Characterization of the Area Explored by a Line-Sweep Sensor on the Plane

Séminaire AID/CIEDS

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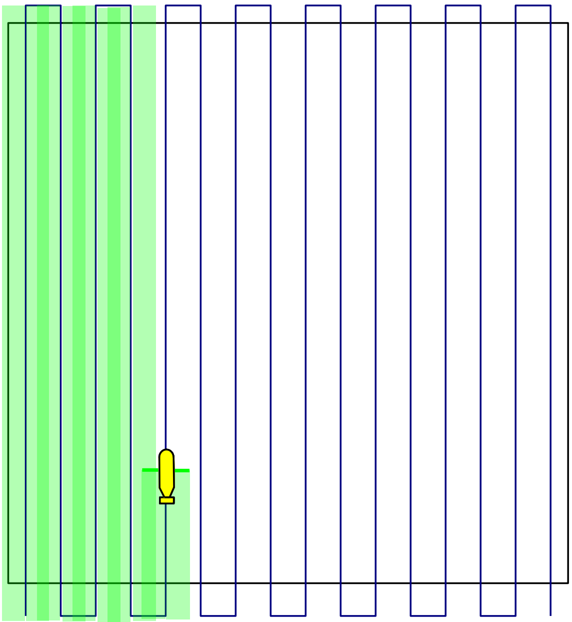
18/04/2023

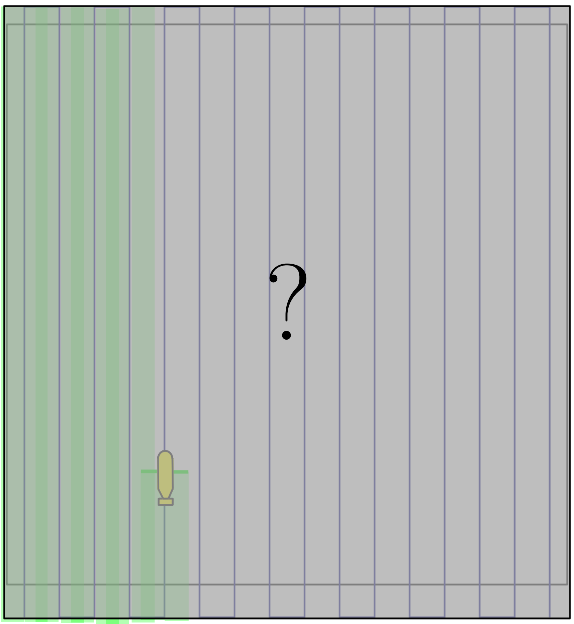
Case of Study



Context

- Unknown environment,
- area covering mission,
- revisiting,
- region avoidance,
- line-sweep exploration.

Case of Study

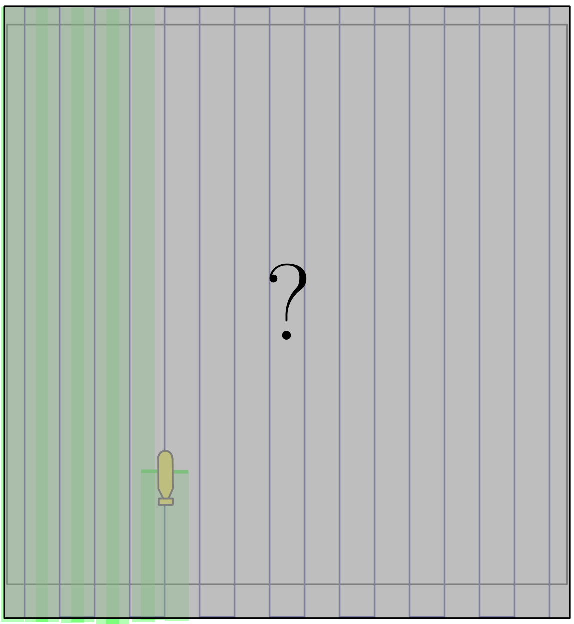


Objectives

Using only proprioceptive data, to estimate:

- Explored area
- Number of views (coverage measure)

Case of Study



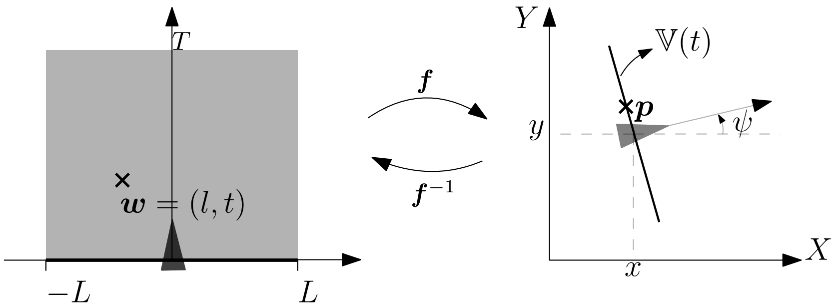
Applications:

- Assess area-covering missions,
- plan other missions to fill possible gaps,
- assess revisiting missions,
- optimal trajectory planning,
- localization in homogeneous environments.

Problem Statement

Visible Area

$$\mathbb{V} : [0, T] \rightarrow \mathcal{P}(\mathbb{R}^2)$$



Problem Statement

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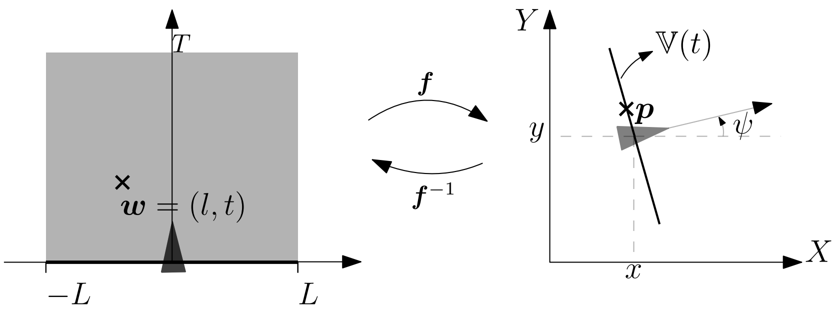


Waterfall Space and Sweep Function

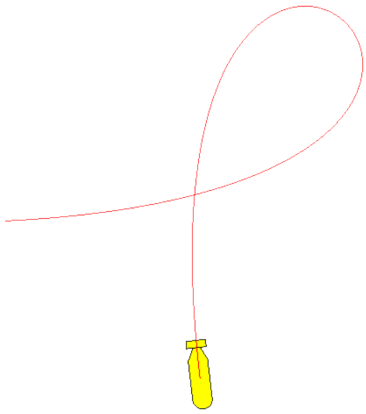
$$W = [-L, L] \times [0, T]$$

$$\boldsymbol{f} : W \to \mathbb{R}^2$$

Problem Statement

Robot's Trajectory

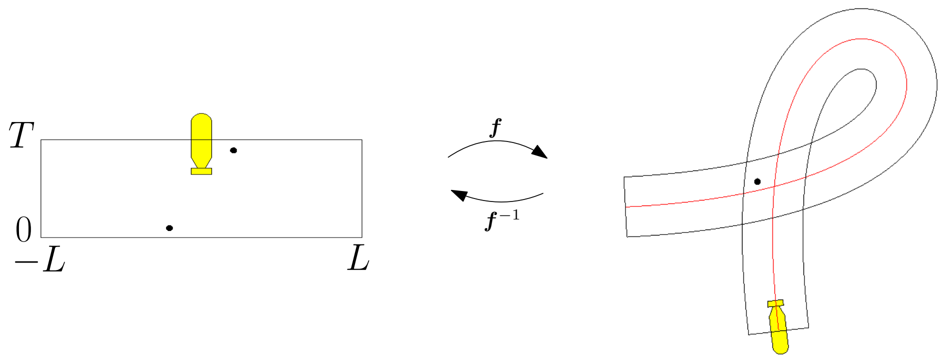
- $\boldsymbol{x} : [0, T] \to \mathbb{R}^2$,
- $\boldsymbol{x}$ is differentiable in $[0, T]$.



Problem Statement

Explored Area

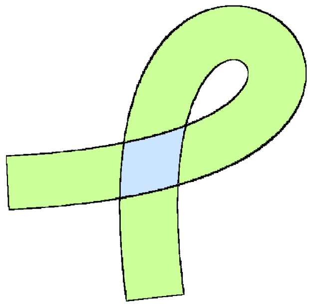
- $W = [-L, L] \times [0, T]$,
- $\mathbb{A}_{\mathbb{E}} = \boldsymbol{f}(W)$,
- Sensor's Contour $\gamma = \boldsymbol{f}(\partial W)$.



Problem Statement

Coverage Measure

$$c_m(\boldsymbol{p}) = \#Ker(\boldsymbol{f} - \boldsymbol{p})$$



Problem Statement

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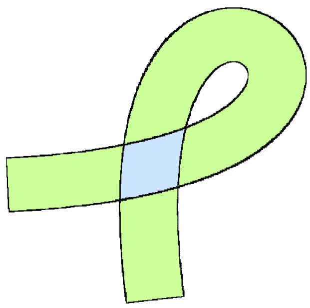
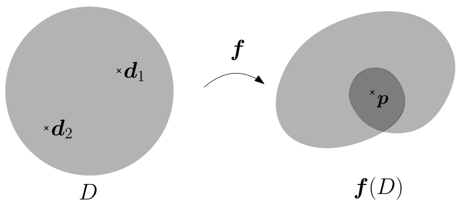


$$\mathbb{A}_{\mathbb{E}} = \{\boldsymbol{p} \in \mathbb{R}^2 | c_m(\boldsymbol{p}) \geq 1\}$$

Problem Approach

Topological Degree

- D is an open subset of $\mathbb{R}^n$,
- $\boldsymbol{f} : \overline{D} \to \mathbb{R}^n$ is continuous,
- $\boldsymbol{p} \in \mathbb{R}^n \setminus \boldsymbol{f}(\partial D)$
- $deg : \ (\boldsymbol{f}, D, \boldsymbol{p}) \to \mathbb{Z}$.



Problem Approach

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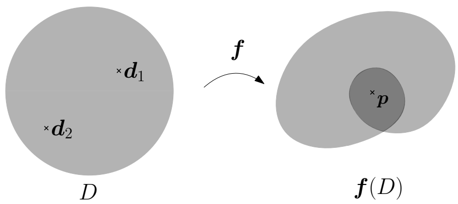
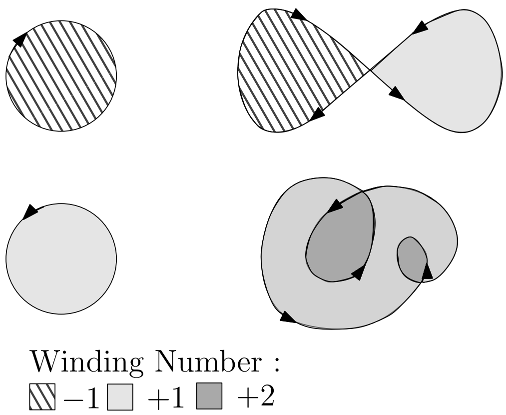


If $det(\boldsymbol{f}'(\boldsymbol{d}))$ is non-zero on each $\boldsymbol{d}$ such that $\boldsymbol{f}(\boldsymbol{d}) = \boldsymbol{p}$,

$$deg(\boldsymbol{f}, D, \boldsymbol{p}) = \sum_{\boldsymbol{d} \in \boldsymbol{f}^{-1}(\boldsymbol{p})} sign(det(\boldsymbol{f}'(\boldsymbol{d})))$$

Problem Approach

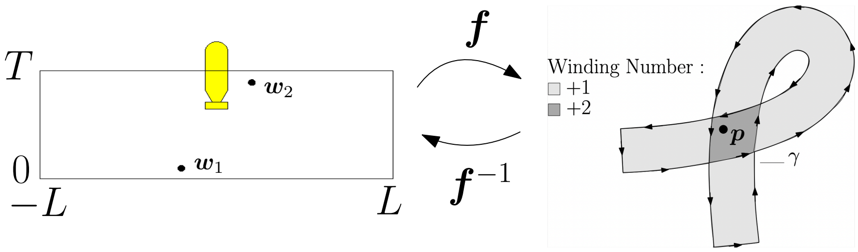
Winding Number



- D is an open subset of $\mathbb{R}^2$,
- $\boldsymbol{f} : \overline{D} \to \mathbb{R}^2$ is continuous,
- $\boldsymbol{p} \in \mathbb{R}^2 \backslash \boldsymbol{f}(\partial D)$,
- $\eta(\boldsymbol{f}(\partial D), \boldsymbol{p}) \in \mathbb{Z}$.

Problem Approach

For any $\boldsymbol{p} \in \mathbb{R}^2$, $c_m(\boldsymbol{p}) = \eta(\gamma, \boldsymbol{p})$

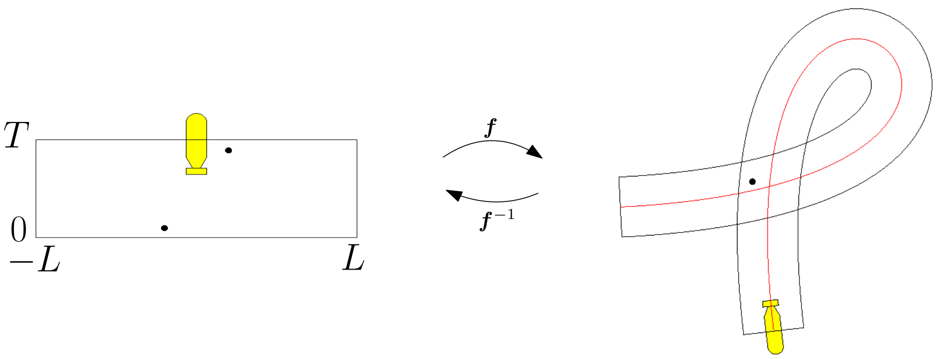


If $det(\boldsymbol{f}'(\boldsymbol{w}))$ is positive on each $\boldsymbol{w} \in W$ such that $\boldsymbol{f}(\boldsymbol{w}) = \boldsymbol{p}$,

$$\eta(\gamma, \boldsymbol{p}) = \sum_{\boldsymbol{w} \in \boldsymbol{f}^{-1}(\boldsymbol{p})} sign(det(\boldsymbol{f}'(\boldsymbol{w}))) = \#Ker(\boldsymbol{f} - \boldsymbol{p})$$

Costa Vianna M.L., Goubault E., Jaulin L., Putot S. (2022). Estimating the Coverage Measure and the Area Explored by a Side-Scan Sonar. *OCEANS 2022*

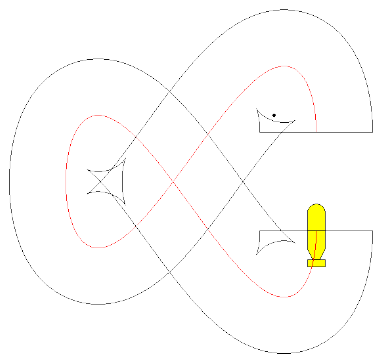
Sweeping backwards



If $det(\boldsymbol{f}'(\boldsymbol{w}))$ is positive on each $\boldsymbol{w} \in W$ such that $\boldsymbol{f}(\boldsymbol{w}) = \boldsymbol{p}$,

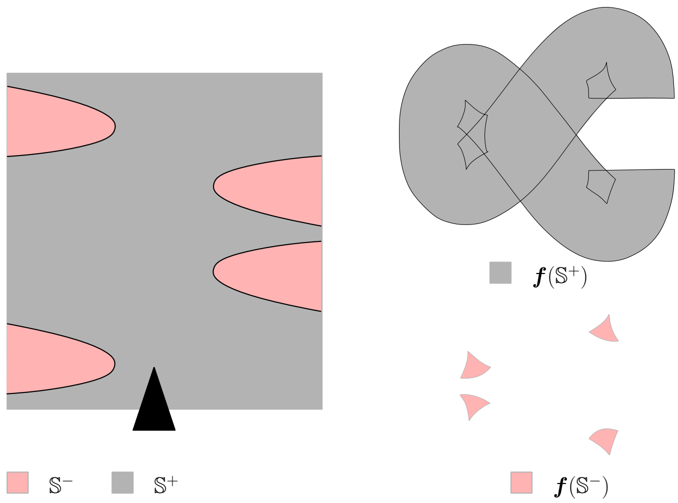
$$c_m(\boldsymbol{p}) = \eta(\gamma, \boldsymbol{p})$$

Sweeping backwards



$$\eta(\gamma, \boldsymbol{p}) = \sum_{\boldsymbol{w} \in \boldsymbol{f}^{-1}(\boldsymbol{p})} sign(det(\boldsymbol{f}^{'}(\boldsymbol{w}))) = +1 - 1 + 1 = +1 \neq \#Ker(\boldsymbol{f} - \boldsymbol{p})$$

Sweeping backwards



$$\mathbb{S}^+ = \{\boldsymbol{w} \in W | det(\boldsymbol{f}'(\boldsymbol{w})) > 0)\},\ \gamma^+ = \boldsymbol{f}(\partial\mathbb{S}^+)$$

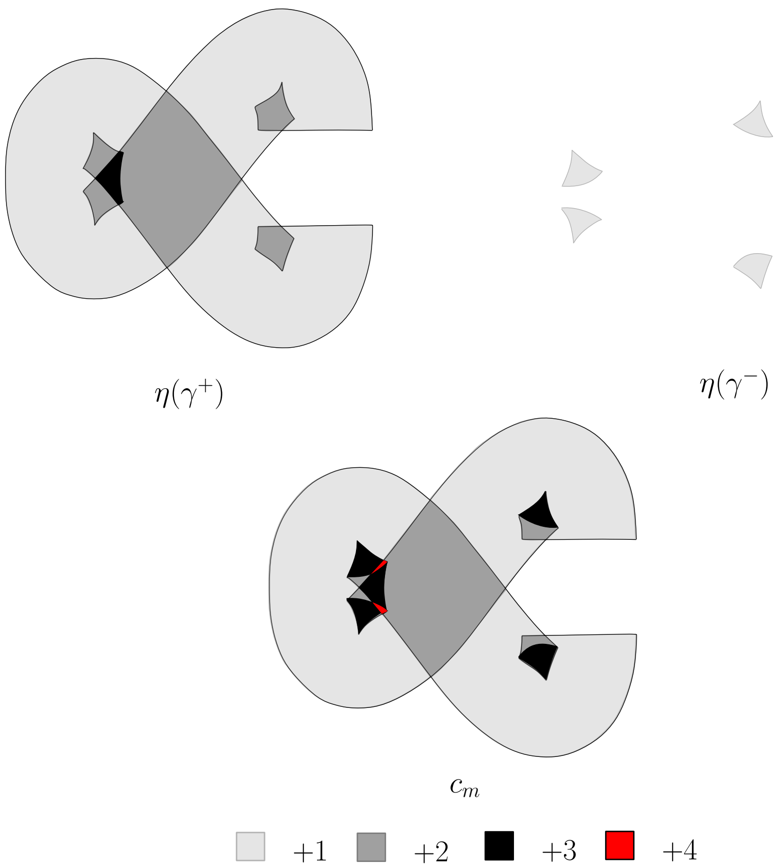
$$\mathbb{S}^- = \{\boldsymbol{w} \in W | det(\boldsymbol{f}'(\boldsymbol{w})) < 0)\},\ \gamma^- = \boldsymbol{f}(\partial\mathbb{S}^-)$$

Sweeping backwards

$$c_m(\boldsymbol{p}) = \#Ker(\boldsymbol{f} - \boldsymbol{p}) = \#Ker\ (\boldsymbol{f} - \boldsymbol{p})_{|\mathbb{S}^+} + \#Ker\ (\boldsymbol{f} - \boldsymbol{p})_{|\mathbb{S}^-}$$

$$c_m(\boldsymbol{p}) = \sum_{\boldsymbol{w} \in \boldsymbol{f}^{-1}_{|\mathbb{S}^+}(\boldsymbol{p})} +1 \quad + \sum_{\boldsymbol{w} \in \boldsymbol{f}^{-1}_{|\mathbb{S}^-}(\boldsymbol{p})} +1 = \eta(\gamma^+, \boldsymbol{p}) + \eta(\gamma^-, \boldsymbol{p})$$

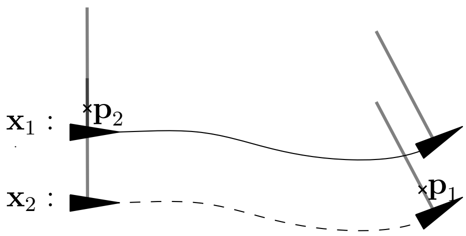
Sweeping backwards



Uncertain Trajectory

Uncertain Robot's Trajectory

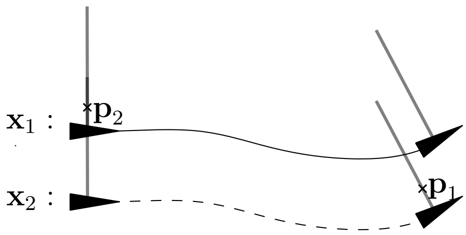
- $[\boldsymbol{x}] \in \mathcal{P}([0, T] \to \mathbb{R}^2)$,
- $[\boldsymbol{v}] \in \mathcal{P}([0, T] \to \mathbb{R}^2)$,
- $\boldsymbol{x}^* \in [\boldsymbol{x}]$,
- $\boldsymbol{v}^* \in [\boldsymbol{v}]$.



Uncertain Trajectory

Uncertain Coverage Measure

- $\boldsymbol{x}^* \in [\boldsymbol{x}]$,
- $[\gamma] \in \mathcal{P}(S^1 \rightarrow \mathbb{R}^2)$,
- $\gamma^* \in [\gamma]$.

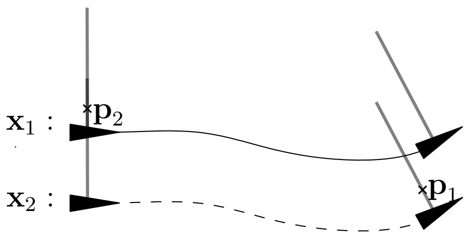


Uncertain Trajectory

Uncertain Coverage Measure

$$[c_m] : \mathbb{R}^2 \to \mathbb{N}_0$$

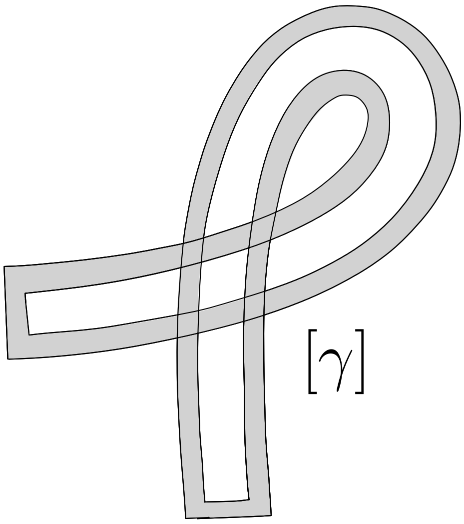
$$\forall \mathbf{x} \in [\mathbf{x}] \ , \ c_{m|\mathbf{x}}(\mathbf{p}) \in [c_m](\mathbf{p})$$



Uncertain Trajectory

Uncertain Coverage Measure

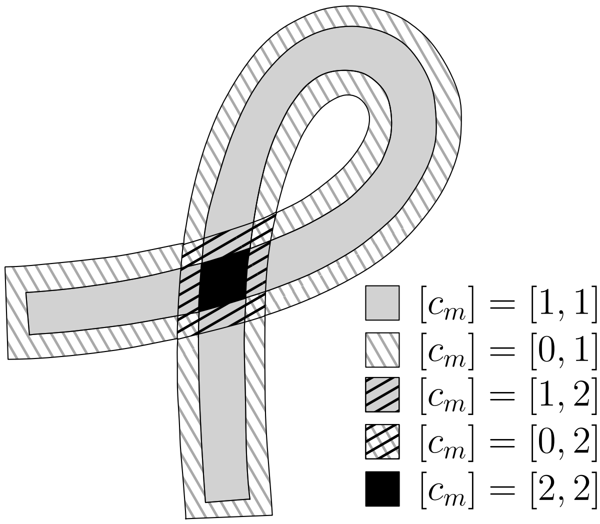
$$[\gamma] = [\gamma_{lb}, \gamma_{ub}]$$



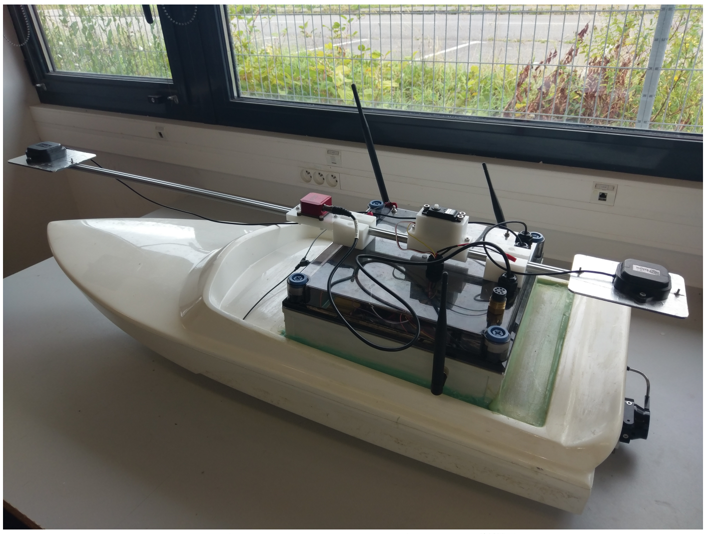
Uncertain Trajectory

Uncertain Coverage Measure

$$[c_m](\boldsymbol{p}) = [\eta(\gamma_{lb}, \boldsymbol{p}), \eta(\gamma_{ub}, \boldsymbol{p})]$$

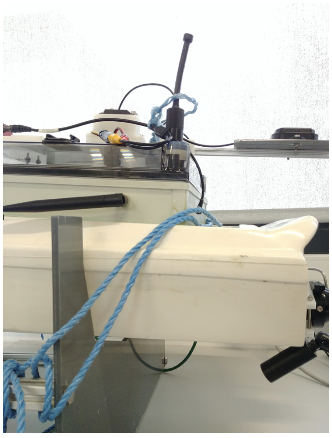


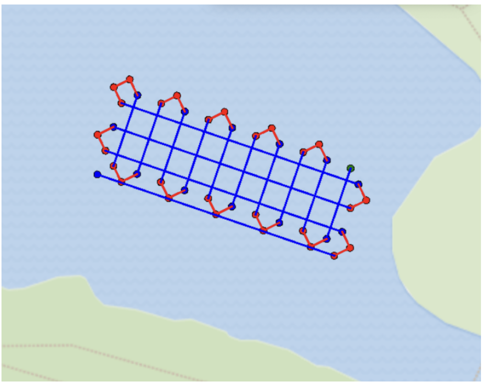
Speboat Guerlédan



Data

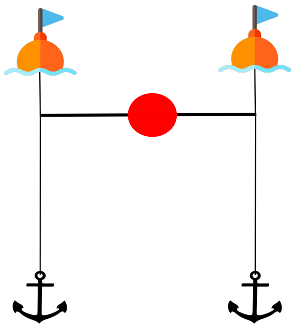
- GPS,
- IMU,
- Remote Controled,
- Autonomous via IHM.





Speboat Guerlédan

Buoy Search



Thank You!