Characterization of the Area Explored by an Autonomous Robot

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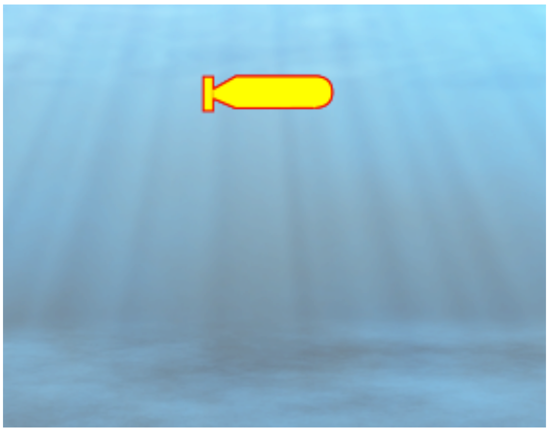
Introduction

Autonomy

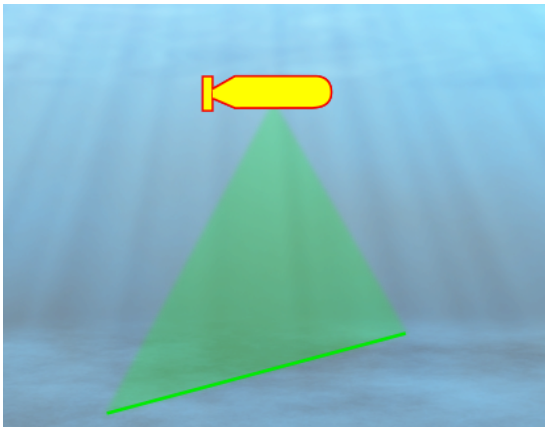
- Navigation sensors (Proprioceptive)
  - IMU, GPS, DVL . . .



Introduction

Autonomy

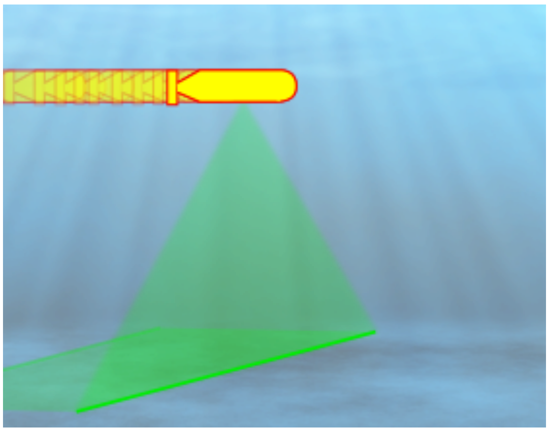
- Navigation sensors (Proprioceptive)
  - IMU, GPS, DVL ...
- Observation sensors (Exteroceptive)
  - Camera, sonar/lidar, temperature, salinity ...



Introduction

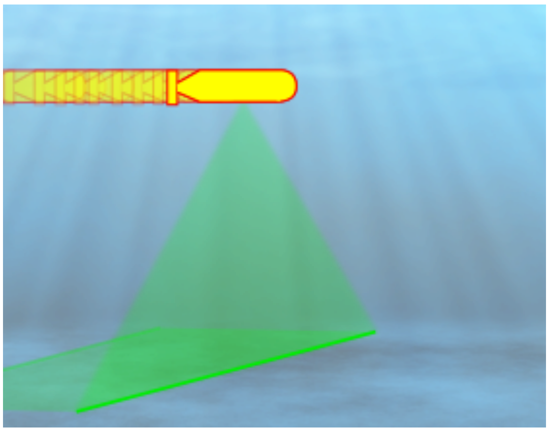
Explored Area

The explored area is the union of the visible areas over the whole trajectory.



Problem

- Compute the explored area.
- Compute the number of times each point in the environment has been explored.



Applications

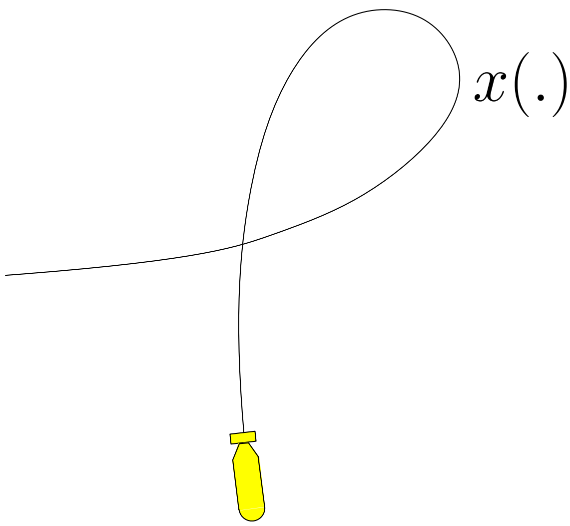
- Assess area-covering missions
  - Determine if a mission is complete
  - Plan other missions to fill possible gaps
- **Guarantee** that if a target is not detected, the target does not exist.

1. Introduction
2. **Problem Statement**
3. Problem Approach
4. Implementation
5. Results
6. Conclusions and Future Work

Problem Statement

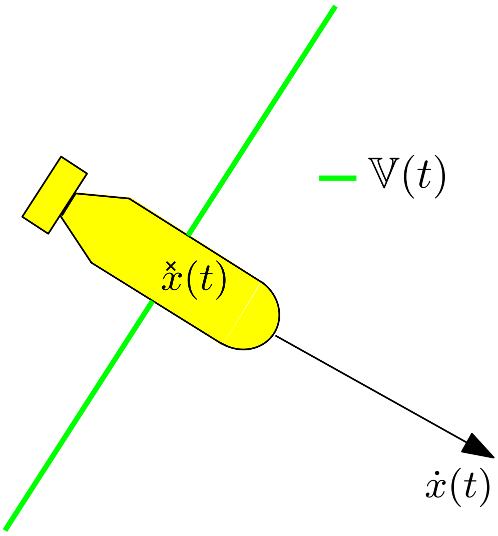
Hypothesis

- $x(.) : \mathbb{R} \to \mathbb{R}^2$
- $T = [0, T_{max}]$
- $x(.)$ is continuous in T.
- $x(.)$ and $\dot{x}(.)$ are known.



Problem Statement

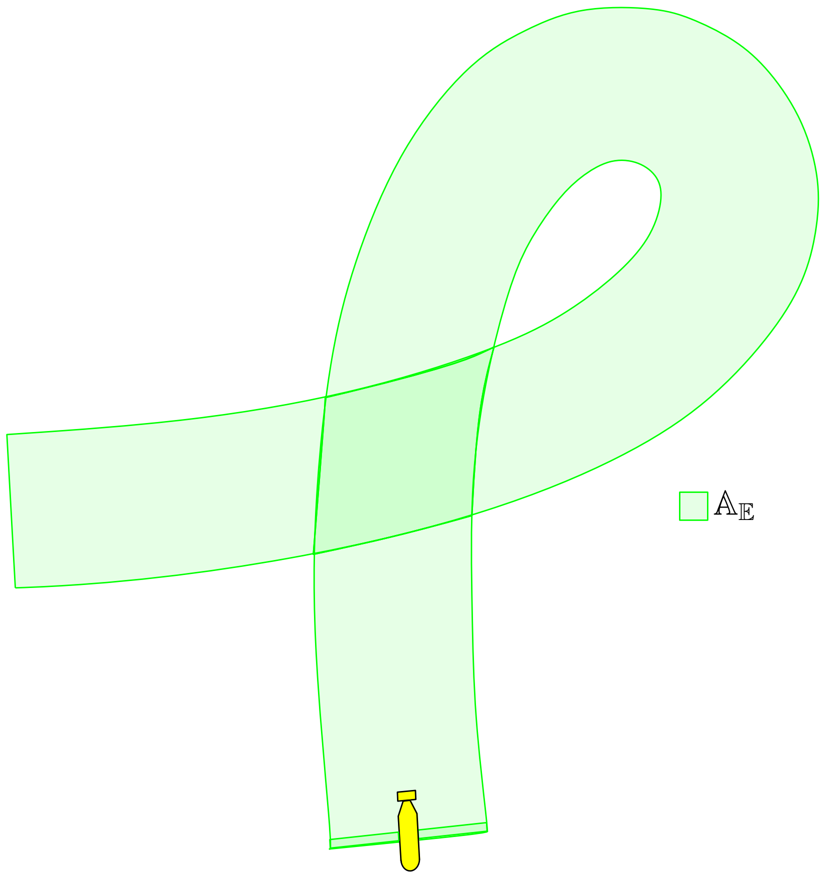
- $\mathbb{V}(x(t), \dot{x}(t))$ is the visible area at time t.



Problem Statement

$\mathbb{A}_{\mathbb{E}}$ is the explored area

$$\mathbb{A}_{\mathbb{E}} = \bigcup_{t \in T} \mathbb{V}(t)$$



Problem Statement

Entries

- $x(.)$, the robot's trajectory.
- $\dot{x}(.)$, the robot's trajectory derivatives.
- T, time interval.
- $\mathbb{V}(.)$, visible area

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- $\mathbb{V}(.)$, visible area

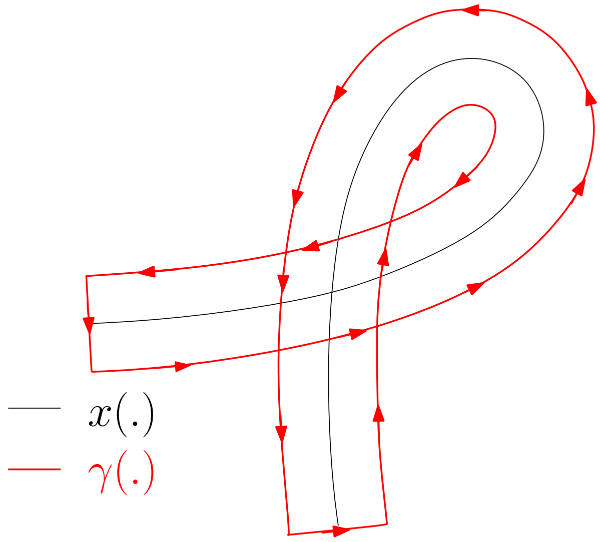
Desired Output

- Guaranteed approximation of the explored area $\mathbb{A}_{\mathbb{E}}$.
- Number of times each point in $\mathbb{A}_{\mathbb{E}}$ was seen during the mission.

Sonar Contour

From the robot's trajectory $x(.)$ and the knowledge of the range of visibility of each observation sensor, the sonar contour $\gamma(.)$ can be defined as illustrated.

- $\gamma(.) : \mathbb{R} \rightarrow \mathbb{R}^2$
- $T_\gamma = [0, 1]$
- $\gamma(.)$ is continuous in T_γ.
- $\gamma(0) = \gamma(1)$.



Parameterization

- $x(.) : \mathbb{R} \to \mathbb{R}^2$
- $T = [0, T_{max}]$
- $x(.)$ is continuous in T.

- $\gamma(.) : \mathbb{R} \to \mathbb{R}^2$
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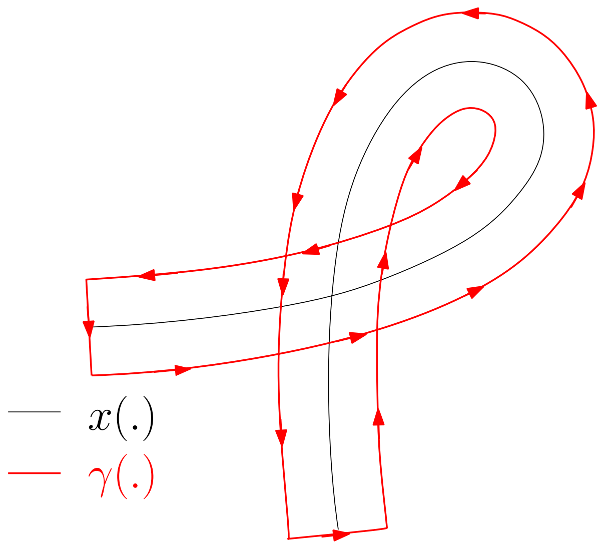


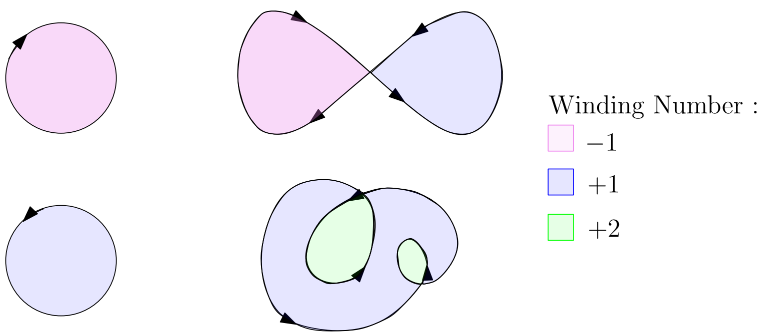
The parameterization of $x(.)$ and $\gamma(.)$ are not the same.

- $x(.)$ is parameterized by time $t \in T$.
- $\gamma(.)$ is parameterized by $\tau \in T_\gamma$.

Winding Number

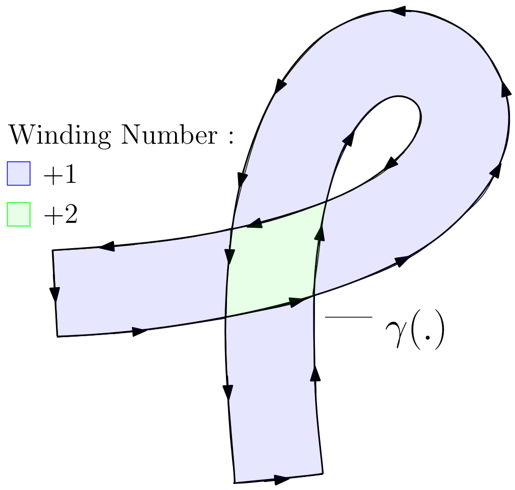
Winding Number

The winding number $\eta(\gamma(.), p)$ of a closed curve $\gamma(.)$ in the plane around a given point p is an integer representing the total number of times that curve travels counterclockwise around the point. [2]



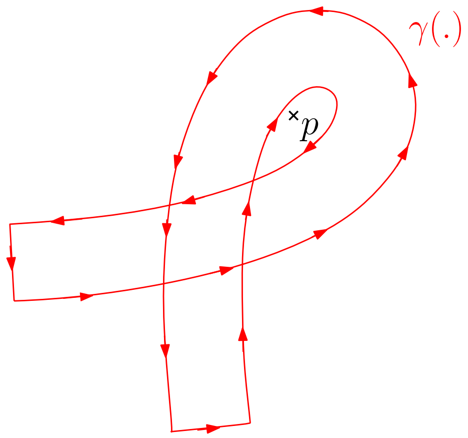
Problem Approach

$$\mathbb{A}_{\mathbb{E}} = \{z \in \mathbb{R}^2 | \eta(\gamma(.), z) \neq 0\}$$



Winding Number Calculation

Given a closed contour $\gamma(.)$ and a point p, what is the winding number $\eta(\gamma(.), p)$?



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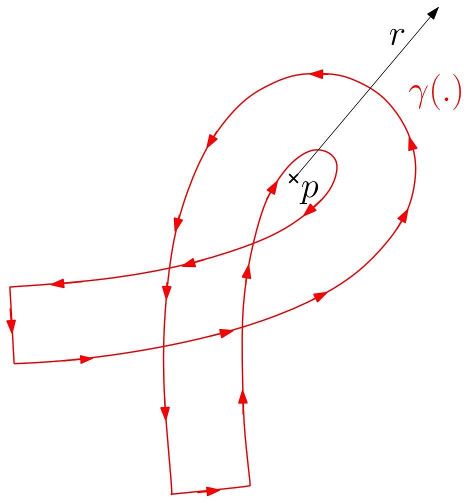

Dan Sunday's algorithm [4]

- Choose an infinite ray cast r from the point being checked .

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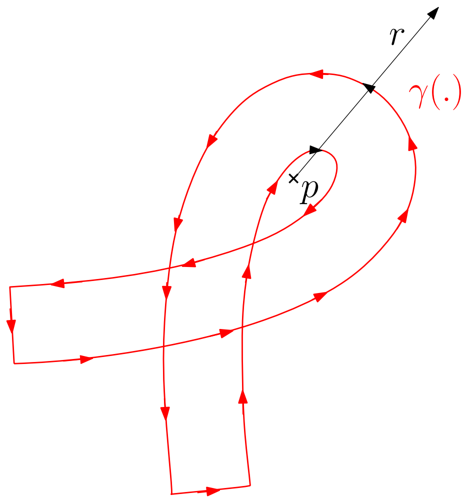

Dan Sunday's algorithm [4]

- Choose an infinite ray cast r from the point being checked .
- Identify intersections of r with $\gamma(.)$ and directions.

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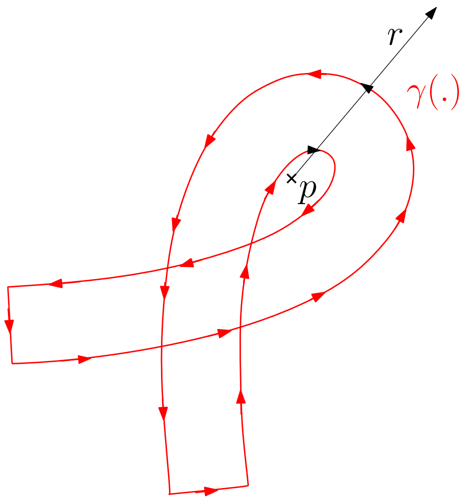

$\eta(\gamma(.), p) = 1 - 1 = 0$

Dan Sunday's algorithm [4]

- Choose an infinite ray cast r from the point being checked .
- Identify intersections of r with $\gamma(.)$ and directions.
- Compute $\eta(\gamma(.), p)$

Problem Approach

Entries

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Desired Output

- Guaranteed approximation of the explored area $\mathbb{A}_\mathbb{E}$.
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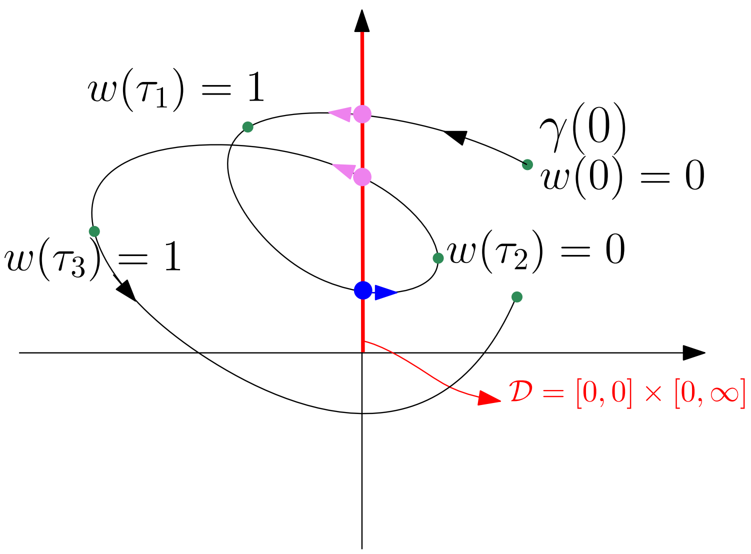
Proposed solution

$$\eta(\gamma(.), .)$$

Turning Number

- The objective of the turning number is to provide a sequential method for calculating the winding number w.r.t 0.
- $w(\tau) \in \mathbb{Z}$ is the turning number associated to $\gamma(\tau)$.

Turning Number

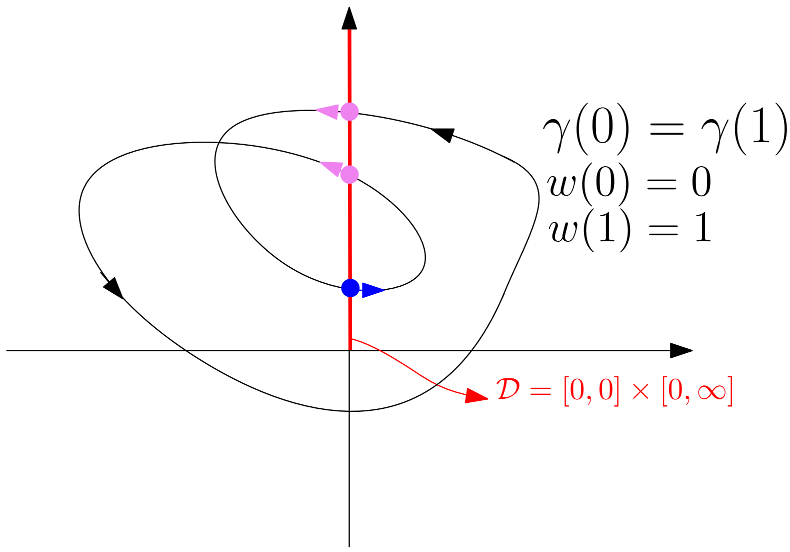


- $T^+(\tau) = \{t \in [0, \tau] \mid \gamma(t) \in \mathcal{D}$ and $\gamma_1(t - \delta) > 0\}$
- $T^-(\tau) = \{t \in [0, \tau] \mid \gamma(t) \in \mathcal{D}$ and $\gamma_1(t - \delta) < 0\}$
- $w(\tau) = \#T^+(\tau) - \#T^-(\tau)$

Problem Approach

The turning number allows a recursive calculation of the winding number.

$$\text{If } \gamma(0) = \gamma(1) \Rightarrow w(1) = \eta(\gamma(.), 0)$$



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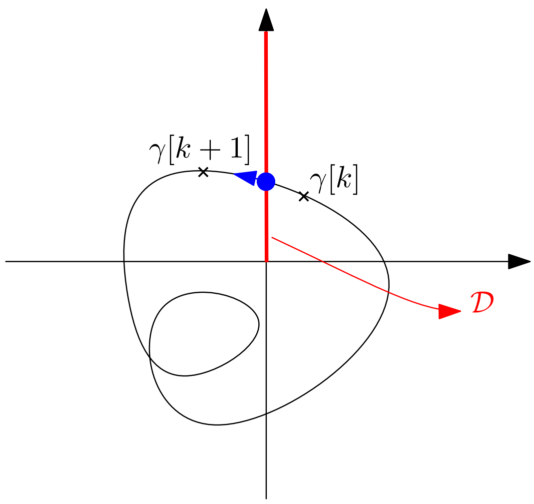
Implementation

Numerical Representation of the Contour

Numerical representation of the contour

Discretization

- $T_\gamma = [0, 1]$
- $\gamma(.)$ is continuous in T_γ.
- $\gamma(.)$ is evaluated at $\tau = \delta k$, where, $k = 0, 1, \ldots, \frac{1}{\delta}$ and $\delta \in \mathbb{R}$ is the discretization step.



Tubes

Definition

A tube is an envelope of trajectories, [3], [1] .

- $[\gamma](.) : \mathbb{R} \rightarrow \mathbb{IR}^n$
- $\gamma(.) \in [\gamma](.)$ if $\forall \tau \in T_\gamma$, $\gamma(\tau) \in [\gamma](\tau)$

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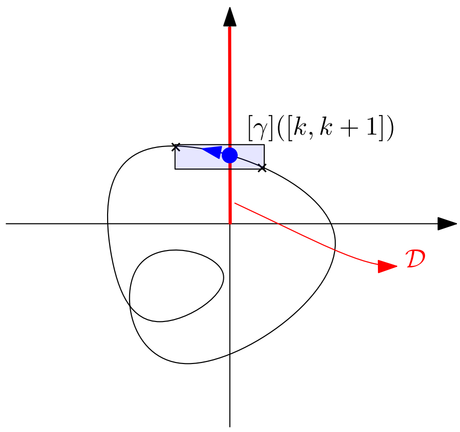
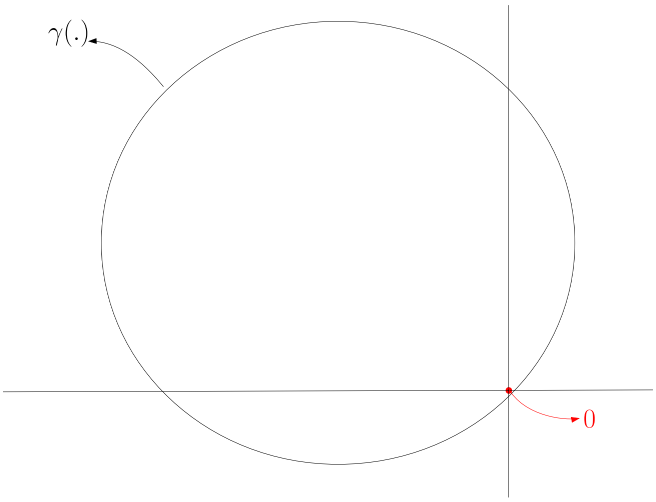
Example:
$[\gamma]([k, k+1])$ is the smallest box enclosing all solutions for $\gamma(\tau)$, such that, $\gamma(.) \in [\gamma](.)$ and $\tau \in [k\delta, (k+1)\delta]$.



Numerical representation of the contour

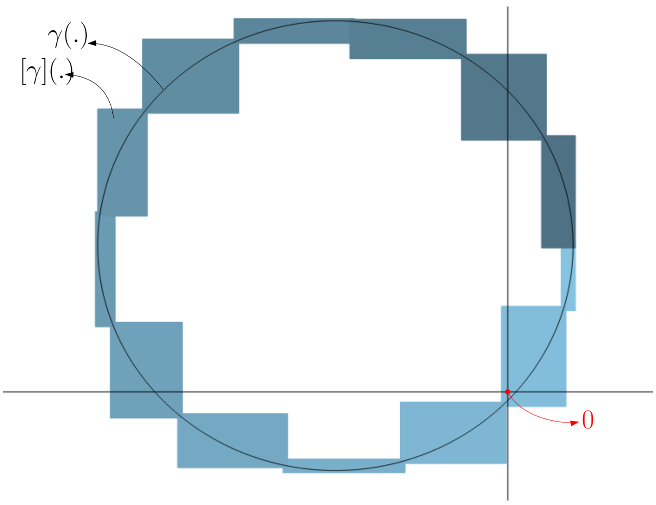
Example

- $\eta(\gamma(.), 0) \in \mathbb{Z}$



Numerical representation of the contour

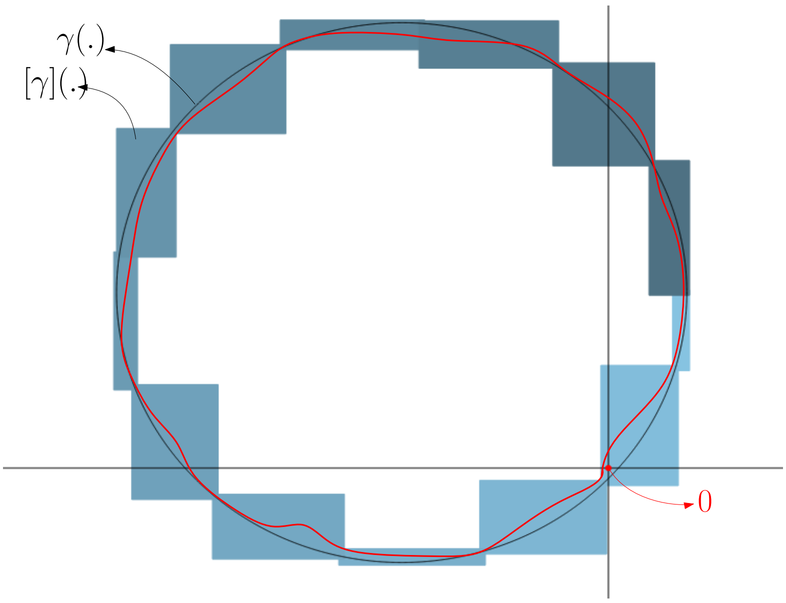
- $\gamma(.) \in [\gamma](.)$
- $\dot{\gamma}(.) \in [\dot{\gamma}](.)$
- $\eta([\gamma](.), 0) \in \mathbb{IZ}$
- $\eta(\gamma(.), 0) \in \eta([\gamma](.), 0)$



Numerical representation of the contour

Example

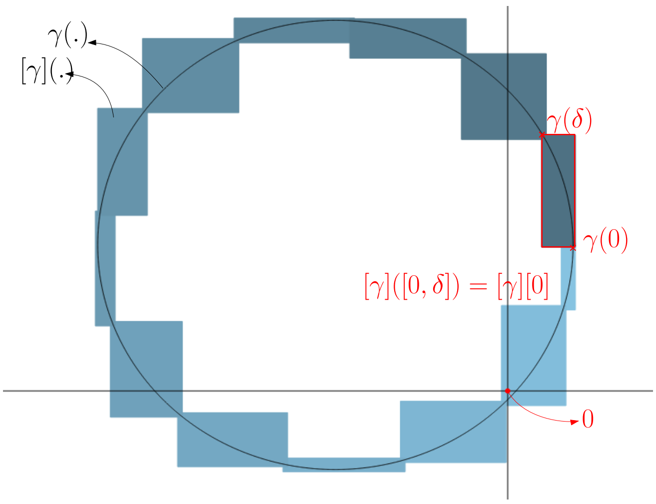
- $\eta([\gamma](.), 0) = [0, 1]$
- $\eta(\gamma(.), 0) = 1 \in \eta([\gamma](.), 0)$



Numerical representation of the contour

Discrete representation

$$[\gamma][k] = [\gamma]([k\delta, (k+1)\delta])$$
where, $k = 0, 1, \dots, \frac{1}{\delta} - 1$



Implementation
Algorithm

Algorithm

The winding number $\eta([\gamma](.), 0)$ can be sequentially computed using the turning number.

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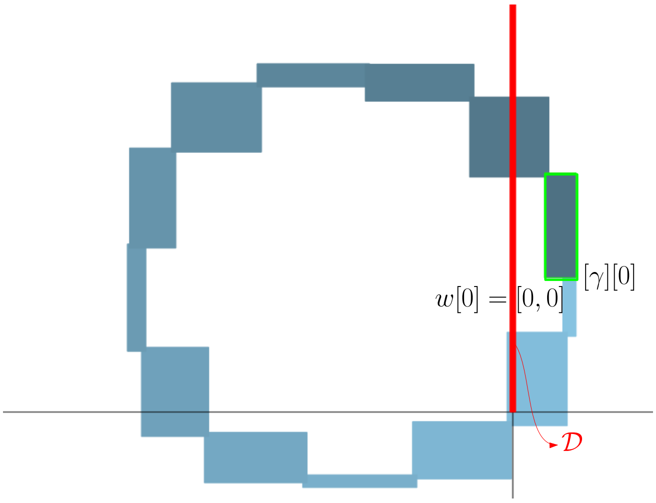
Initialization

- We assume that $[\gamma][0] \cap \mathcal{D} = \emptyset$

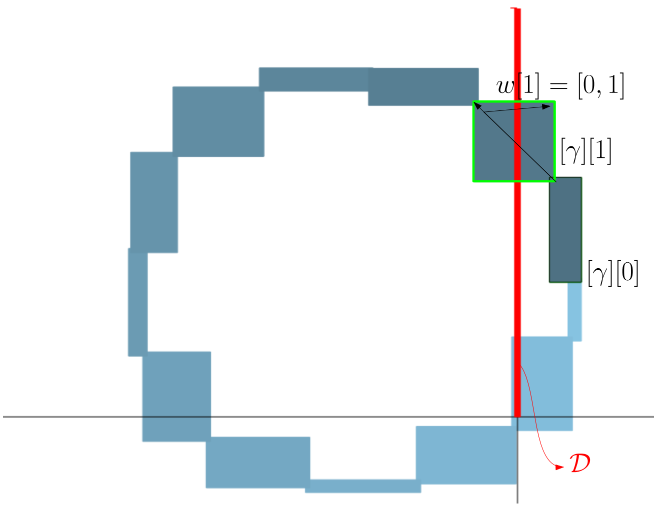
$$w[0] = [0, 0]$$



Algorithm

Rule 1

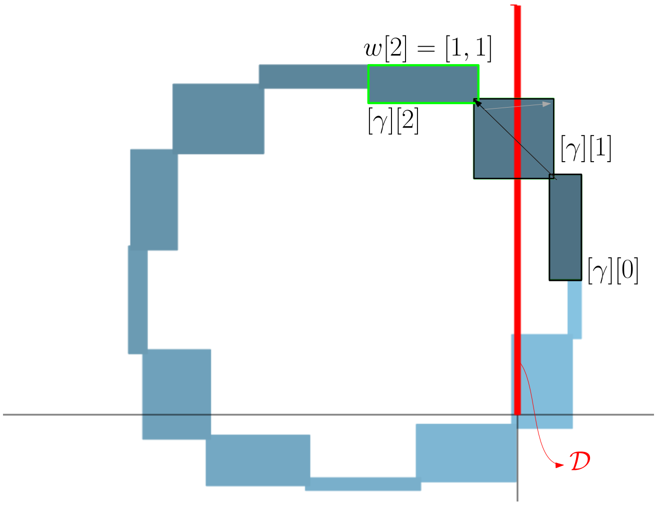
$$\left.\begin{array}{c} [\gamma][k] \cap \mathcal{D} \neq \emptyset \\ 0 \notin [\gamma][k] \\ [\gamma][k-1] \cap \mathcal{D} = \emptyset \\ [\gamma_1][k-1] \subset \mathbb{R}^+ \end{array}\right\} \quad w[k] = (w[k-1] + [0,1])$$



Algorithm

Rule 2

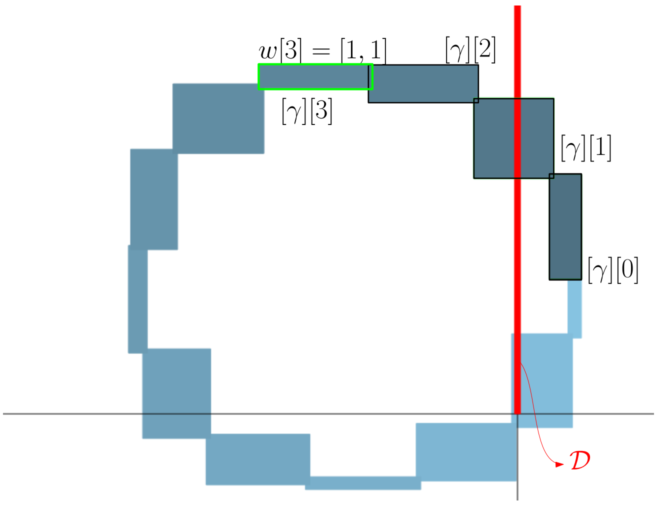
$$\left.\begin{array}{c}[\gamma][k]\cap\mathcal{D}=\emptyset\\ [\gamma_1][k]\subset\mathbb{R}^-\\ [\gamma][k-1]\cap\mathcal{D}\neq\emptyset\\ 0\notin[\gamma][k-1]\end{array}\right\}\quad [w][k]=[w^-[k-1]+1,w^+[k-1]]$$



Algorithm

Rule 3

$$\left.\begin{array}{r}[x][k] \cap \mathcal{D} = \emptyset \\ [x][k-1] \cap \mathcal{D} = \emptyset\end{array}\right\} \quad [w][k] = [w][k-1]$$



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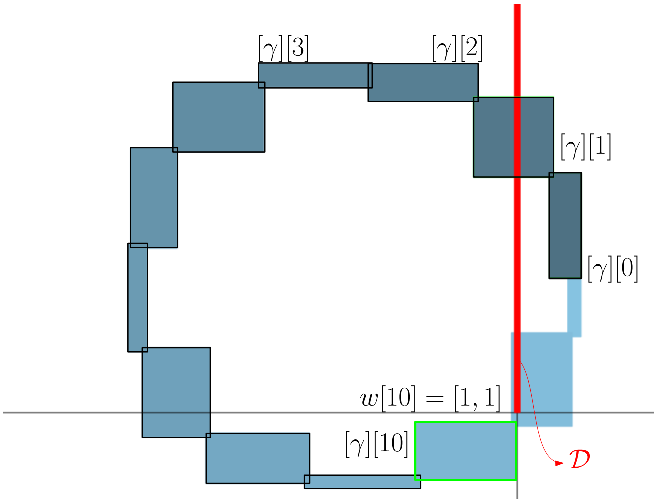
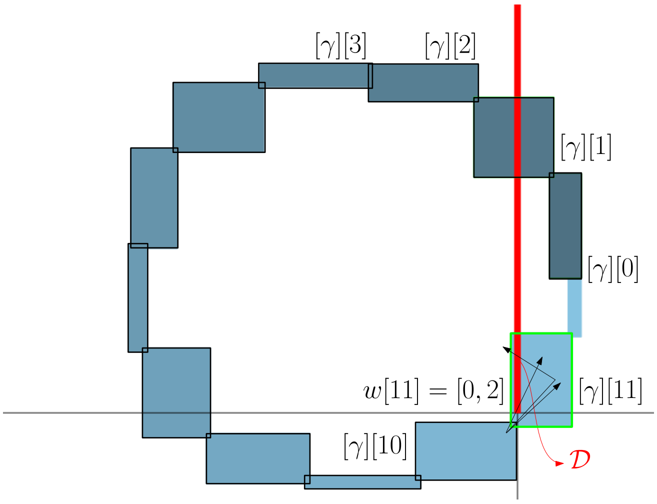


Algorithm

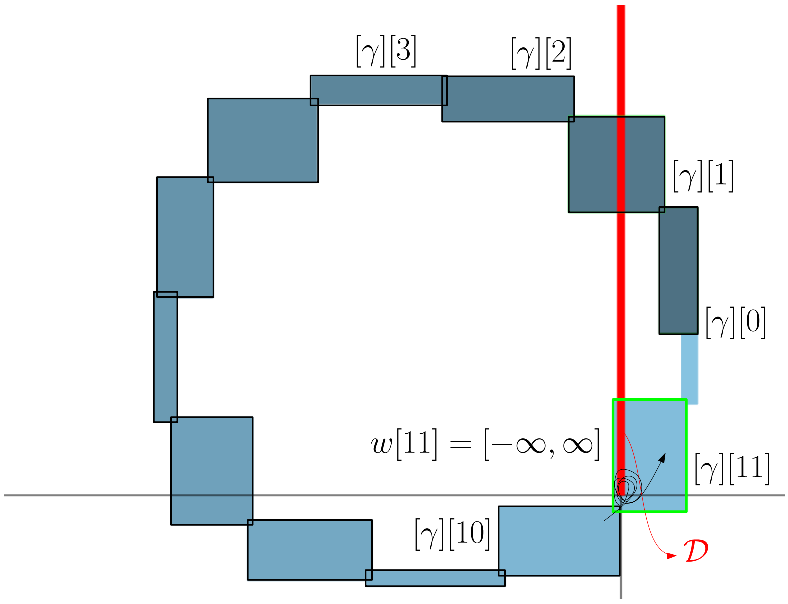
Rule 4

$$\left.\begin{array}{c}[\gamma][k-1]\cap\mathcal{D}=\emptyset\\ 0\in[\gamma][k]\\ 0\notin[\dot{\gamma}][k]\end{array}\right\}\quad [w][k]=[w][k-1]+[-1,1]$$



Algorithm

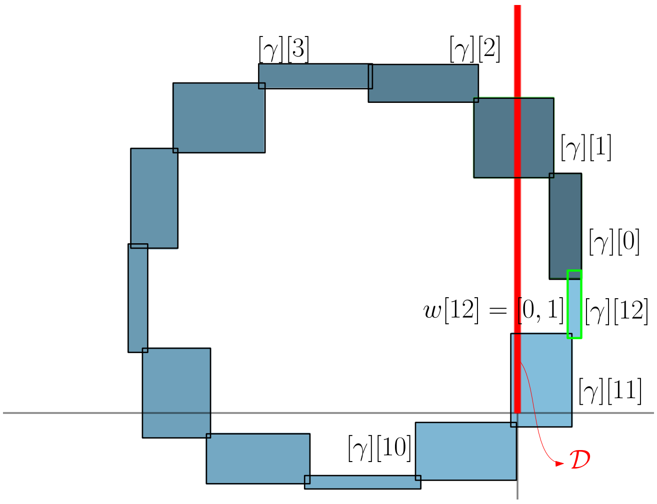
If $0 \in [\dot{\gamma}][k]$



Algorithm

Rule 5

$$\left.\begin{array}{c}[\gamma][k] \cap \mathcal{D} = \emptyset \\ [\gamma_1][k] \subset \mathbb{R}^+ \\ 0 \in [\gamma][k-1]\end{array}\right\} \quad [w][k] = [w^-[k-1], w^+[k-1]-1]$$



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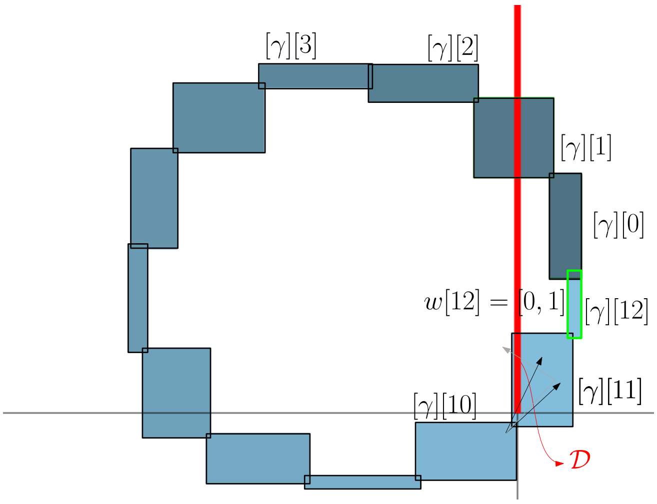

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$$\eta([\gamma](.), 0) = [0, 1]$$
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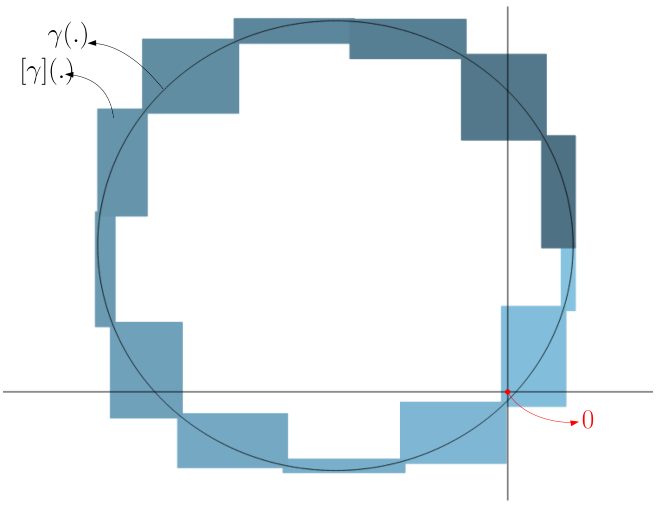
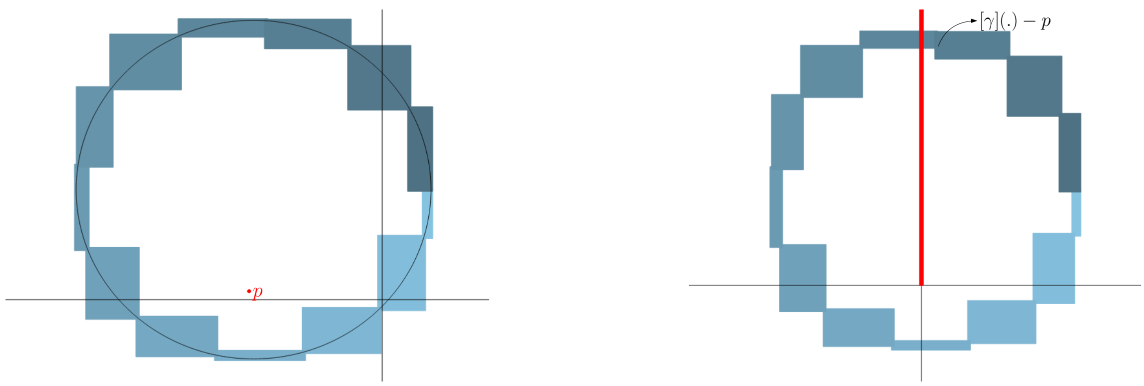


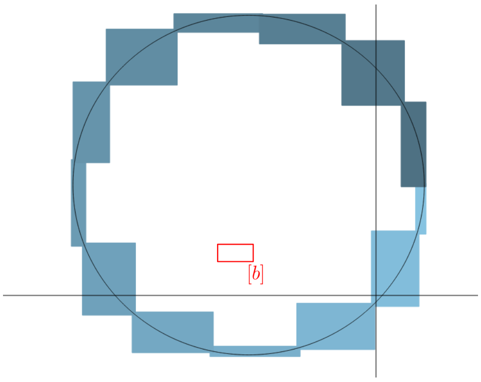
Algorithm

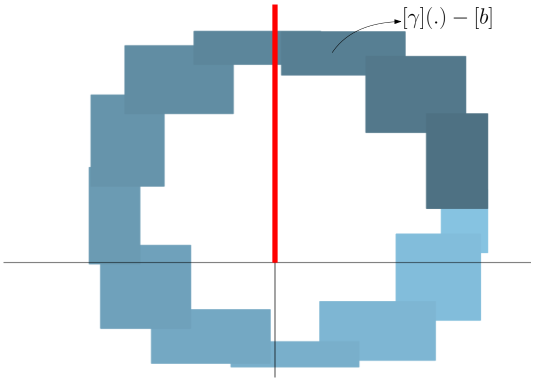
- $p \in \mathbb{R}^2$
  - $\eta([\gamma](.), p) = \eta([\gamma](.) - p, 0)$



Algorithm

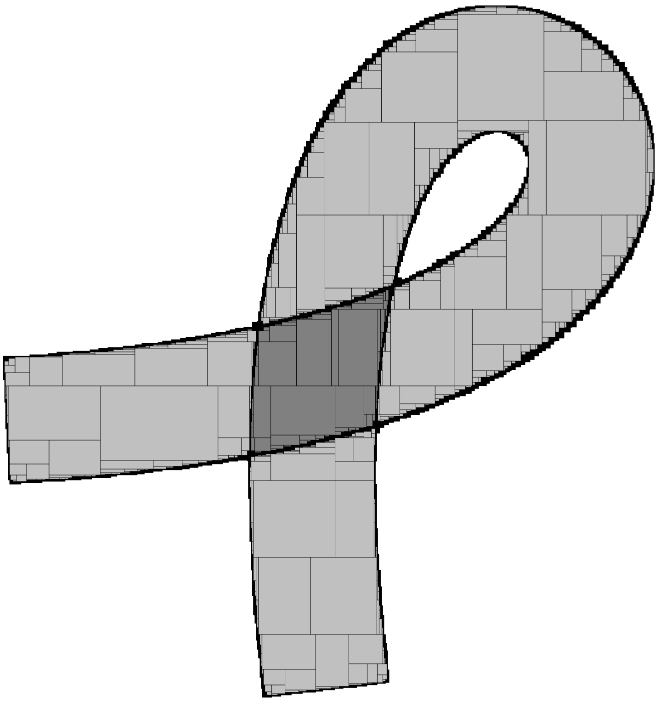
- $p \in \mathbb{R}^2$
  - $\eta([\gamma](.), p) = \eta([\gamma](.) - p, 0)$
- $[b] \in \mathbb{IR}^2$
  - $\eta([\gamma](.), [b]) = \eta([\gamma](.) - [b], 0)$





Results

$$\mathbb{A}_{\mathbb{E}} = \{z \in \mathbb{R}^2 | \eta(\gamma(.), z) \neq 0\}$$



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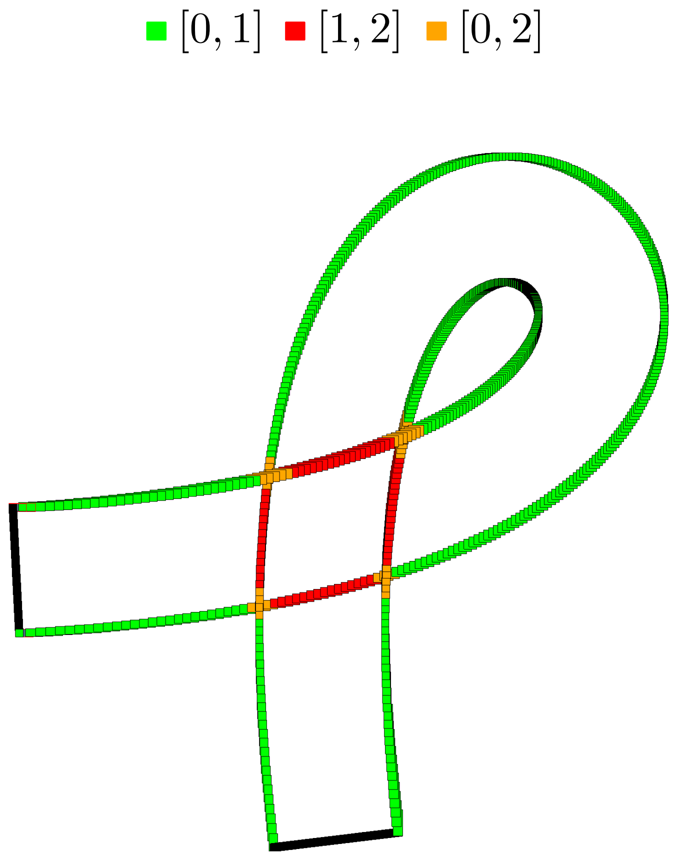

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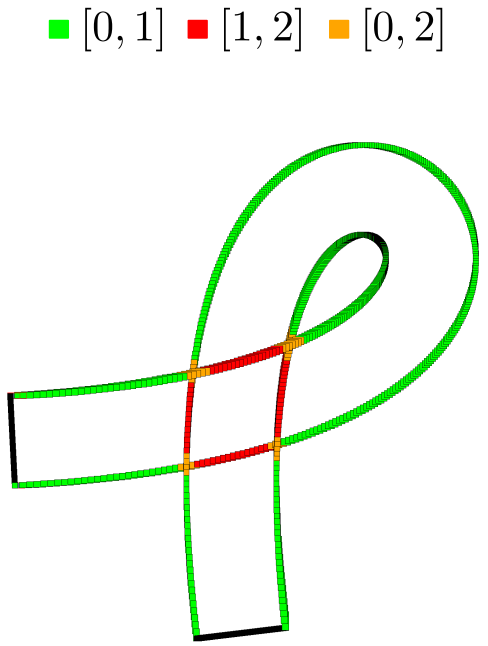

Detection of points of self intersection.

$$\eta^+ \geq \eta^- + 2$$

Conclusions

Using the topological properties of the robot's exteroceptive sensors contour, we are able to

- determine the are explored during a mission,
- determine the number of times a point in the space was in the robot's range of visibility.

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Using the topological properties of the robot's exteroceptive sensors contour, we are able to

- determine the are explored during a mission,
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The explored area can be quickly computed using the property of continuity of winding numbers in the space and through the identification of self intersections on the contour.

Future Work

- Add uncertainty to the robot's trajectory and therefore to the explored area.
- Explore the property of self intersection of the current algorithm.

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Solving non-linear constraint satisfaction problems involving time-dependant functions.
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[2] S. G. Krantz.
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[4] Dan Sunday.
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