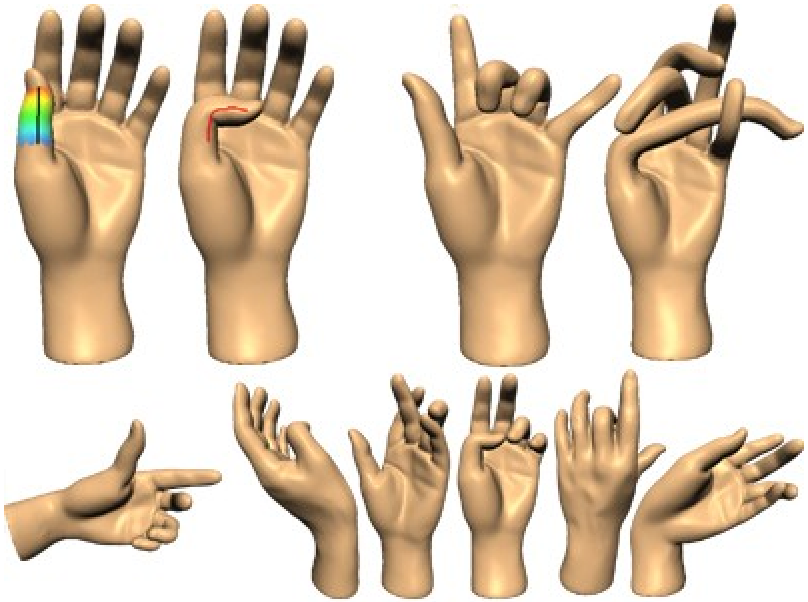
INF555



Fundamentals of 3D

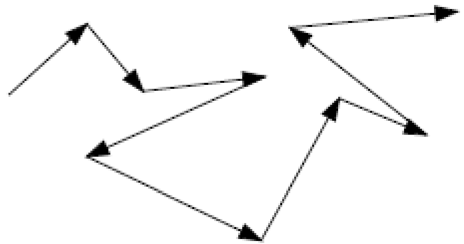
Lecture 8: Introduction to meshes

Les maillages

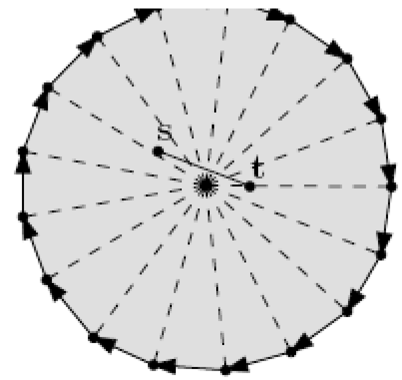
Frank Nielsen
nielsen@lix.polytechnique.fr

23 Novembre 2011

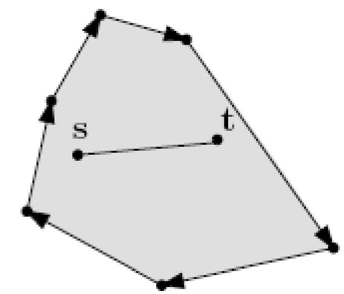
Polygons



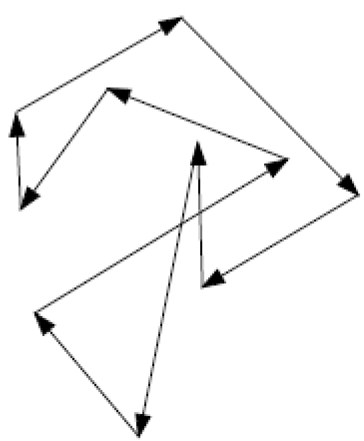
Polygonal chain



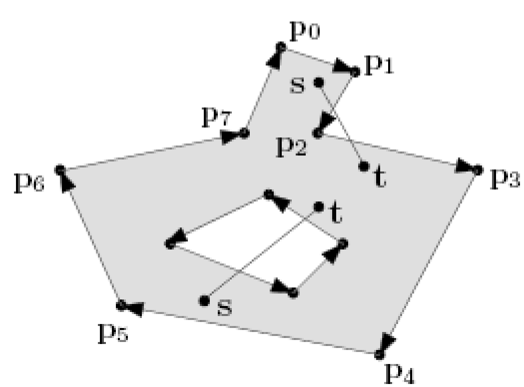
Regular (convex) polygon



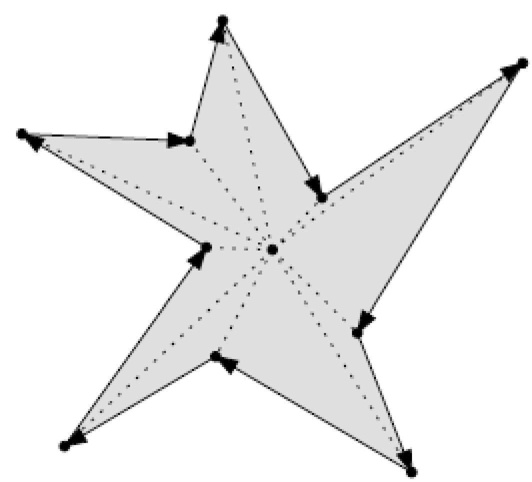
Convex polygon



Nonsimple polygon

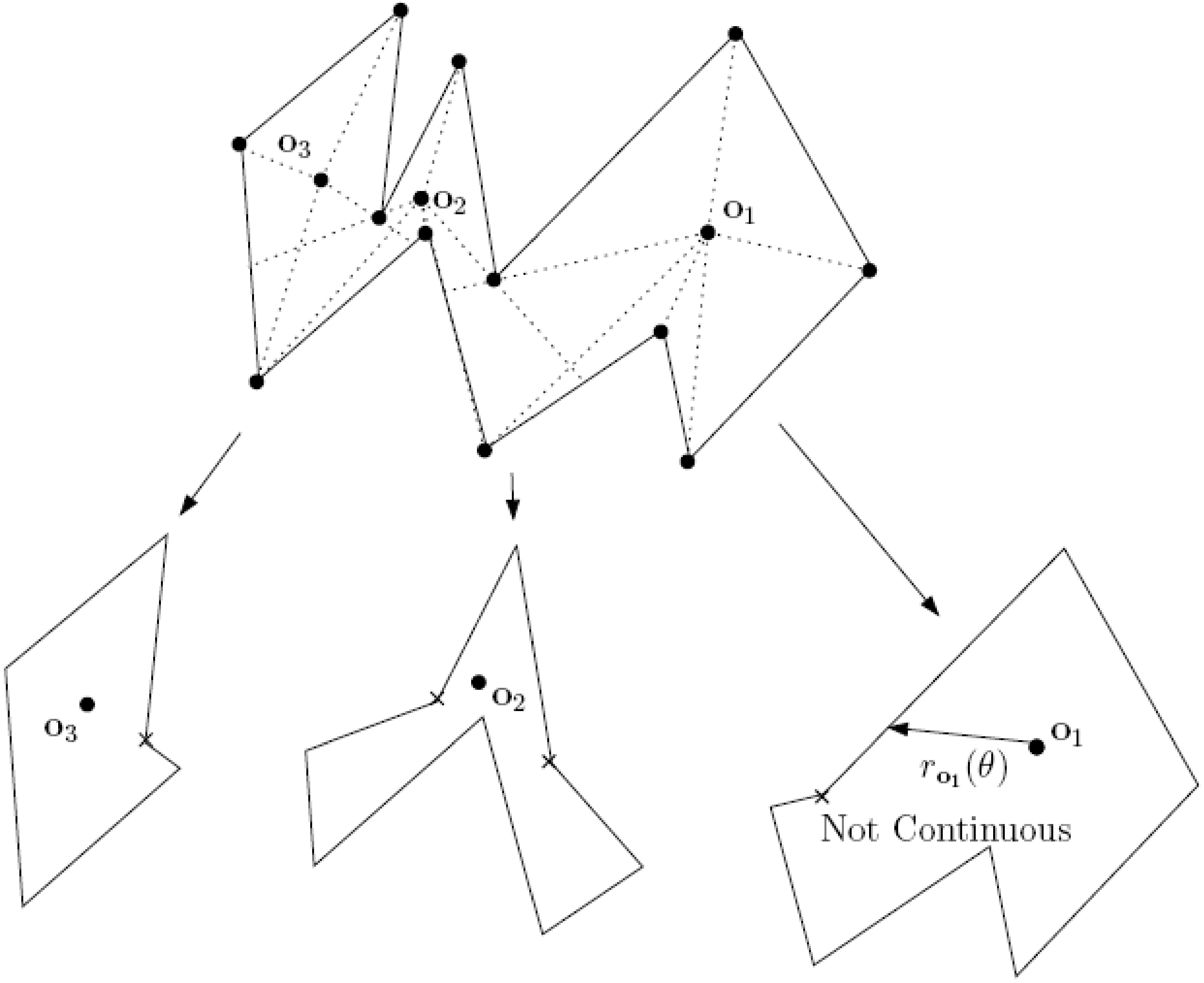


Nonconvex polygon with hole

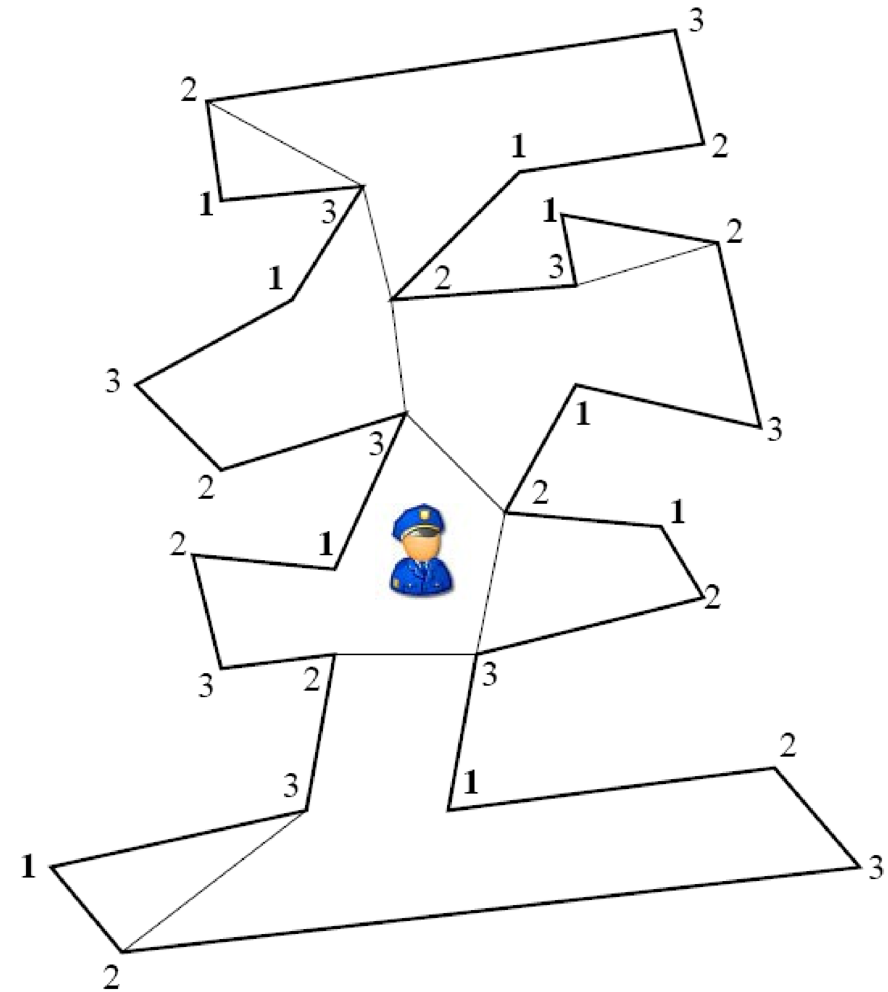


Star-shaped polygon

Polygons: Star-shaped decomposition

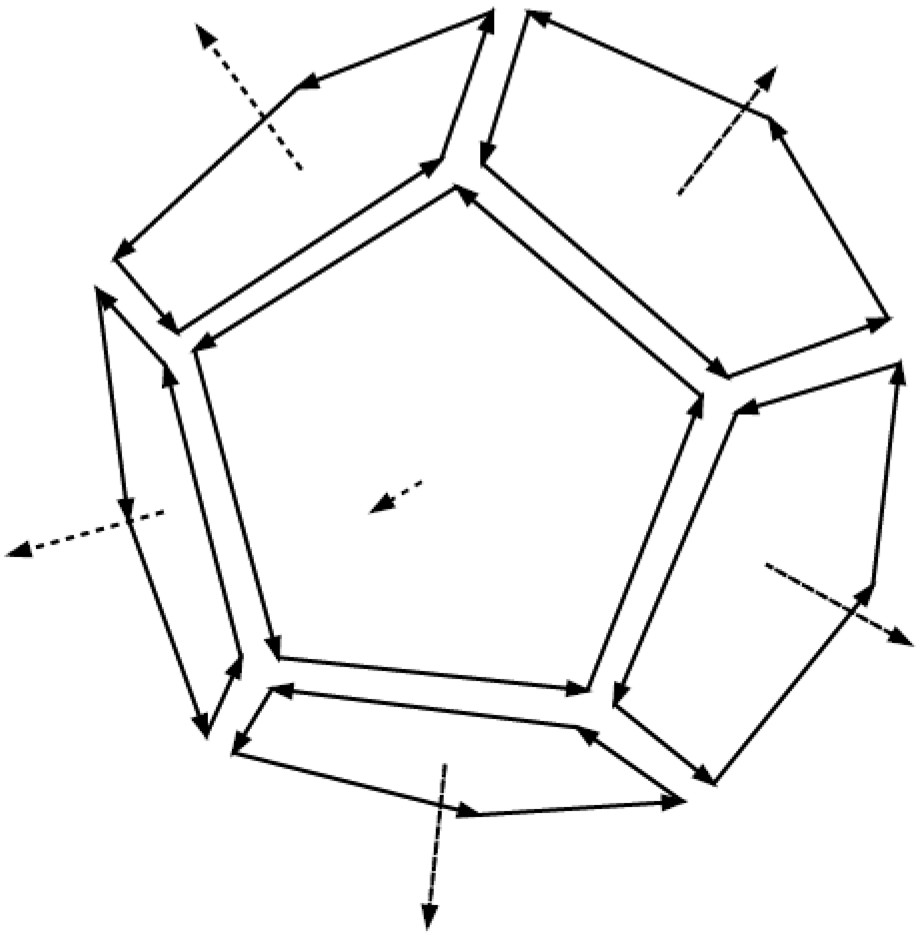


Polygons: Star-shaped decomposition



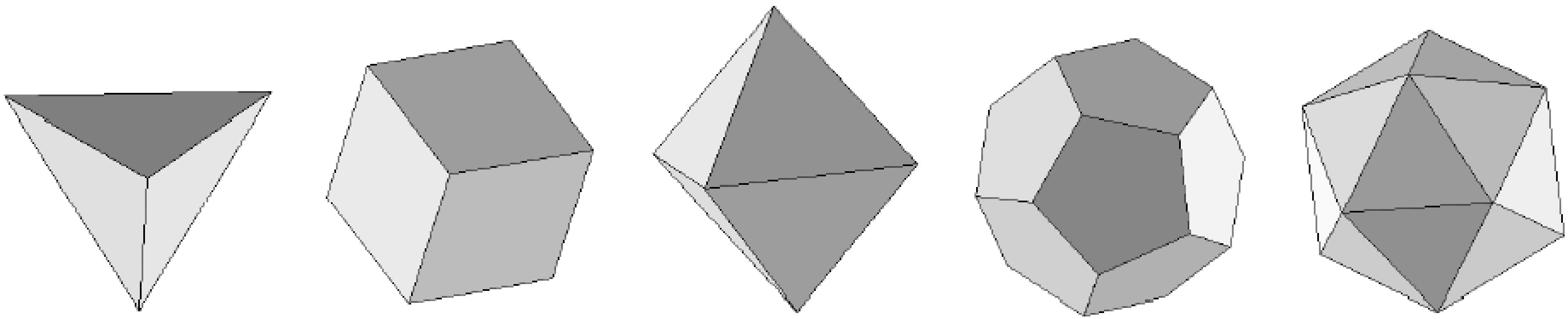
Art gallery, illumination problems, robots'race, etc.
Place guards...

3D : Orienting the fact edges for *outer normals*



Polyhedron, convex polyhedra

Platonic solids: 5 convex polyhedra

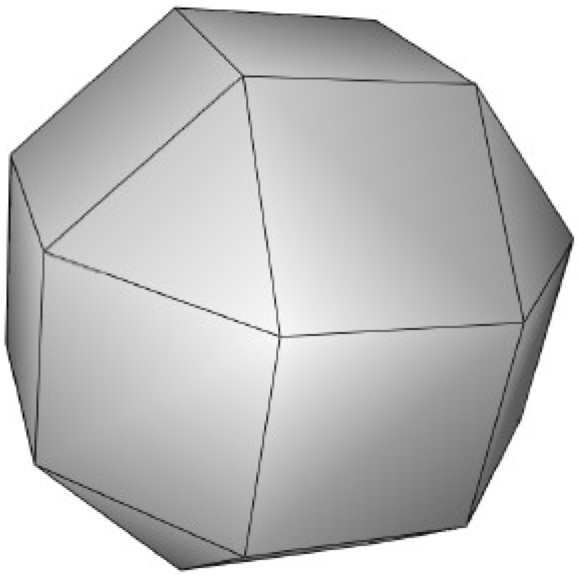


(a) (b) (c) (d) (e)

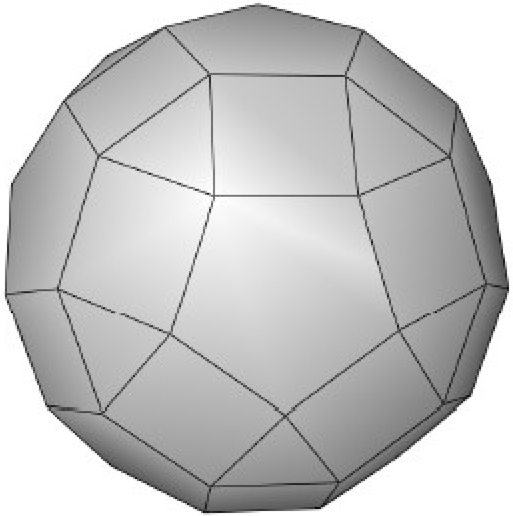
Platonic solid	Schläfli symbol	# Vertices	# Faces	# Edges
Tetrahedron (a)	$(3,3)$	4	4	6
Hexahedron (b)	$(4,3)$	8	6	12
Octahedron (c)	$(3,4)$	6	8	12
Dodecahedron (d)	$(5,3)$	20	12	30
Icosahedron (e)	$(3,5)$	12	20	30

Identical faces
(group of symmetry)

Uniform polyhedra



rhombicuboctahedron



rhombicosidodecahedron

- faces=regular polygons (not necessarily the same),
- <u>isometry mapping of its vertices</u> (=symmetry)

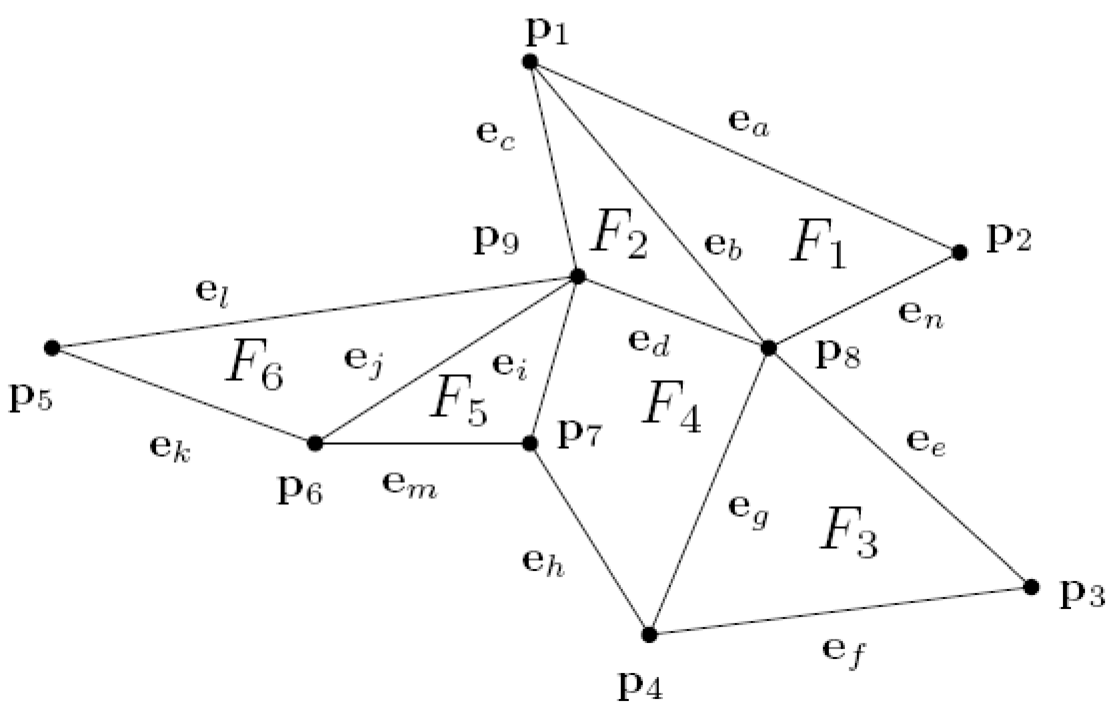
75 uniform finite polyhedra/ if you like origami

Science Museum, London

Meshes: Notations

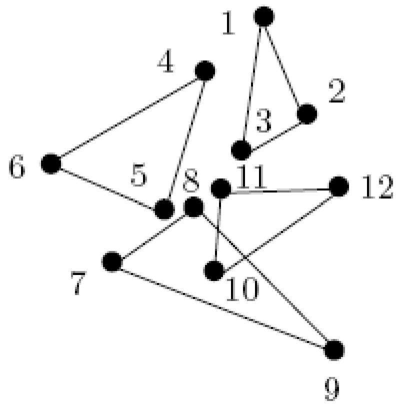
- vertices
- edges
- faces

$$\mathcal{M} \quad \begin{array}{ll} |\mathcal{V}(\mathcal{M})| = 9 & \mathcal{V}(\mathcal{M}) = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7, \mathbf{p}_8, \mathbf{p}_9\} \\ |\mathcal{E}(\mathcal{M})| = 14 & \mathcal{E}(\mathcal{M}) = \{\mathbf{e}_a, \mathbf{e}_b, \mathbf{e}_c, \mathbf{e}_d, \mathbf{e}_e, \mathbf{e}_f, \mathbf{e}_g, \mathbf{e}_h, \mathbf{e}_i, \mathbf{e}_j, \mathbf{e}_k, \mathbf{e}_l, \mathbf{e}_m, \mathbf{e}_n\} \\ |\mathcal{F}(\mathcal{M})| = 6 & \mathcal{F}(\mathcal{M}) = \{F_1, F_2, F_3, F_4, F_5, F_6\} \end{array}$$

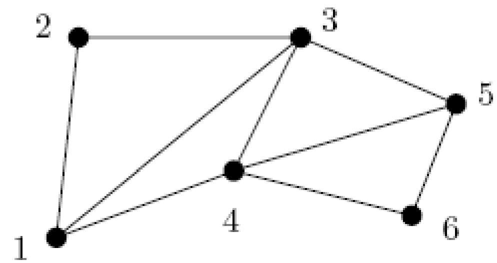


$$E(\mathbf{p}_8) = \{\mathbf{e}_b, \mathbf{e}_d.\mathbf{e}_g, \mathbf{e}_e, \mathbf{e}_n\} \qquad N(\mathbf{p}_8) = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_9\}$$

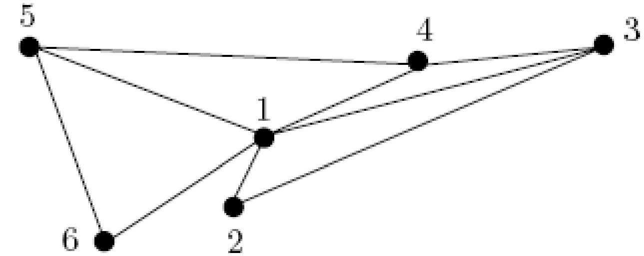
Meshes: Connectivity (= structuring)



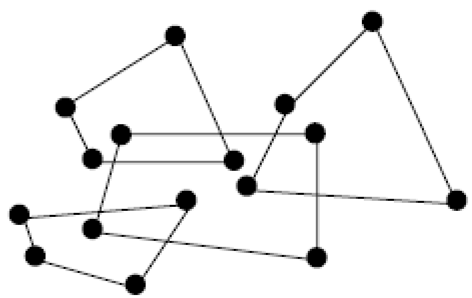
Triangle Soup



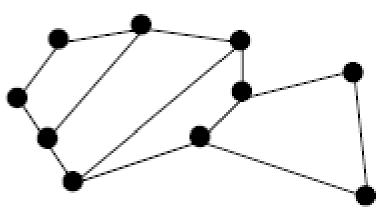
Triangle Strip



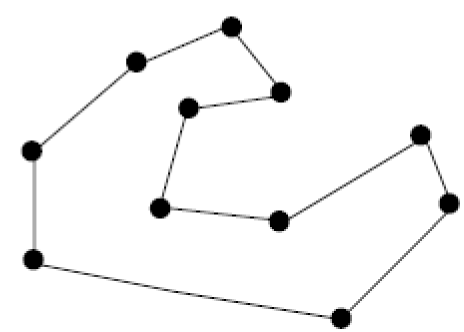
Triangle Fan (Umbrella)



Quad Soup

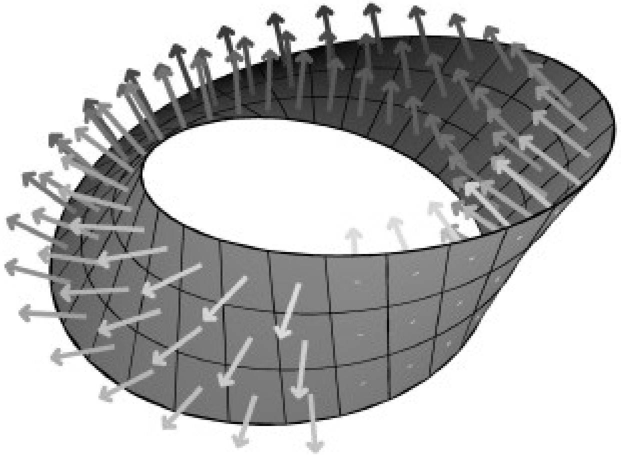


Quad Strip

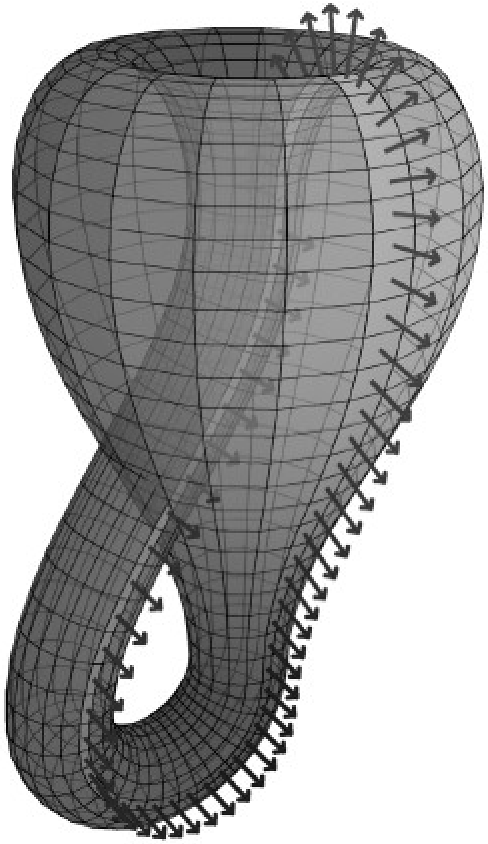


Polygon

Meshes: Non-orientable surfaces

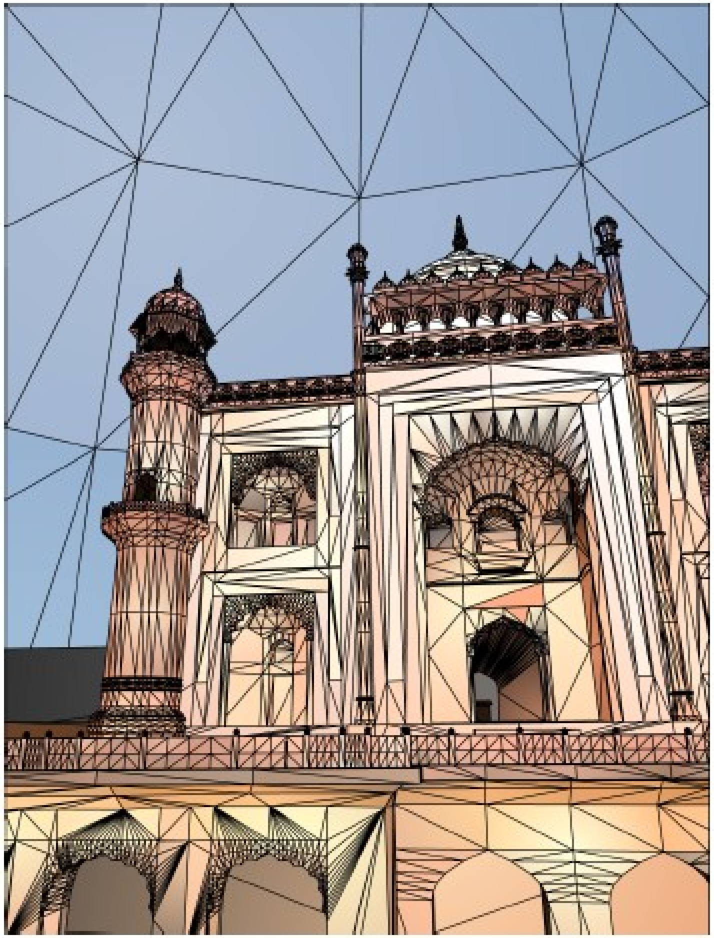


Möbius band

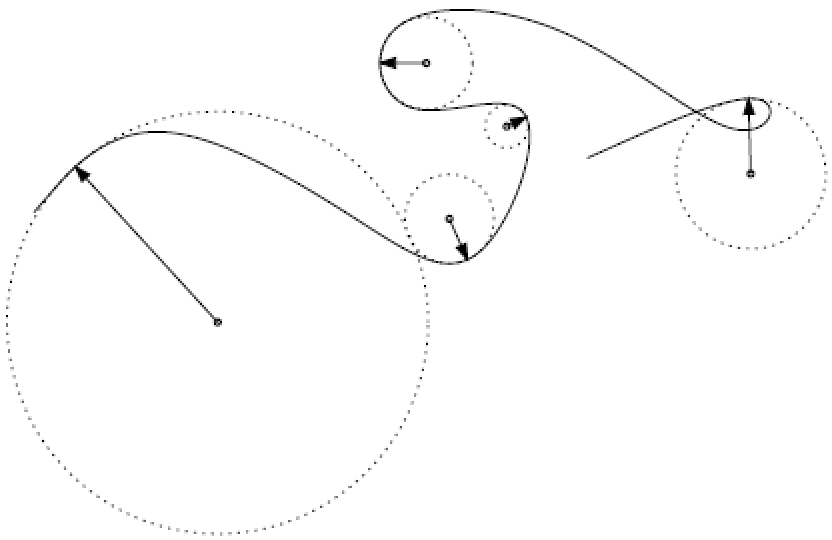


Klein bottle

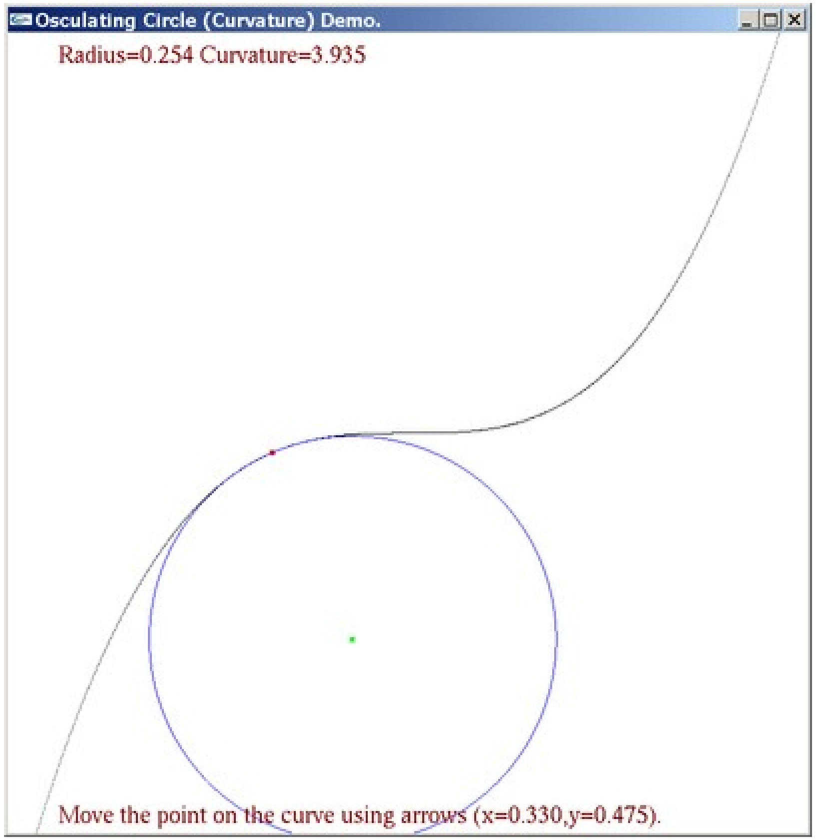
Textured meshes: Realistic computer graphics



Osculating circles and curvature



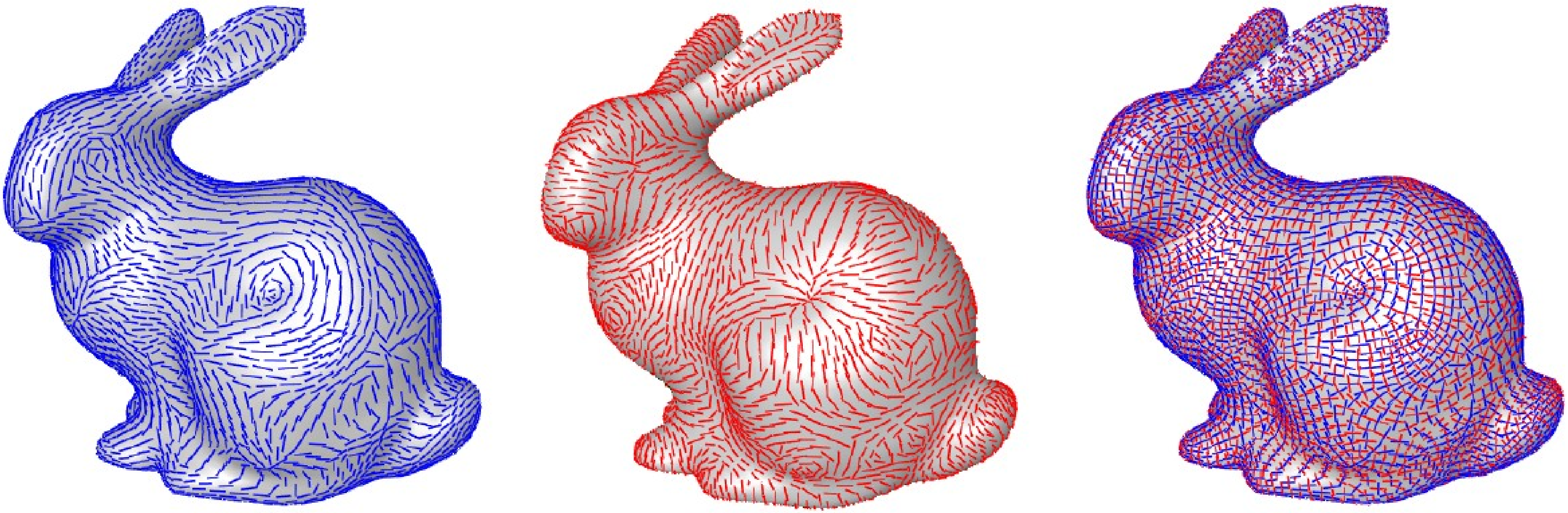
$$\rho(\mathbf{p}) = \frac{1}{R(\mathbf{p})}$$



Curvature is the inverse of the radius of the osculating circle.

Mesh: Sectional curvatures and principal directions

Directions are perpendicular to each other



Minimum curvature   Maximum curvature

sectional curvatures.

Intersection of a surface *S* with a plane containing point **p** and its normal:
2D curve that can be analyzed using the osculating circles.

Mesh: Gaussian and mean curvatures

Gaussian curvature:

$$\rho_G = \rho_{\max} \times \rho_{\min}$$

Mean curvature:

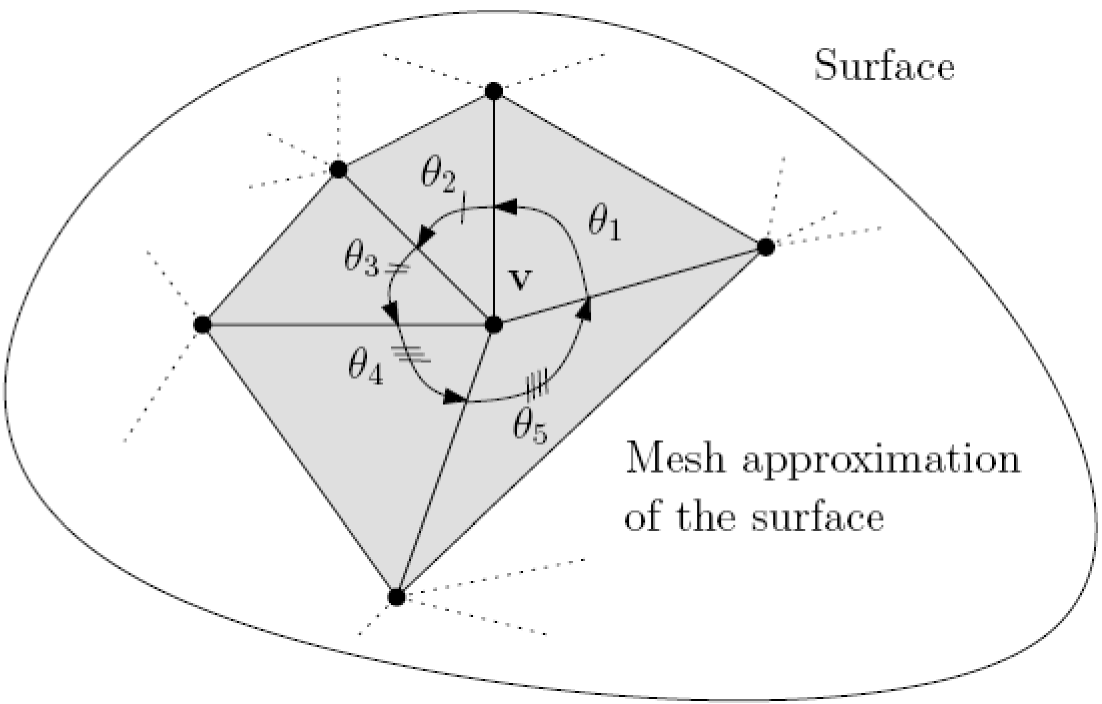
$$\frac{\rho_{\max}+\rho_{\min}}{2}$$

→ In Riemannian geometry, many ways to define curvatures (and torsions)

Mesh: Integral Gaussian curvature/angle excess (Deviation from flatness)

$$\int\int_{A\in T(\mathbf{v})} \rho_G(A)dA \simeq -\theta(\mathbf{v}).$$

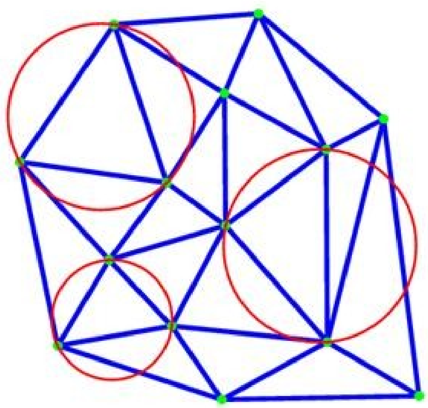
$$\rho(\mathbf{v}) = 2\pi - \sum_{i=1}^{|T(\mathbf{v})|} \theta_i.$$



Mesh: Ingredients of topology

Euler's formula is a topological invariant:

$$\#\text{Vertices} - \#\text{Edges} + \#\text{Faces} = 2.$$



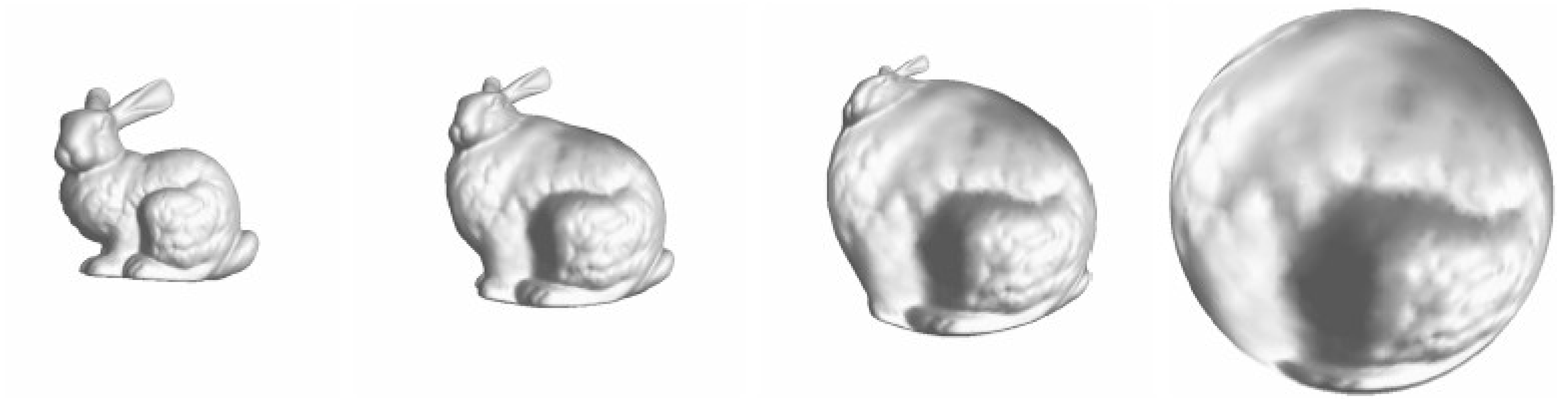
Closed triangulated manifold:

$$\#\text{Vertices} \leq \frac{2}{3}\#\text{Edges}, \qquad \text{and} \qquad \#\text{Vertices} \leq 2\#\text{Faces} - 4.$$

$$\#\text{Edges} \leq 3\#\text{Vertices} - 6, \qquad \text{and} \qquad \#\text{Edges} \leq 3\#\text{Faces} - 6.$$

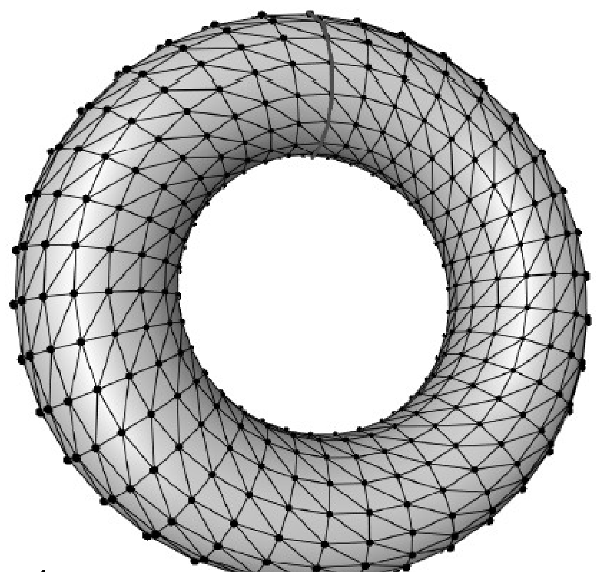
$$\#\text{Faces} \leq \frac{2}{3}\#\text{Edges}, \qquad \text{and} \qquad \#\text{Faces} \leq 2\#\text{Vertices} - 4.$$

Mesh topology: Genus, polyhedra with holes (topology=global property)

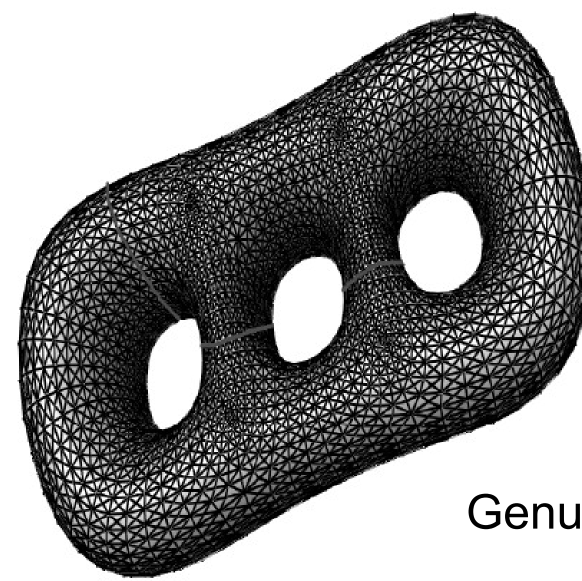


Genus 0

$$\#\text{Vertices} - \#\text{Edges} + \#\text{Faces} = 2 - 2\#\text{Genus}$$

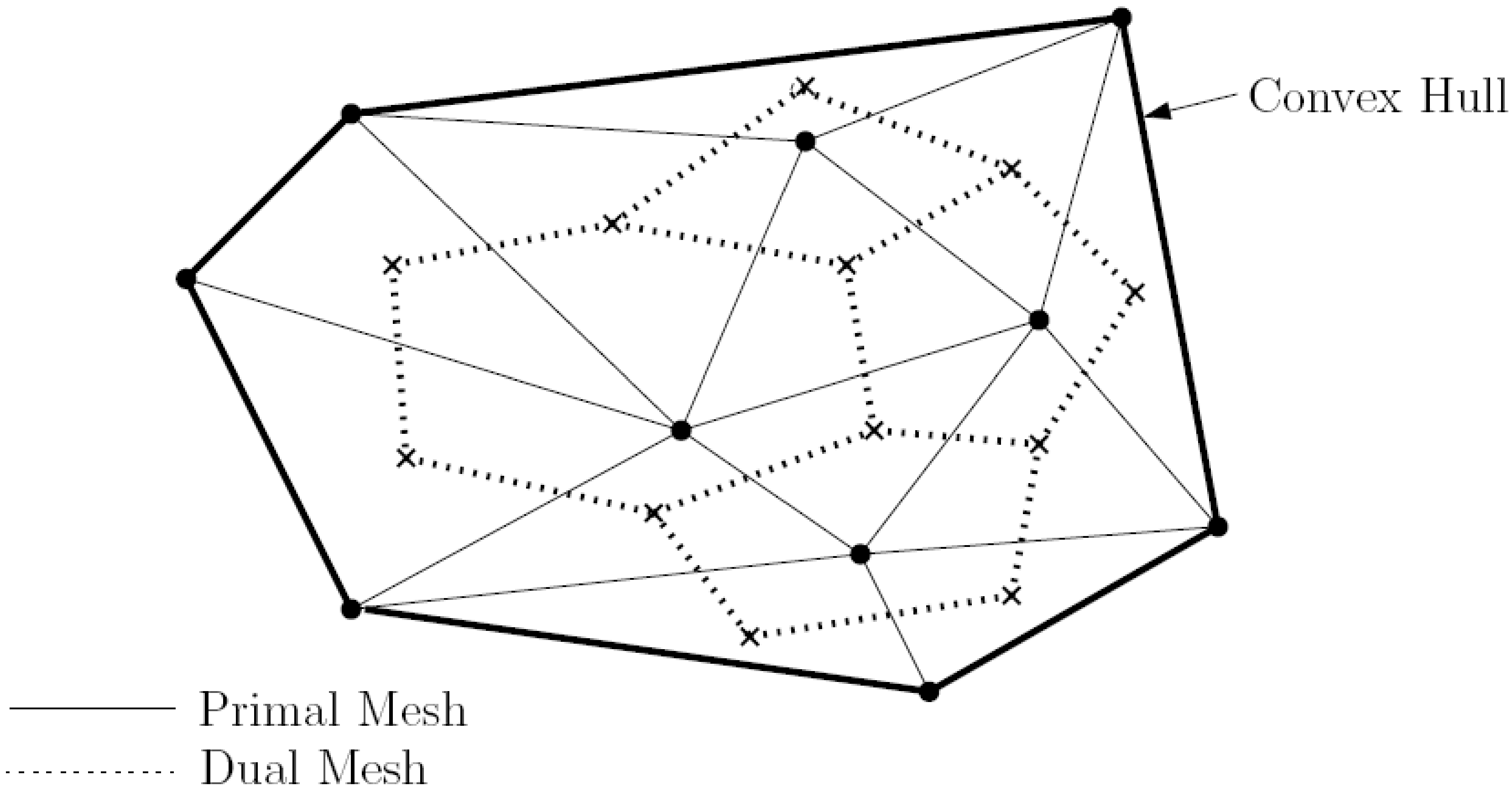


Genus1

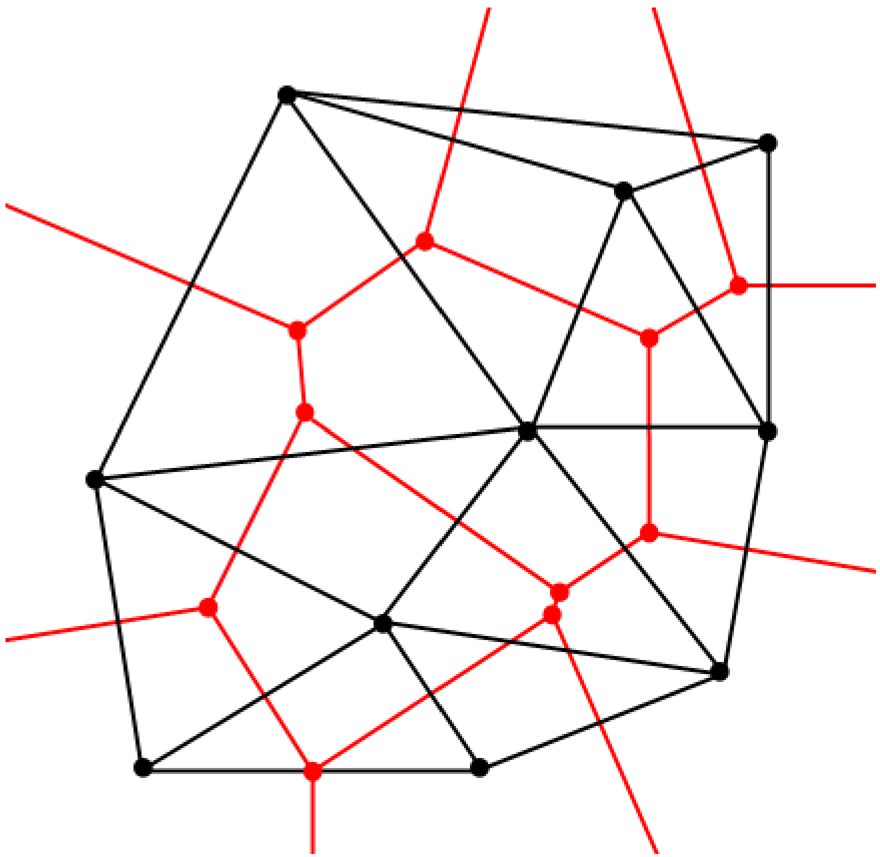


Genus 3

Mesh: Primal/Dual graph representations

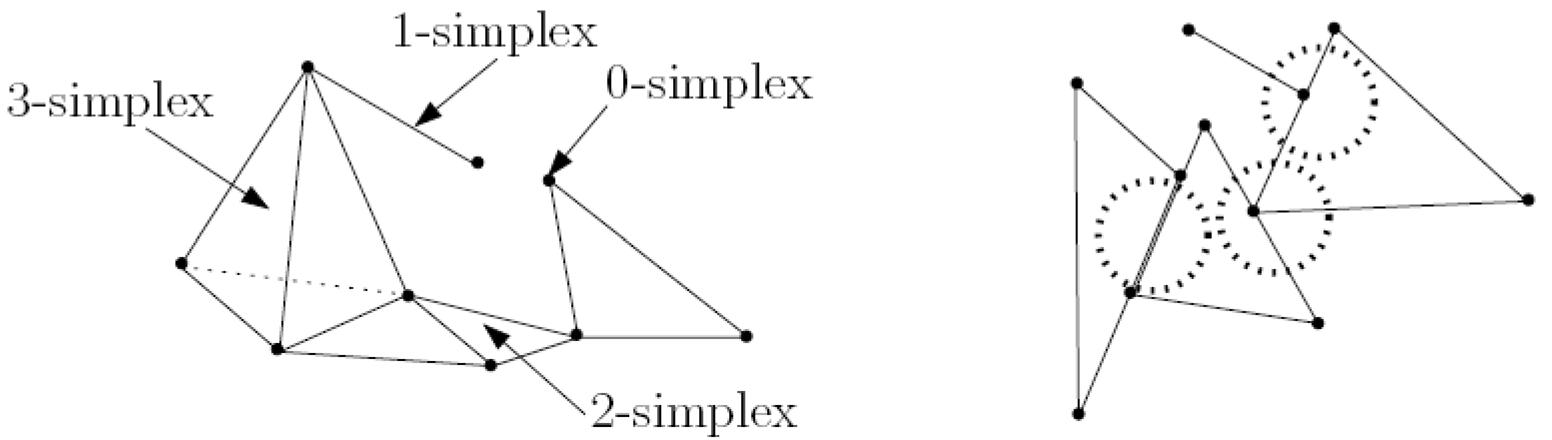


Mesh: Primal/Dual graph representations



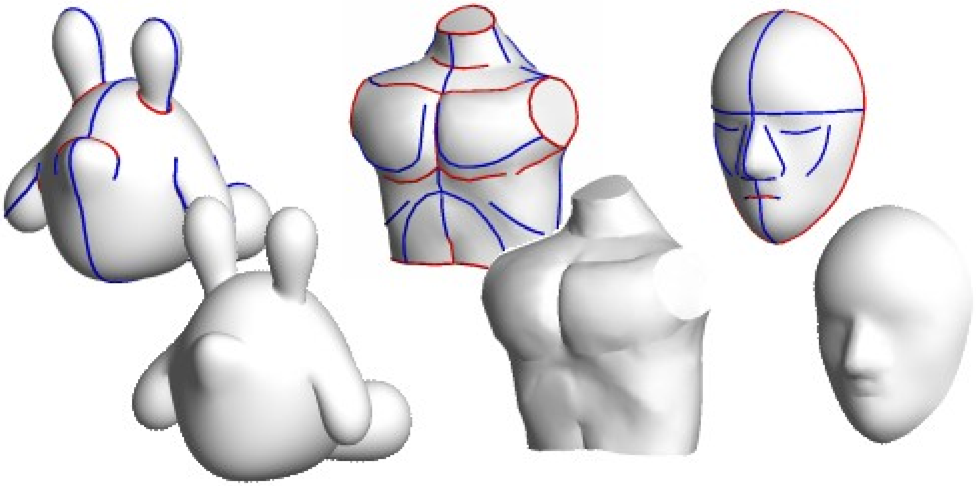
Primal Voronoi/Dual Delaunay triangulation

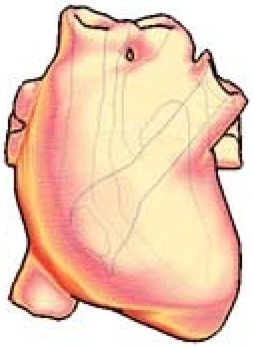
Simplicial complexes: Building blocks (LEGO-type)



(a) Simplicial complex (b) Nonsimplicial complex

Sketching meshes: Pen computing
Meshes for the masses.
→ Difficult to design

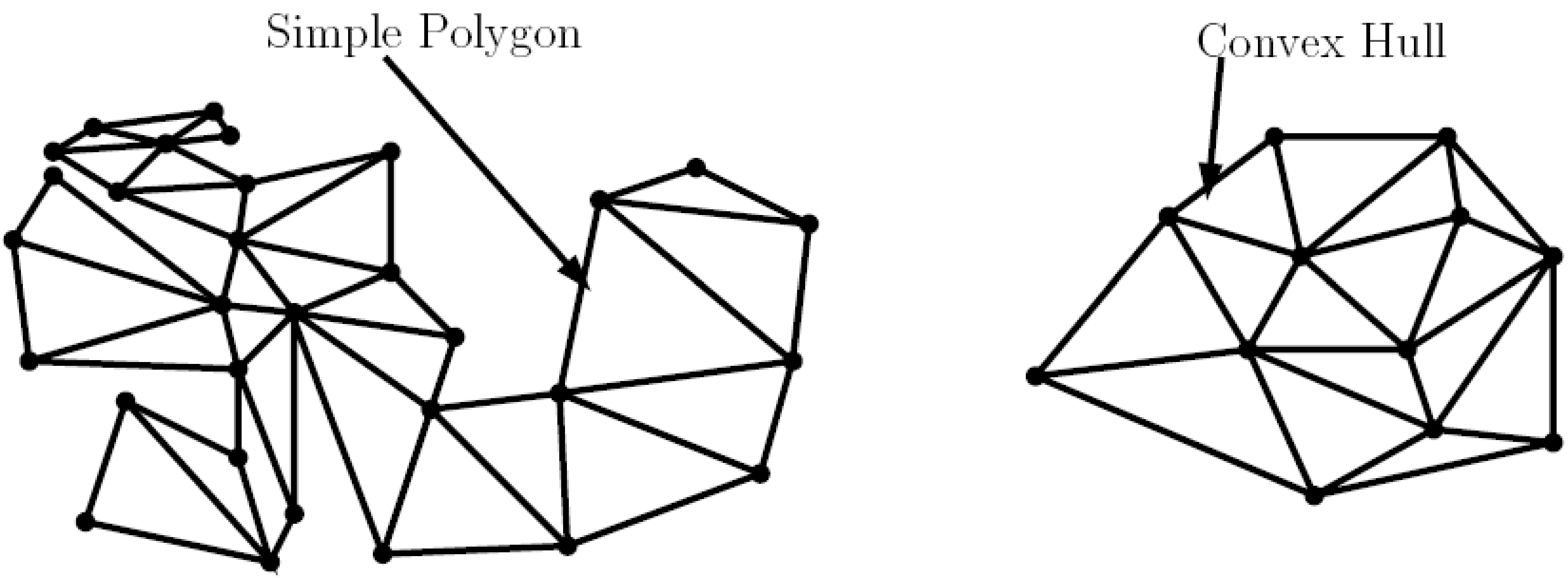




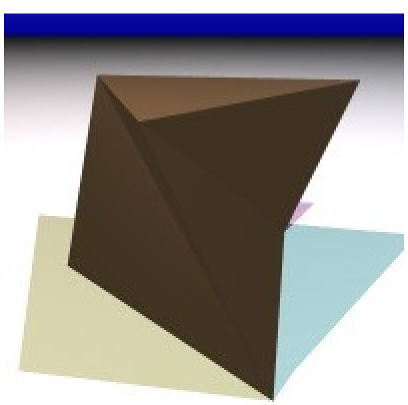
http://www.sonycsl.co.jp/person/nielsen/PT/vteddy/vteddy-desc.html

Triangulation meshes:

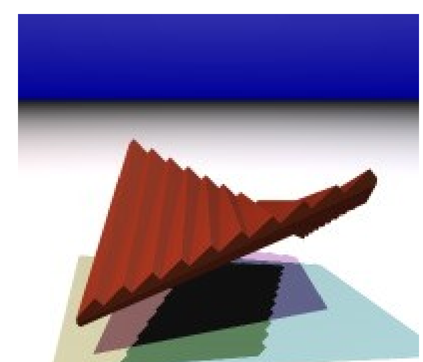
Always possible in 2D (but difficult)



NOT Always possible in 3D!!! (require additional Steiner points)



Untetrahedralizable Objects



SCHONHARDT'S POLYHEDRON CHAZELLE'S POLYHEDRON

Meshes: Procedural modeling/ city(buildings)

L-system (Lindenmayer) process

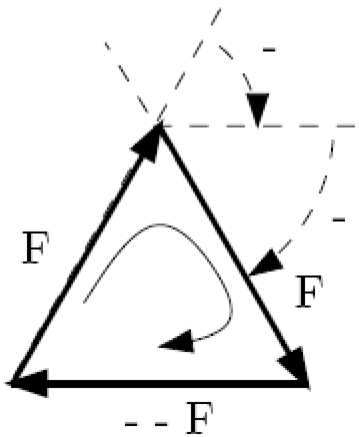
$$\text{START} \longrightarrow A$$
$$A \longrightarrow B$$
$$B \longrightarrow AB$$

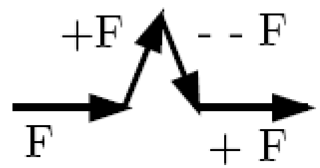
Fibonacci's sequences

Step	String	String Length
0	A	1
1	B	1
2	AB	2
3	BAB	3
4	ABBAB	5
5	BABABBAB	8
6	ABBABBABABBAB	13
7	BABABBABABBABBABABBAB	21

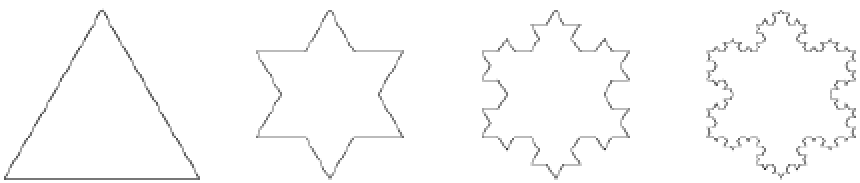
Grammar, language, parsing, etc.

L-system (Lindenmayer) process / LOGO





$$\mathrm{START} \longrightarrow F--F--F$$
$$F \longrightarrow F+F--F+F$$

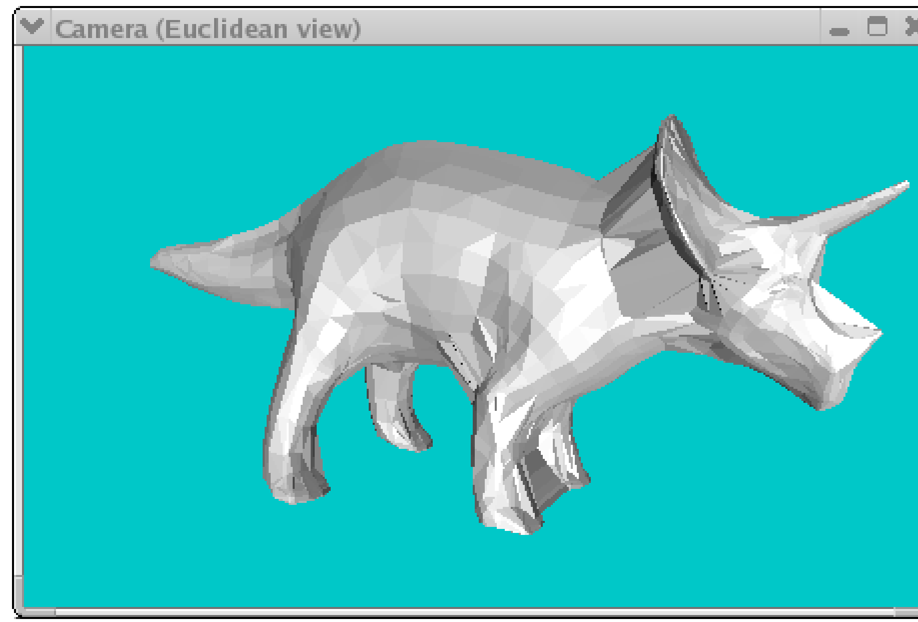


Data-structures for meshes: Indexed face list

(This is the PPM equivalent for 3D objects)

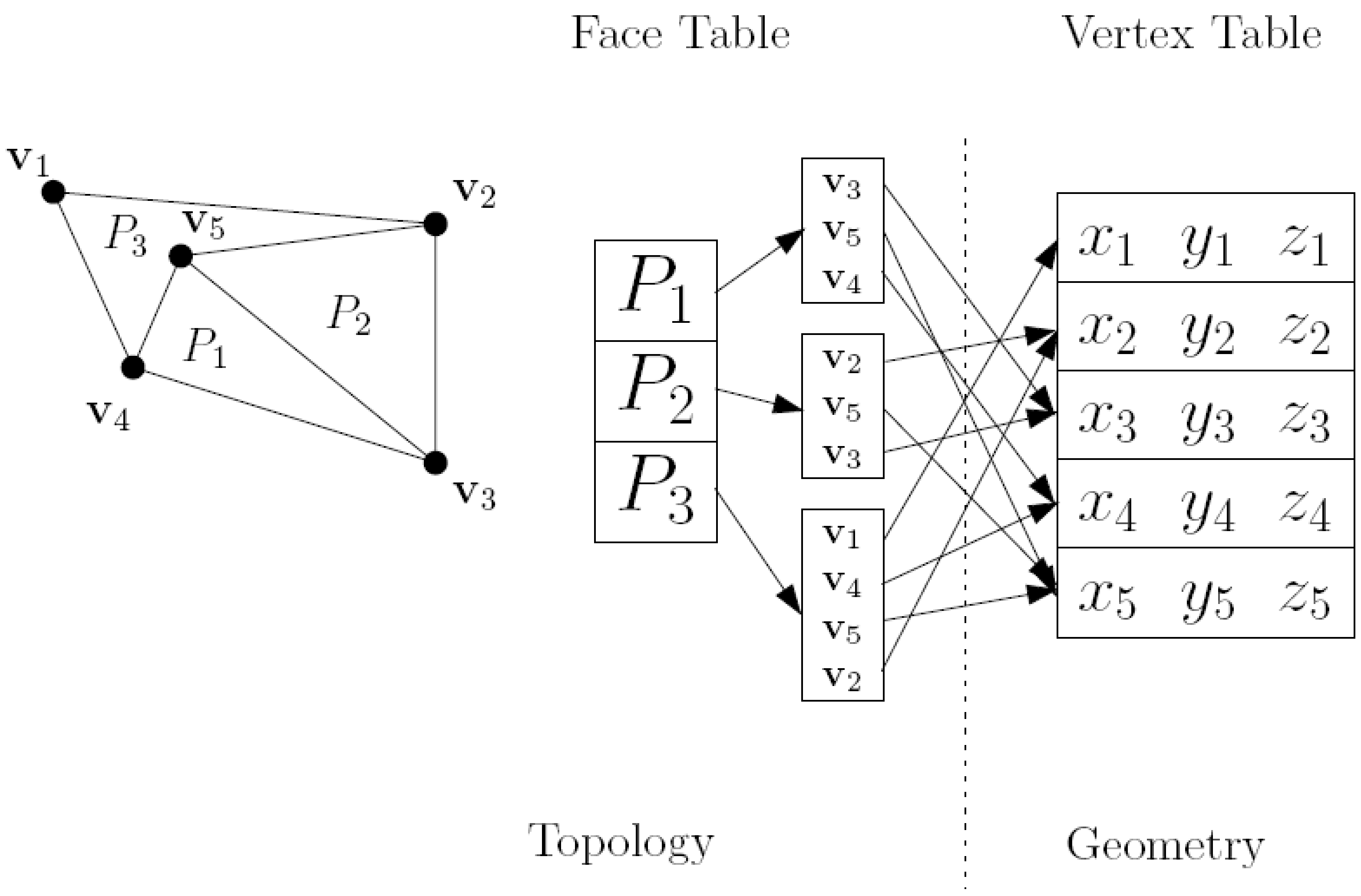
Object Oriented Graphics Library (OOGL) / OFF format

```
OFF
# Geomview OOGL format cube.off
# #Vertices #Faces #Edge
8 6 0
# Vertex table
−0.500000 −0.500000 0.500000
0.500000 −0.500000 0.500000
−0.500000 0.500000 0.500000
0.500000 0.500000 0.500000
−0.500000 0.500000 −0.500000
0.500000 0.500000 −0.500000
−0.500000 −0.500000 −0.500000
0.500000 −0.500000 −0.500000
# Face index table (first vertex index: 0)
4 0 1 3 2
4 2 3 5 4
4 4 5 7 6
4 6 7 1 0
4 1 7 5 3
4 6 0 2 4
```



Geomview.org viewer

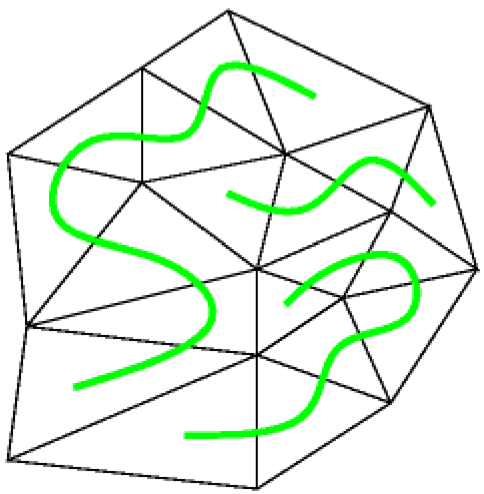
Data-structures for meshes: Indexed face list

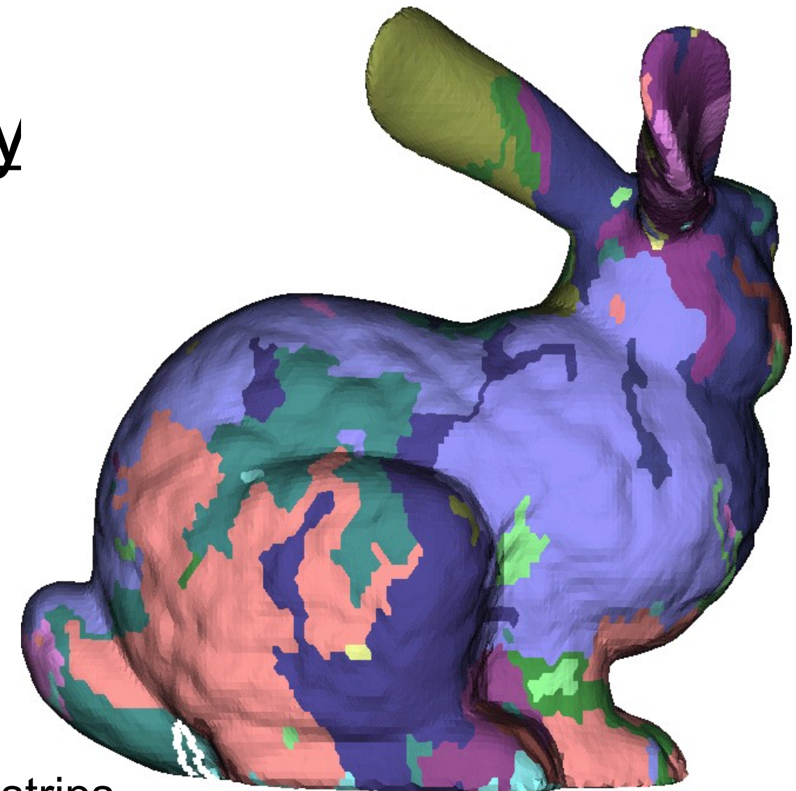


→ If we change vertex coordinates, all faces updated simultaneously !

Optimizing bandwidth: Triangle/quad strips

Compress mesh vertices
→ Compress <u>mesh connectivity</u>





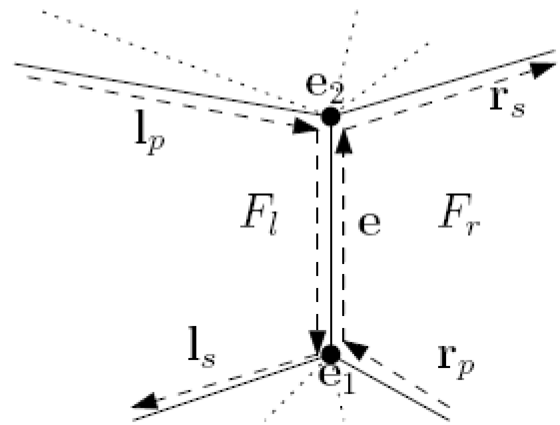
Bunny with 150 strips

GreedyStripMesh($\mathcal{M}$)

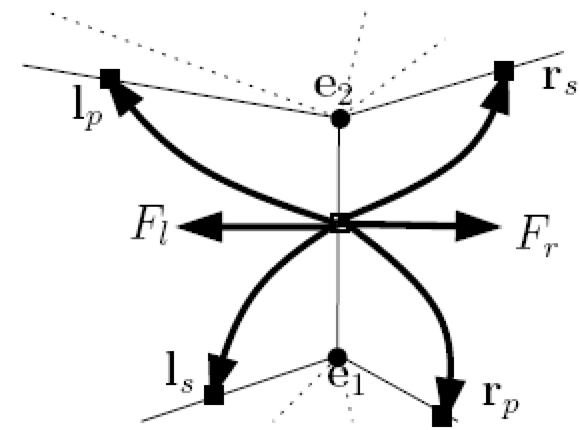
$\triangleleft$ Overview of the greedy stripping method $\triangleright$
while there remains triangles in $\mathcal{M}$
  do Pick a triangle T of $\mathcal{M}$ that has minimum number of adjacent triangles
    For each edge $\mathbf{e}$ of T, build the strip passing through T and $\mathbf{e}$
    Choose the longuest strip and remove its triangles from $\mathcal{M}$

Many data-structures for meshes....

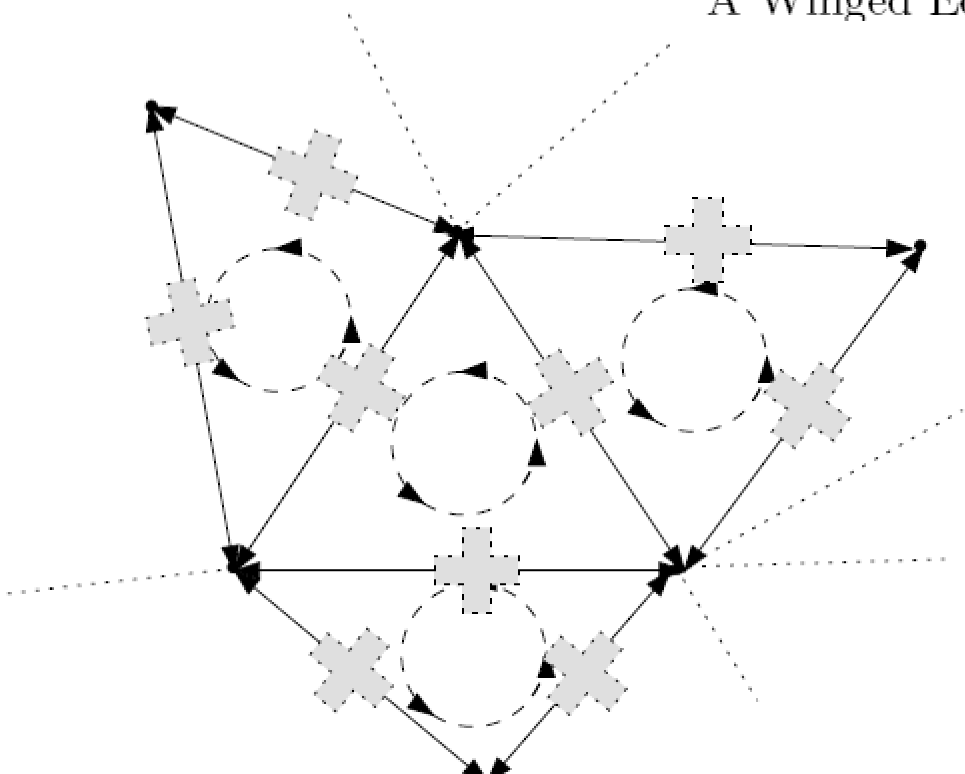
Winged edges
Half edges
Quad edges

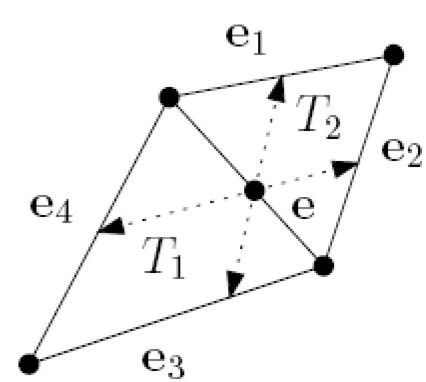


A Winged Edge



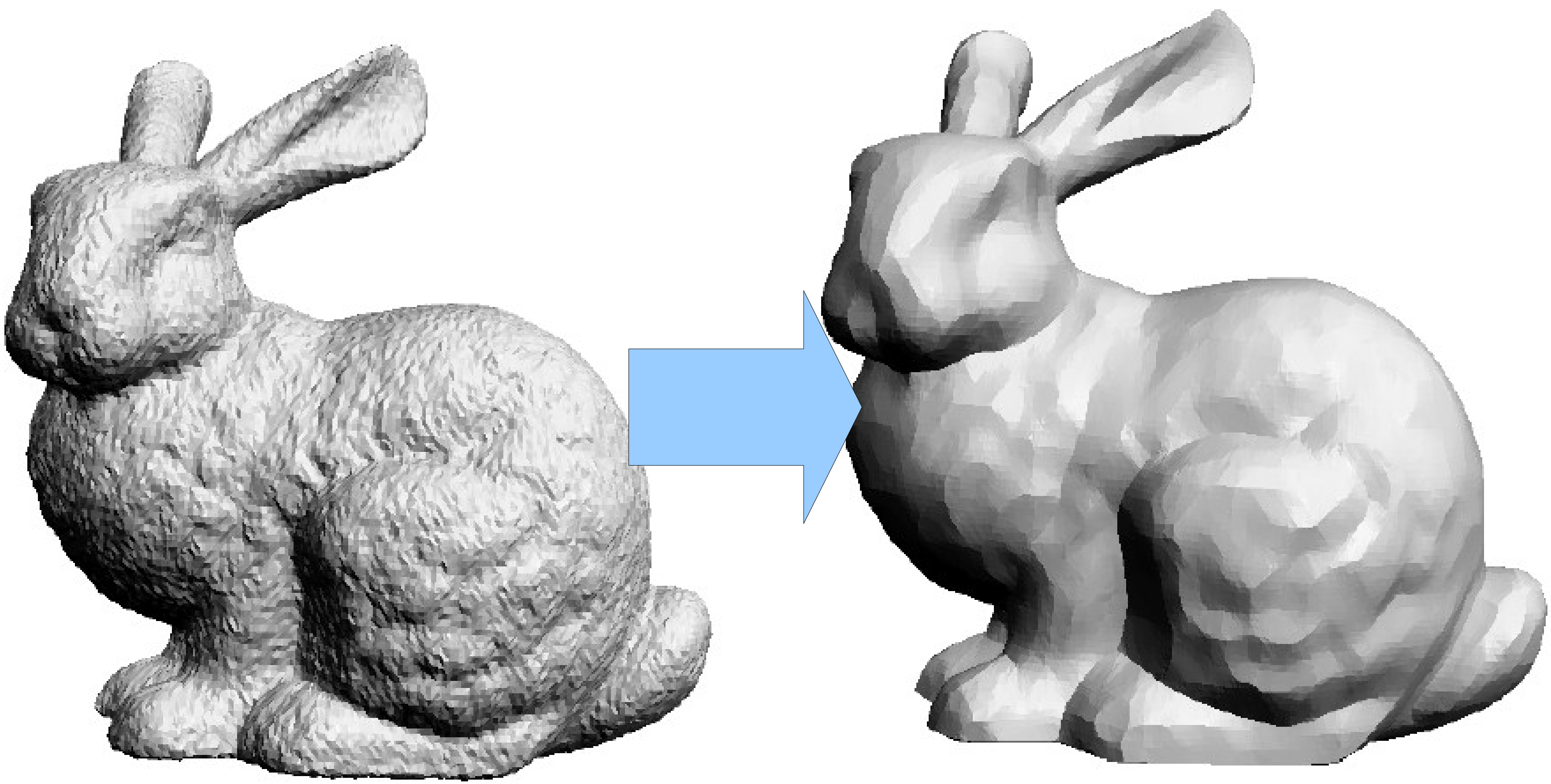
Pointer References



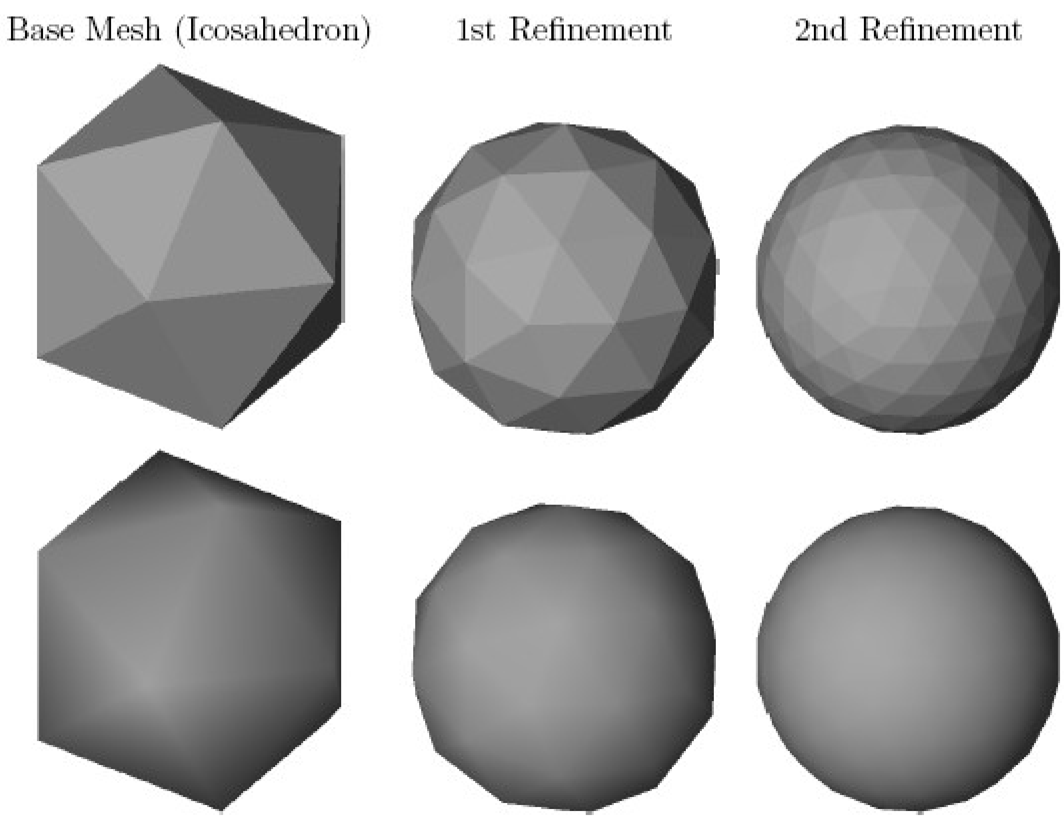


(Discrete) Laplacian smoothing on meshes

$$\mathbf{v} \leftarrow \mathbf{v} + \frac{\lambda}{|N(\mathbf{v})|} \sum_{i=1}^{|N(\mathbf{v})|} (N(\mathbf{v}, i) - \mathbf{v})$$

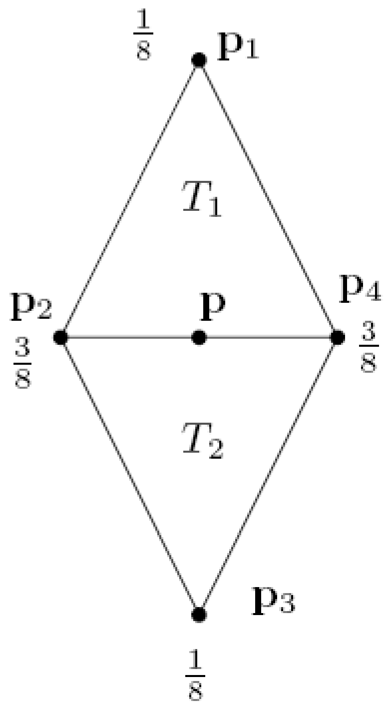


Surface subdivision: Refining → Limit surface is smooth



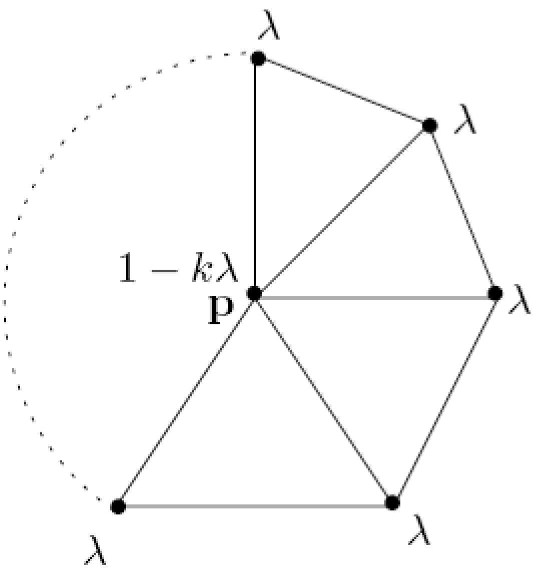
Surface subdivision: Dr Loop scheme

New vertex creation



$$\mathbf{p} = \frac{1}{8}\mathbf{p}_1 + \frac{3}{8}\mathbf{p}_2 + \frac{1}{8}\mathbf{p}_3 + \frac{3}{8}\mathbf{p}_4$$

Vertex relocation



$$\mathbf{p} = (1 - k\lambda)\mathbf{p} + \lambda \sum_{i=1}^{k} N(\mathbf{p}, i)$$

$$\lambda = \frac{1}{k}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{k}\right)^2\right)$$

Simplified to

$$\lambda = \begin{cases} \frac{3}{16} & k = 3, \\ \frac{3}{8k} & k > 3. \end{cases}$$

SUBDIVISION($\mathcal{M}, k$)

$\triangleleft$ General subdivision framework $\triangleright$
$\triangleleft$ Depend on whether the scheme is approximating/interpolating and primal/dual $\triangleright$
for $\mathbf{e} \in \mathcal{M}$
  do $\triangleleft$ for each edge $\triangleright$
    Create new vertex using the vertex creation mask
    $\triangleleft$ Could be several for n-adic subdivision $\triangleright$
for $\mathbf{v} \in \mathcal{M}$
  do Move original vertex using the vertex displacement mask
Reconnect all vertices as a triangular mesh based on $\mathcal{M}$

Limit position, smoothness properties

$$\lambda_\infty = \frac{1}{\frac{3}{8}\lambda + k}$$

Kobbelt's subdivision (+edge flipping)

$$\mathbf{p} = (1 - \lambda n)\mathbf{p} + \lambda \sum_{i=1}^{|N(\mathbf{p})|} N(\mathbf{p}, i).$$

$$\lambda = \frac{4 - 2\cos\frac{2\pi}{n}}{9n}$$

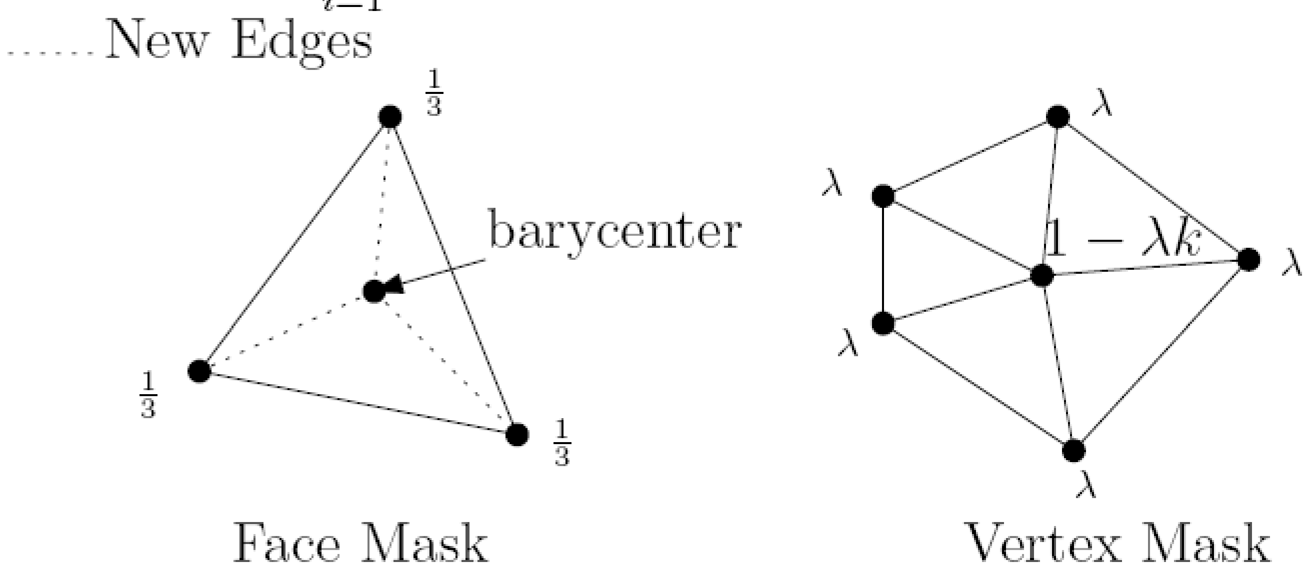
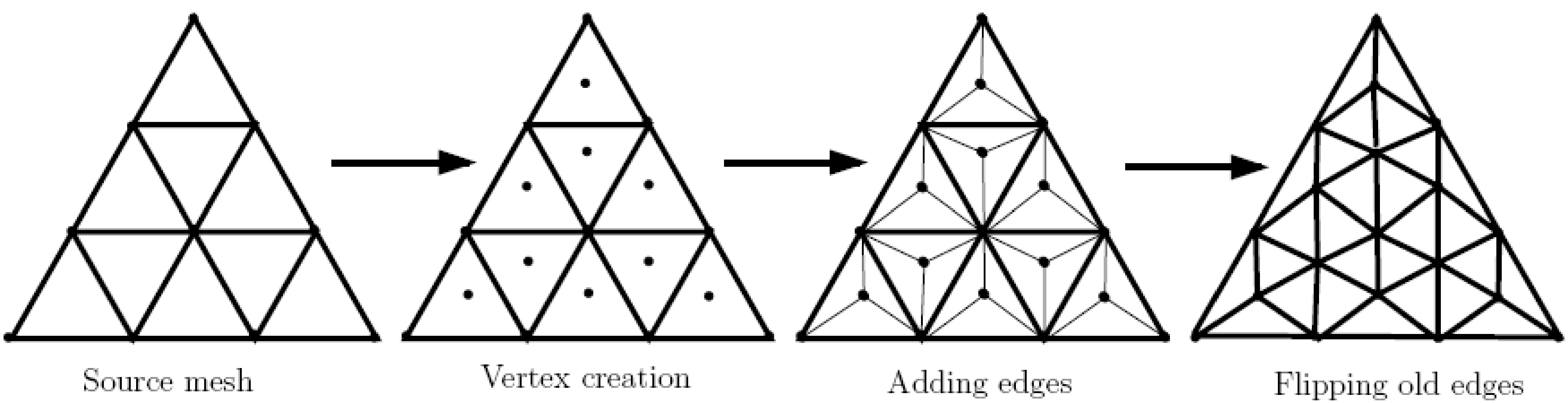
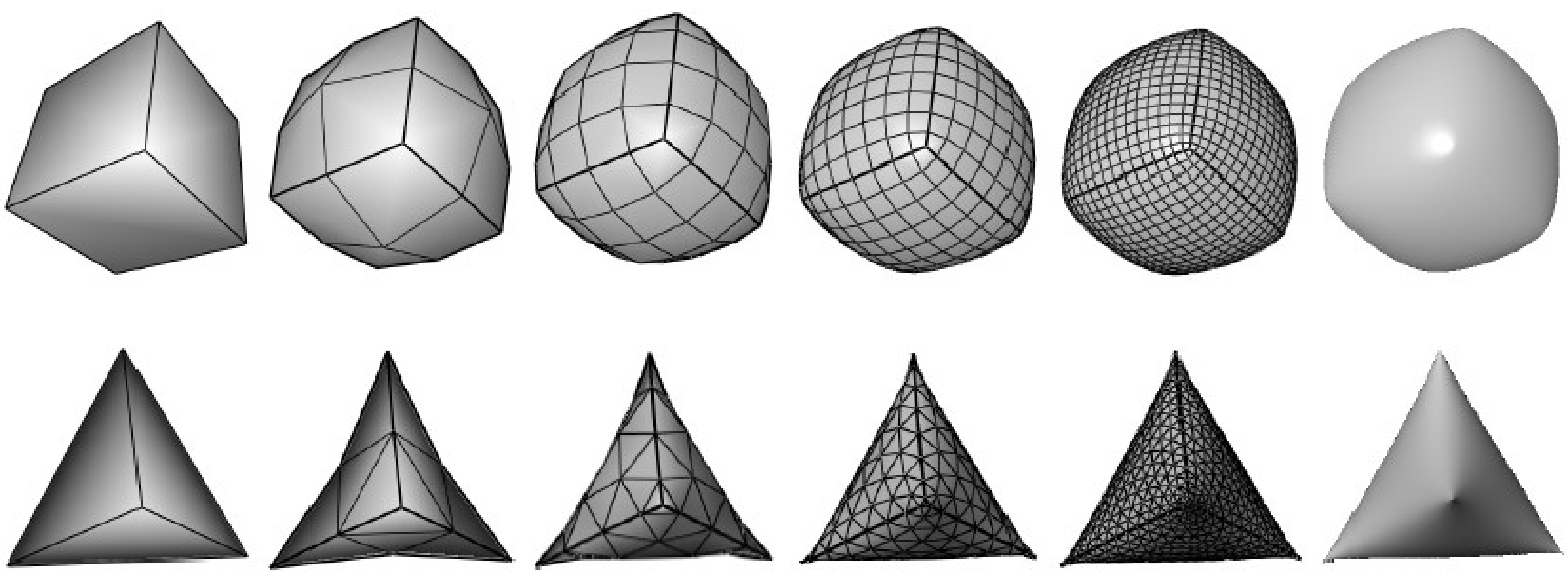


FIGURE 5.41 *The face and vertex masks for the $\sqrt{3}$-subdivision.*



Mesh subdivision: Combinatorial explosion !

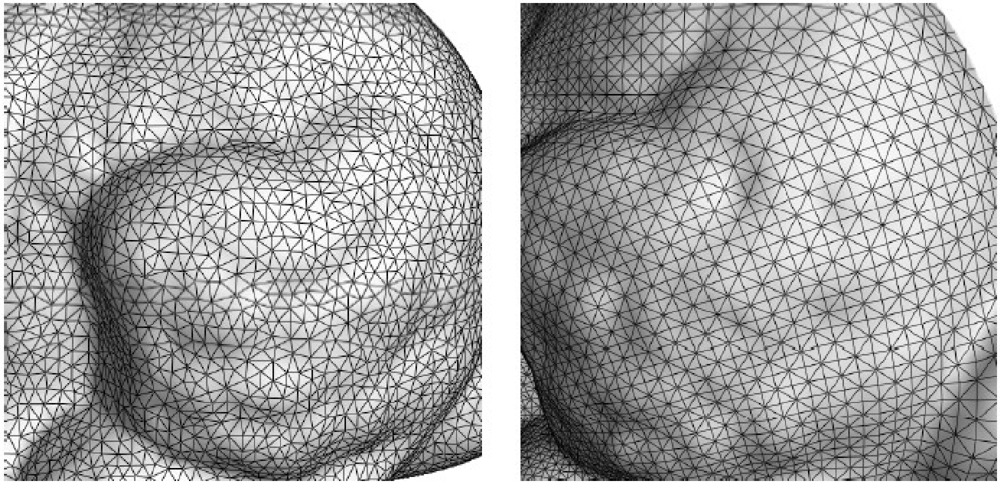
	Cube			Tetrahedron		
Level	#Vertices	#Faces	#Edges	#Vertices	#Faces	#Edges
0	8	6	12	4	4	6
1	26	24	48	10	16	24
2	98	96	192	34	64	96
3	386	384	768	130	256	384
4	1538	1536	3072	514	1024	1536
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8	393218	393216	786432	131074	262144	393216

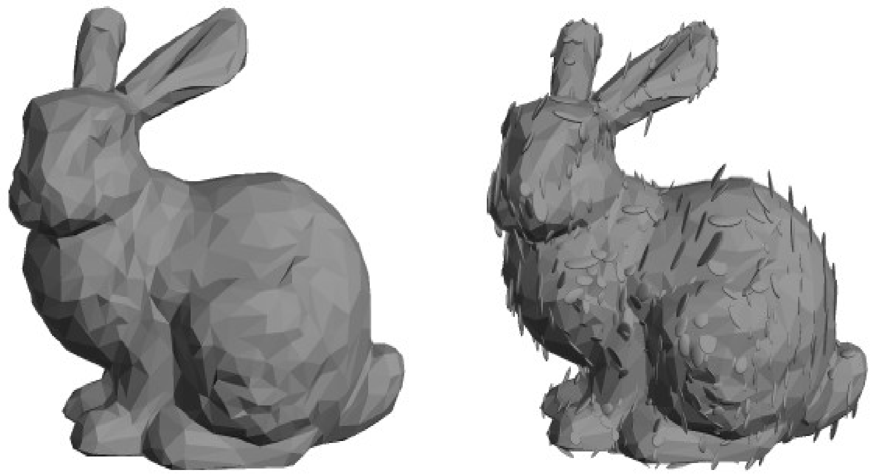


Remeshing

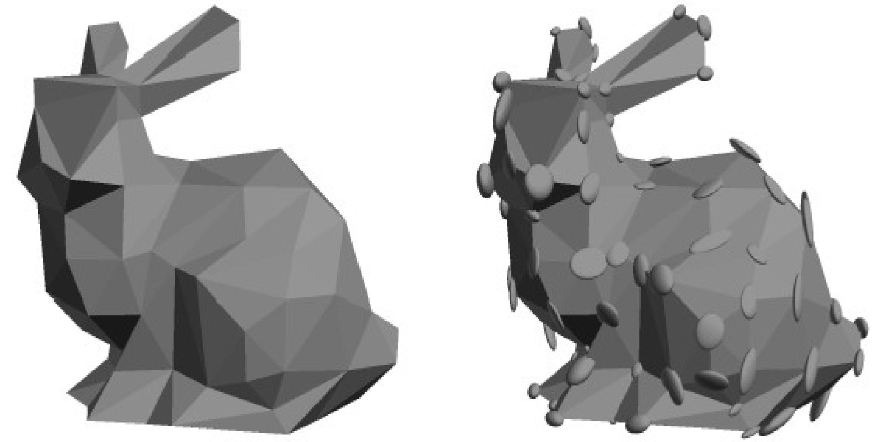
Decimating mesh

Mesh simplification

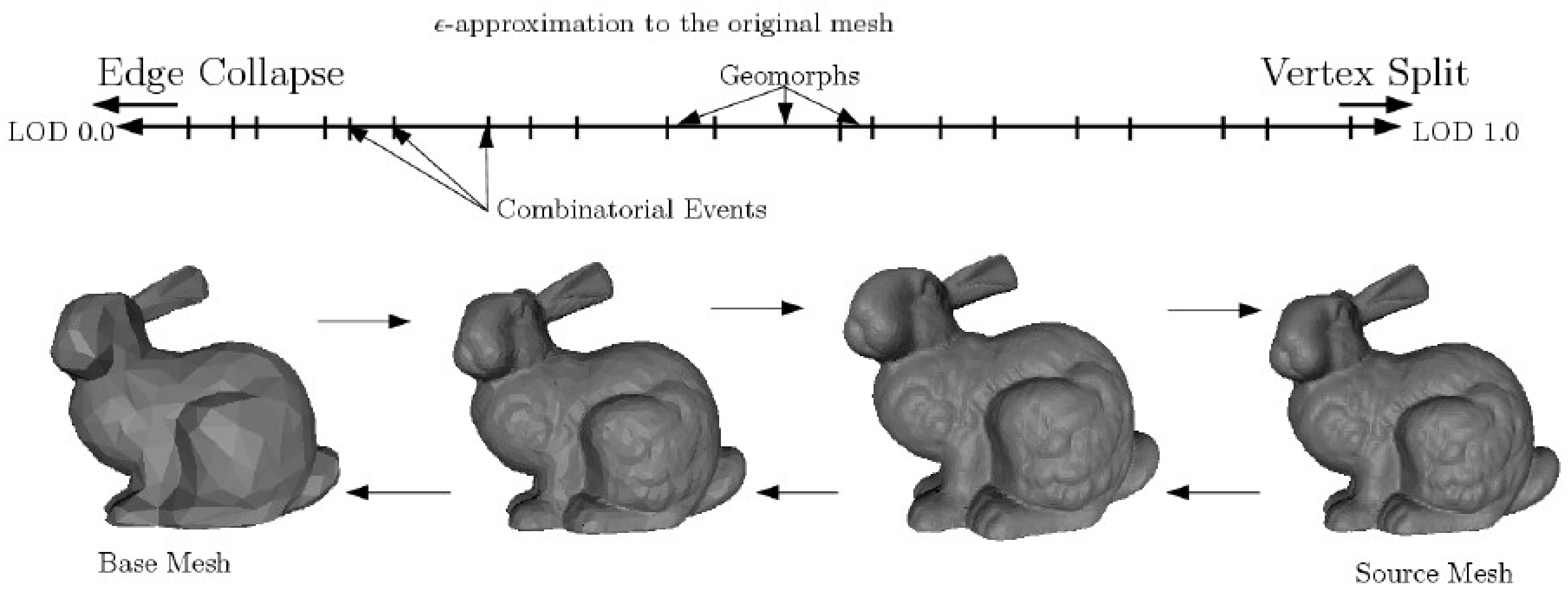




(a) (b)



Progressive mesh representations



Level of details/streaming...

Parameterization and texture mapping

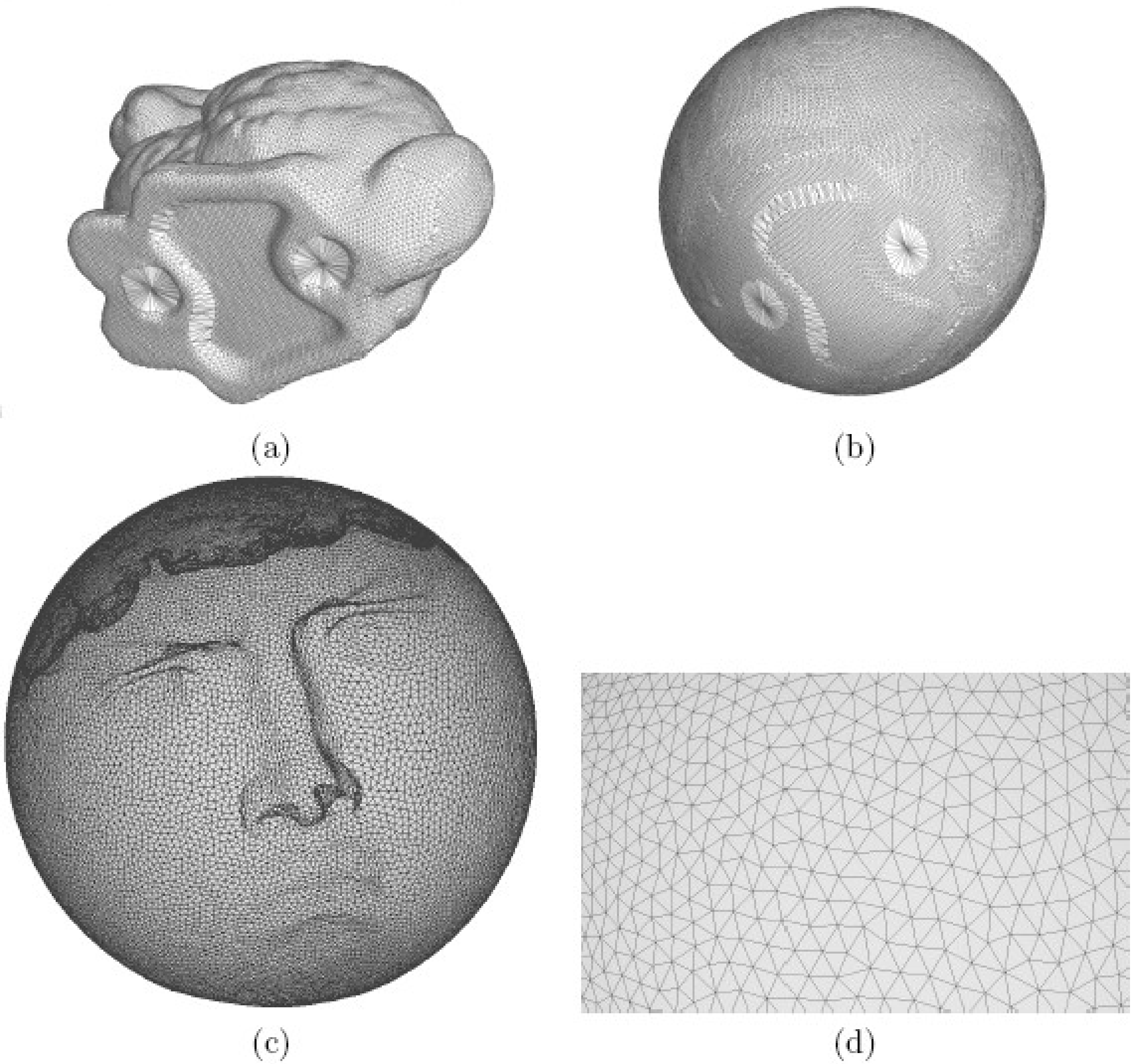
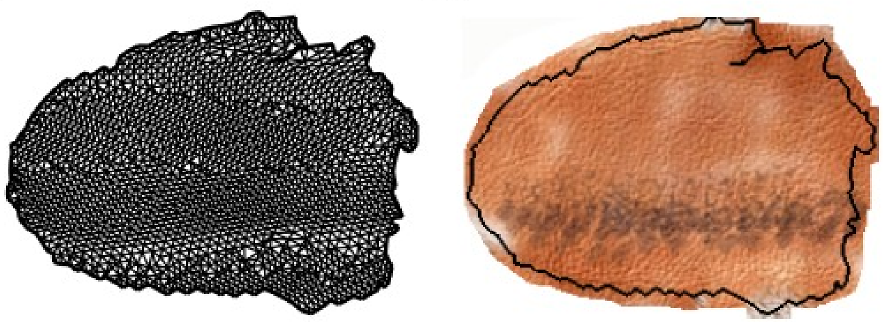


Image texture is 2D

Parameterization and texture mapping

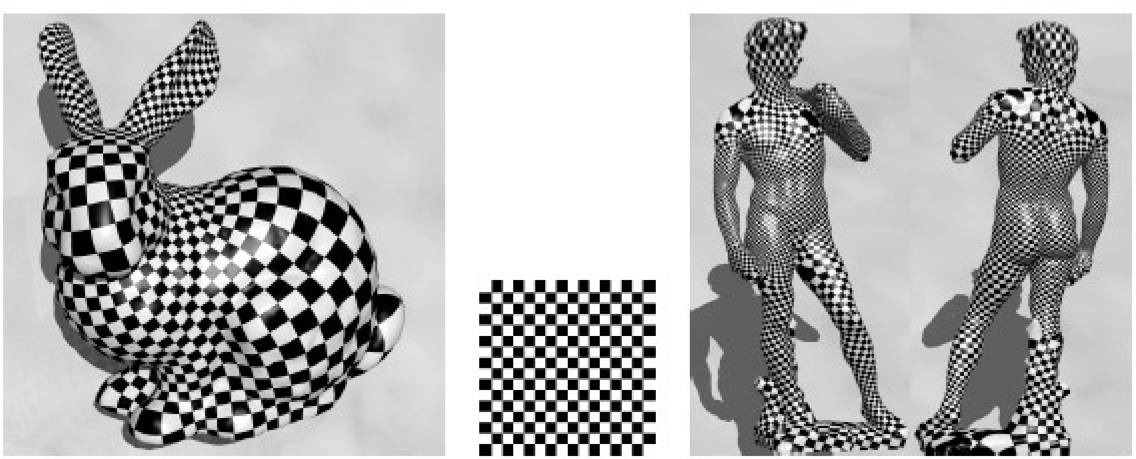
(a) (b) (c)

Atlas



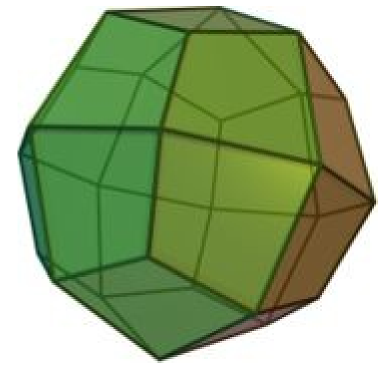
(d) (e)

Minimize distortion



Conformal mapping
(preserve angles)

H- and V- representations of polytopes

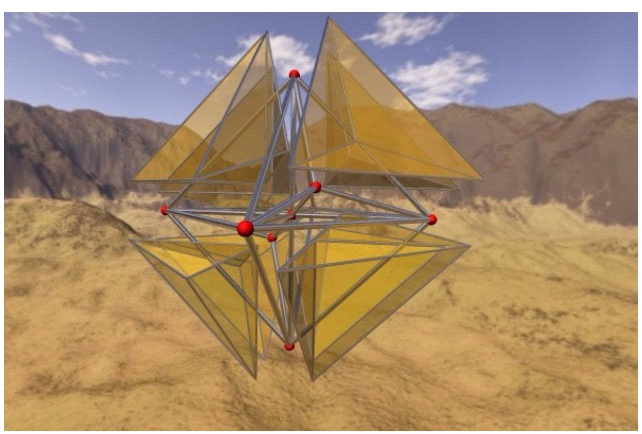


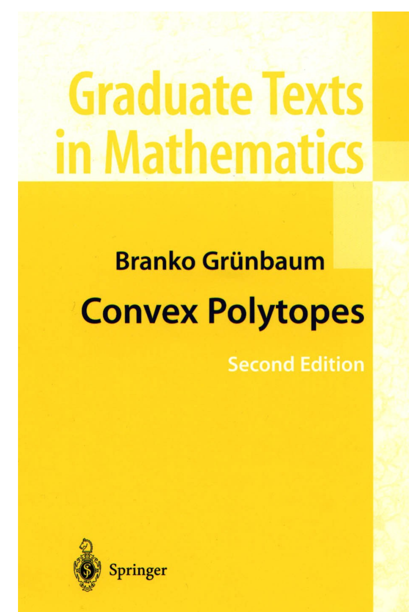
Half-spaces representation

$$P = \{x \in \mathbb{R}^n : \; Ax \leq b\}$$

Vertex representation (convex hull)

$$\bar{P} = \{x \in \mathbb{R}^n : x = \sum_i \lambda_i x_i, \sum_i \lambda_i = 1, \lambda_i \geq 0\}$$





http://www.math.tu-berlin.de/polymake/index.html