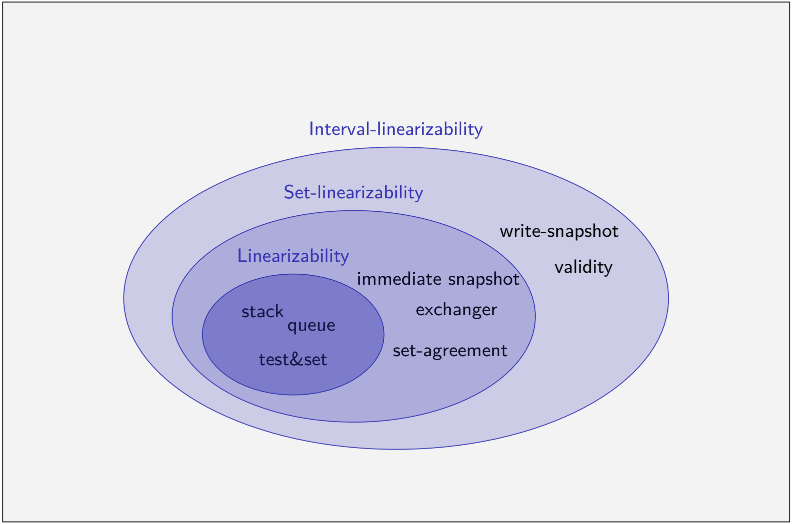

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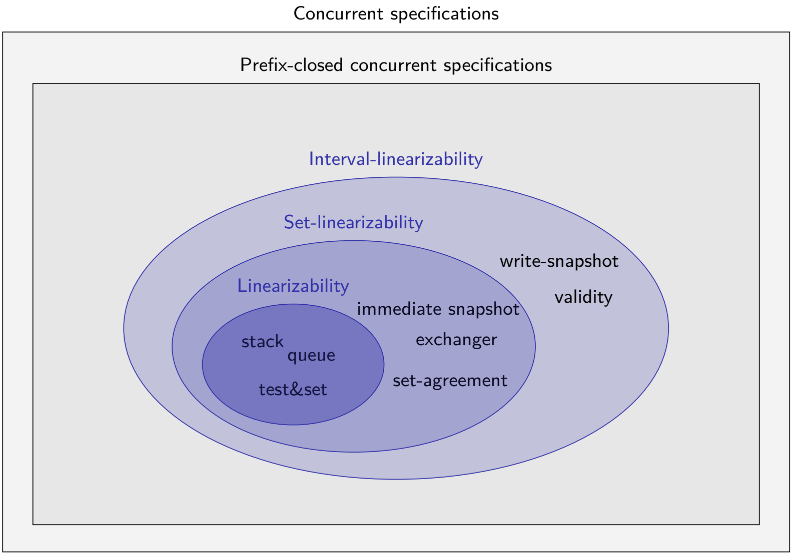
Concurrent specifications
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Interval-linearizability
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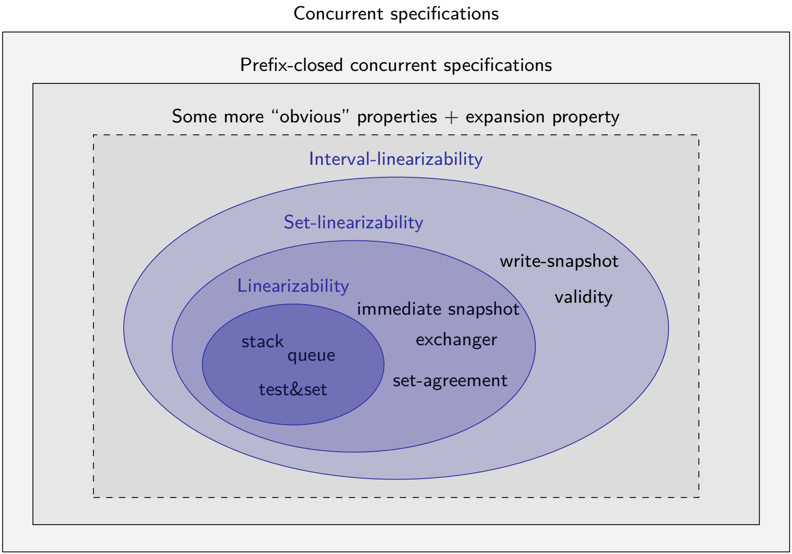
Concurrent specifications
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Some more “obvious” properties + expansion property
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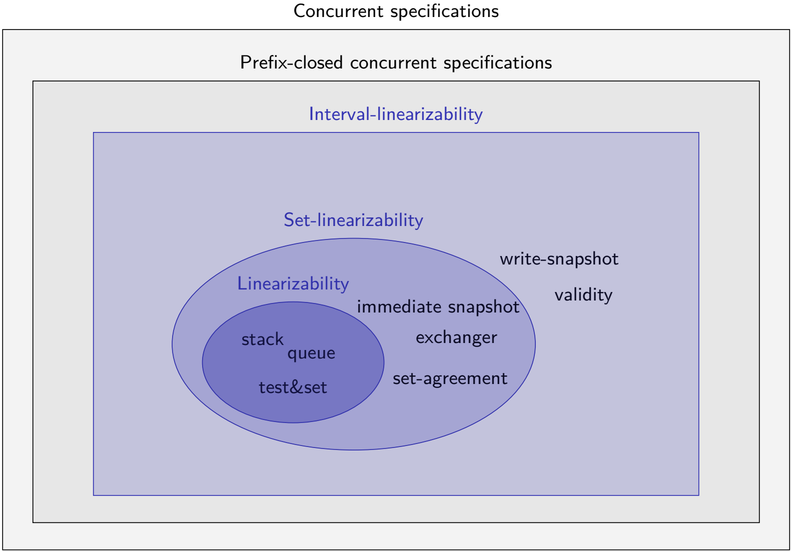

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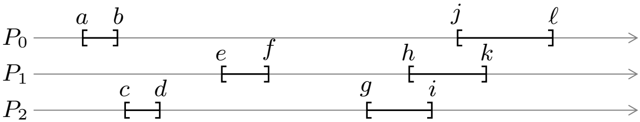

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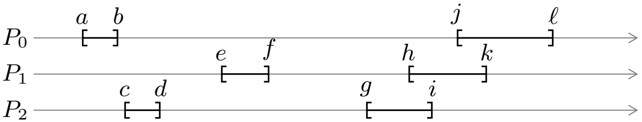
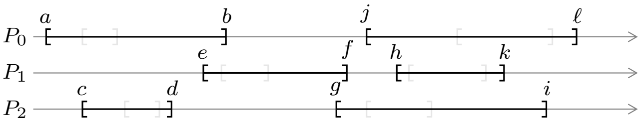


if we *expand* the intervals,



then the resulting trace is still correct.

Example: the Exchanger object

Similar to the one available in Java[1]: *"A synchronization point at which threads can pair and swap elements within pairs".*
Here, we consider a wait-free variant.

[1] `java.util.concurrent.Exchanger<V>`

Example: the Exchanger object

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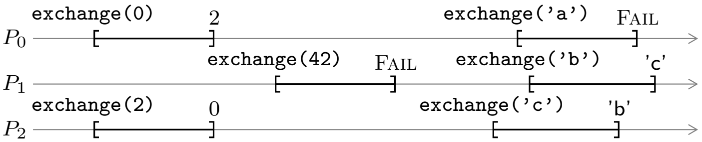
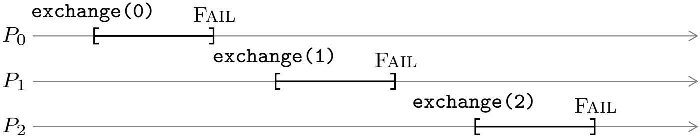
A typical execution of the exchanger looks like this:



[1] java.util.concurrent.Exchanger<V>

Example: the Exchanger object

The following execution is correct:



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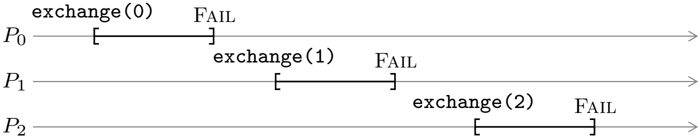
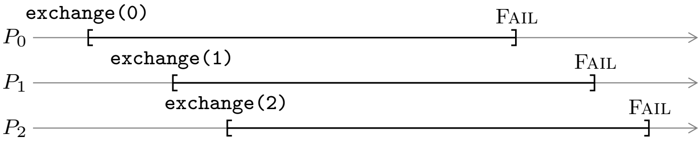


Hence, according to the expansion property,



should be considered correct too!

Results

- In a reasonable computational model:

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The semantics $[\![P]\!]$ of any program P has the expansion property.

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- In a reasonable computational model:

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The semantics $[\![P]\!]$ of any program P has the expansion property.

- Linearizability-based techniques can only produce specifications which satisfy the expansion property.

Theorem

For every sequential specification σ, Lin(σ) *has the expansion property.*

We write ConcSpec for the set of concurrent specifications satisfying the expansion property (and prefix-closure, etc).

Results

$$\begin{tikzcd}[column sep=huge] \mathsf{SeqSpec} \arrow[rr, bend right, "\mathsf{Lin}"'] \arrow[rr, phantom, "\top"] & & \mathsf{ConcSpec} \arrow[ll, bend right, "\mathsf{U}"'] \end{tikzcd}$$

Theorem

The maps Lin *and* U *form a Galois connection: for every* $\sigma \in \mathsf{SeqSpec}$ *and* $\tau \in \mathsf{ConcSpec}$,

$$\mathsf{Lin}(\sigma) \subseteq \tau \qquad \iff \qquad \sigma \subseteq \mathsf{U}(\tau).$$

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Applications:

- By the properties of Galois connections,

$$\mathsf{Lin}(\mathsf{U}(\mathsf{Lin}(\sigma))) = \mathsf{Lin}(\sigma)$$

  This yields a simple criterion to check whether a given specification τ is linearizable: check whether $\mathsf{Lin}(\mathsf{U}(\tau)) = \tau$.

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- The Galois connection for interval linearizability has the following corollary:

Theorem

ConcSpec *is the set of interval-linearizable specifications.*

Thanks!