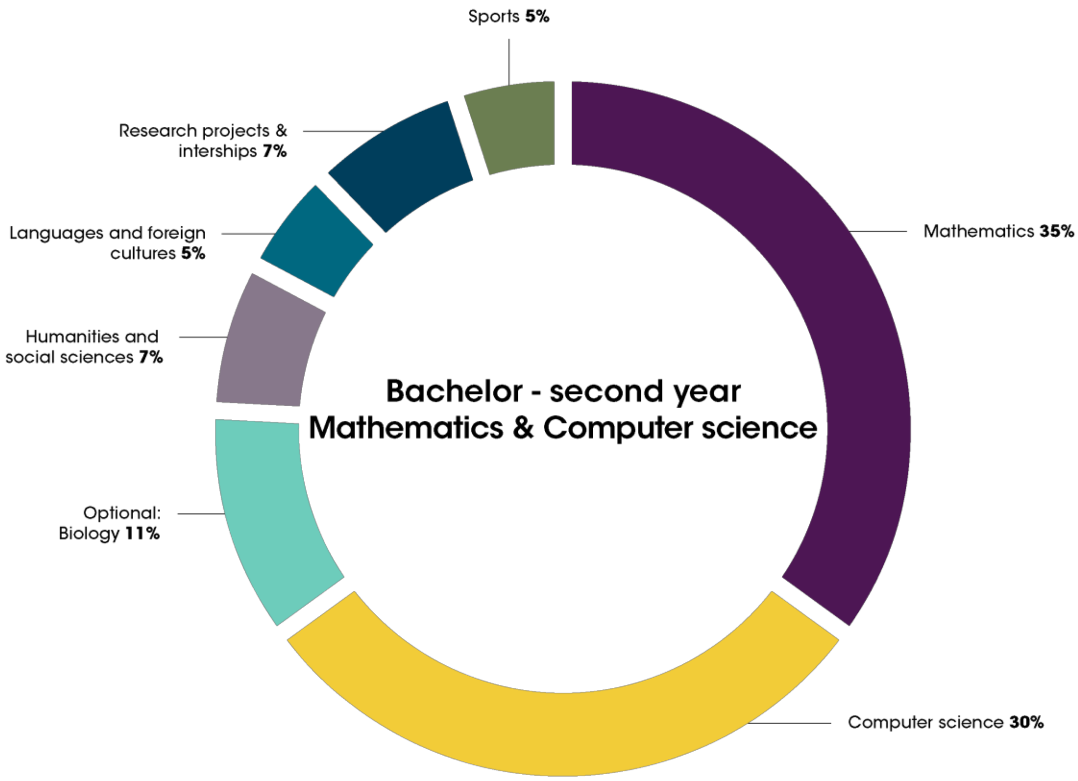
École Polytechnique Bachelor program

Preliminary syllabus
Mathematics & Computer Science - Year 2



- **Semester 3**

18 weeks of classes

Scientific courses:
14 hours of mathematics per week, 252 hours in total, 13 ECTS
10 hours of computer science, 180 hours in total, 9 ECTS
4 hours of optional scientific elective: biology, 72 hours in total, 2 extra ECTS

Other courses:
9 hours of general culture, including outings, 1 ECTS
2 hours of foreign languages, 36 hours in total, 2 ECTS
2 hours of humanities and social sciences, 36 hours in total, 1 ECTS
2 hours of sports, 36 hours in total, 2 ECTS

BIOLOGY (optional elective for semesters 3, 4, 5 and 6) (2 ECTS)

Courses built on experimental approaches

Experimentation is at the heart of progress in biology. The biology option is thus built around experimentation. Half of classes will be practical work, performed over weeks. This allows students to build their knowledge, acquire a sound scientific reasoning, and get familiar with the main techniques of modern biology.

Life sciences and their applications are undergoing a complete revolution. The amount of available data has tremendously scaled up, bringing exciting opportunities but also new challenges, which require skills and techniques from sciences, like informatics, physics or mathematics.

The Bachelor program provides excellent formation in these areas. The biology option offers the opportunity to add a strong corpus of knowledge in modern biology, paving the way to carriers in bioengineering or biotechnology. During the two years of major, students will follow courses on molecular biology, cellular biology and its integration at the organism level, genomics.

MATHEMATICS (13 ECTS)

Analysis

The course Analysis 201 is the logical continuation of Analysis 101 in the sense that it mainly develops further the concepts and techniques that were introduced in the latter course. More precisely, some additional notions from Topology are presented; still, the key idea to describe this course is the fact that the functions studied have now several real variables, a more realistic tool if one keeps in mind the goal of modeling phenomena from Physics. Both Differential Calculus and Integration will be covered here in this framework.

Algebra

The main goal of the course Algebra 201 is to provide additional tools in order to study more deeply linear maps between vector spaces; the goal is to obtain good models for these maps up to suitable equivalence. There are two possibilities, which are both covered in the course. This first one is to develop new concepts, for instance duality, in the general context of mere vector spaces. The second one is to enrich the structures of vector spaces to structures from bilinear algebra, which is available in most applications (e.g. in Quantum Mechanics).

Probability

The topics covered in this course will be the following:

Elementary probabilities, discrete and continuous random variables; discrete sample spaces and discrete random variables, expectation, moments, generating function; real random variables, distribution function and density functions, simulation of random variables Moments, change of variables; inequalities (Markov, Bienaymé Chebyvhev, Jensen); concentration inequality. Random vectors, multivariate distribution, moments, variance and covariance; joint density function, marginal density functions, sums of continuous random variables, conditional density functions, Gaussian random vectors; weak law of large numbers simulation of random variables.

Statistics of finite samples

The topics covered in this course will be the following:

Statistical models, goals and performance criteria: data, models, parameters and statistics, decision theoretic framework; exponential family: the one parameter case, the multiparameter case, properties, sufficient statistics, methods of estimation: basic heuristics (minimum contrast estimates, estimating functions, the plug-in and extensions), least-squares and weighted least-squares.

Maximum likelihood in multiparameter exponential family, algorithmic issues (coordinate ascent, the EM algorithm); Measures of performance, unbiased estimation and risk inequalities, the information inequality, Testing and confidence region: the Neyman-Pearson paradigm, uniformly most powerful test and monotone likelihood ratio models, confidence bounds, intervals and regions, the duality of confidence intervals and hypothesis tests, likelihood ratio procedure, p-value, survey sampling.

Computational mathematics

COMPUTER SCIENCE (9 ECTS)

Object-oriented programming in C++

C++ is one of the most widely-used programming languages in the world, especially for system-level programming. Much of its power derives from its use of objects, packets of data and functionality that model things and concepts in the real world. This course will introduce students to the C++ programming language, and the object-based view of software design.

Design and analysis of algorithms

Algorithms are at the heart of all computation. Following on from the introduction to algorithms in CS112, this course provides a solid foundation in modern algorithmics. Students will develop a deeper knowledge of the fundamental algorithms, an understanding of how they work, and an appreciation of how to implement them efficiently. Crucially, they will learn how to reduce other problems to these fundamentals.

Logic and proofs

This course is an introduction to logic (propositional calculus, first-order logic, deduction systems, computer-aided proofs/proof assistants).

HUMANITIES AND SOCIAL SCIENCES (2 ECTS)

History of Science and Technology

This course aims at enhancing students' understanding of science in relation to social concerns through an initiation to the methods and issues of contemporary researches on the history of science and technology. The course will offer both thematic and chronological approaches to the evolutions of sciences in various societies from the antiquity to modern times:

- A critical approach to the "scientific revolution" of the 17th Century;
- Galileo and the interplay between observation, experience and mathematics;
- The various notions of "science" in the antiquity (Egypt, Babylon, India, and Greece);
- Chinese algorithms: *The Nine Chapters on the Mathematical Art*;
- The geometry of Euclid's *Elements*;
- From mechanics to digital representations of life;
- Science for the people, science for the state from the Enlightenment to the French revolution;
- Science, industry and society in the 19th Century;
- The Big science of the 20th Century.

- **Semester 4**

18 weeks of classes

<u>Scientific courses:</u>
12 hours of mathematics, 216 hours in total, 11 ECTS
12 hours of computer science, 216 hours in total, 11 ECTS
4 hours of optional scientific elective: biology, 72 hours in total, 2 extra ECTS

<u>Other courses:</u>
9 hours of general culture, 1 ECTS
2 hours of foreign languages, 36 hours in total, 2 ECTS
2 hours of humanities and social sciences, 36 hours in total, 1 ECTS
2 hours of sports, 36 hours in total, 2 ECTS

<u>MATHEMATICS (11 ECTS)</u>

Analysis

The course Analysis 202 exploits the new notions from Topology to study in further details functions. For example, one is often led to study functions as limits of simpler ones (e.g. for approximation problems): this is made possible in a rigorous manner thanks to topological ideas. This provides the possibility to use crucial tools in many scientific fields, the most striking one being presumably Fourier series (first designed to solve the heat equation and now ubiquitous in science and, in a hidden manner, in every day life). The second part of the course deals with a much wider class of Differential Equations than the one considered in Analysis 102, opening the way to more complex physical questions.

Algebra

The course Algebra 202 starts from the second part of Algebra 201 in order to study objects from bilinear algebra for their own sake. This is motivated by the fact that these objects, mainly quadratic forms, have themselves fundamental applications (e.g. in Number Theory and Mechanics), but also because they lead to algebraic objects, for instance some special groups of matrices, whose applications in Mathematics and Physics are fundamental, from Number Theory and Geometry to classification of particles.

Computational mathematics

<u>COMPUTER SCIENCE (11 ECTS)</u>

Machine learning

This course describes some of the methods and algorithms used in contemporary machine learning, with a variety of scientific applications. When brought up to scale, this becomes an important part of what is now called Big Data.

Computer architecture

This course investigates the design and organisation of computers at their lowest level. This encompasses computer hardware, and also the operating systems that provide an interface between most programs we write and use with the underlying machine and its network.

Introduction to formal languages

This course introduces different concepts in automata theory and formal languages, including formal proofs, deterministic and non-deterministic automata, regular expressions, regular languages, context-free grammars and languages, and Turing machines.

Introduction to networks

This course will introduce an architecture, a skeleton, for implementing a protocol - and part of that will include creating multiple threads, and synchronized queues.

HUMANITIES AND SOCIAL SCIENCES (2 ECTS)

Philosophy: Science and Technology

The relations between humanity and the world have been substantially transformed by sciences and technology. First, the advances in knowledge have made the world less mysterious and more rational: never before we had as much knowledge about matter and the living. Second, technology has become part of everyone's everyday life, to such an extent that it now seems to be second nature (computers, communication tools, robots, etc.) Yet this achievement has its drawback: it is getting more and more difficult to speak of science in the singular since disciplines became very specialised and the intrusion of technology into life is often accused of breaking the links with nature. A philosophical approach sheds some light on the paradox of a science of which we acknowledge the advances and fear at the same time the discoveries. Through the study of modern and contemporary philosophers, this course examines the expectations placed in science and enables to understand the disappointments they have sometimes caused. On the basis of examples (the birth of modern physics, the increasing power of biology, the emergence of "information societies"...) we will study how scientific reason works. We will also show how the philosophers who have dealt with these technical-scientific changes help us understand their impact on our life.

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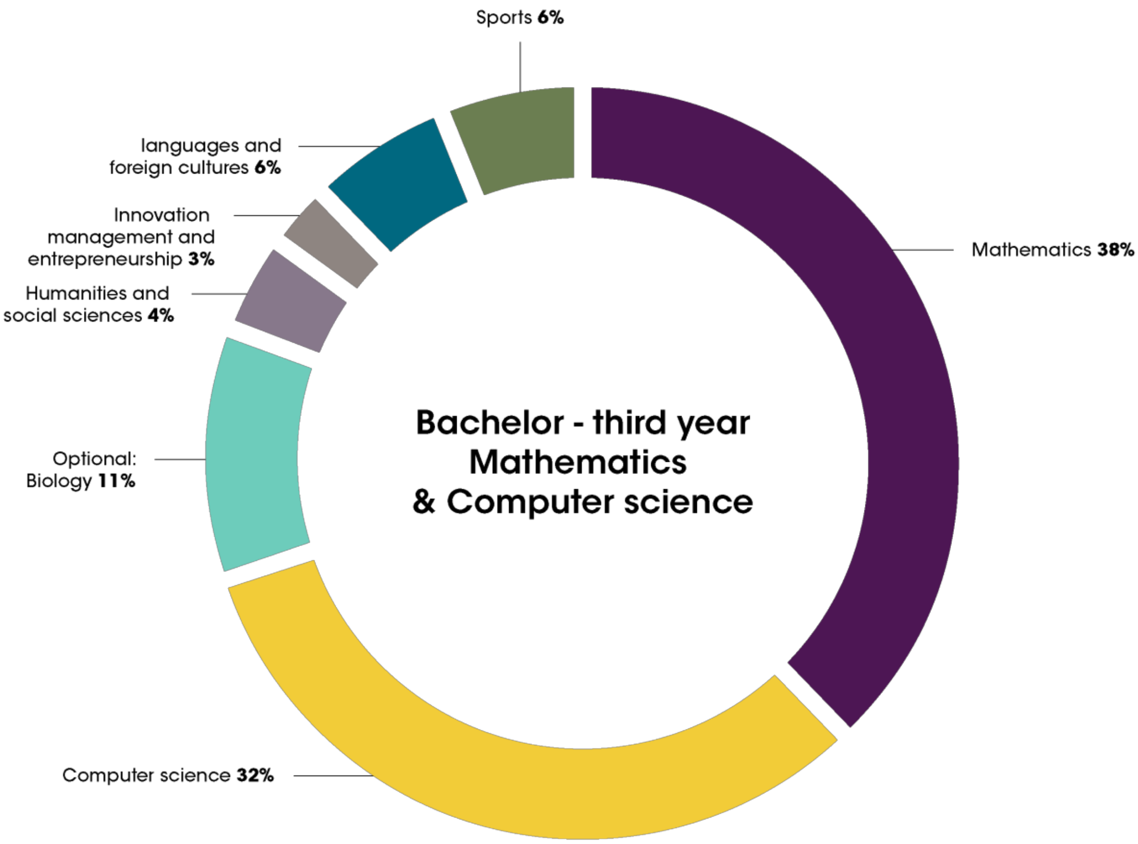

- **Semester 5**

14 weeks of classes

<u>Scientific courses:</u>
14 hours of mathematics per week, 196 hours in total, 14 ECTS
10 hours of computer science, 140 hours in total, 10 ECTS
4 hours of optional scientific elective: biology, 56 hours in total, 2 extra ECTS

<u>Other courses:</u>
7 hours of general culture, 1 ECTS
2 hours of foreign languages, 28 hours in total, 2 ECTS
2 hours of humanities and social science, 28 hours in total, 1 ECTS
2 hours of sports, 28 hours in total, 2 ECTS

MATHEMATICS (14 ECTS)

Measures and integration

The topics covered in this course will be the following:

Measure and Lebesgue integral on R, Finite measure and (abstract) Lebesgue integral \pi, \lambdas systems, monotone classes Convergence monotone; product measure and Fubini's theorem Lp spaces; convergence of random variables: in probability, in Lp, almost surely. Borel-Cantelli theorem; strong law of large numbers; applications; characteristic functions; convergence in law; Paul levy's theorem (without proof); central limit theorem; applications; Monte-Carlo method. L2 space; Hilbert spaces; projection.

Topology and convex optimization

The topics covered in this course will be the following:

Metric spaces, compactness, projection on Hilbert spaces**;** Duality, weak and weak topologies, basic functional analysis tools c onvex sets and their topological properties**;** Convex functions**;** Hahn-Banach, min-max, Polarity, convex duality Convexity and differentiability
Convex optimization (Kuhn-Tucker), linear programming, numerical optimization

Probability and statistics

Stochastic processes
The course will present a part of the theory but also (and mostly) many applications in various areas. A large part of the course will be devoted to modeling. Markov chains Poisson Process Renewal Process; Pure jump Markov Process

Asymptotic statistics

Convergence of random variables (convergence in probability, convergence in distribution, almost sure convergence). Fundamental theorems in statistics: weak and strong law of large numbers, central limit theorems; Lindeberg-Levy theorems, Uniform law of large numbers. Manipulation of convergence (Slutsky lemma, delta-method, variance stabilizing transforms); pivotal functions; confidence regions; estimation; Z-estimation; construction; asymptotic theory (consistency, uniform law of large numbers; asymptotic normality). Maximum likelihood: construction, examples, efficiency - An introduction to linear and logistic regression. Test; likelihood ratio test; asymptotic distribution, rate of separation, multiple tests. Generalized Likelihood ratio test; asymptotic theory; applications to variable selections.

Algebras and arithmetics

COMPUTER SCIENCE (10 ECTS)

Advanced C++ programming

C++ is one of the most important programming languages today. Originally designed as a systems programming language, it is also widely used for embedded and high-performance computing.
This course is an introduction to the C++ programming language, providing the tools to design and develop large software applications.

Compilers

Compilation is the process of transforming high-level programs and abstractions into the binary machine code used in computer processors. This course introduces the principles and techniques of compilation, with parsers, interpreters, and translators, as well as topics in code optimisation and semantic analysis. Students will build a compiler for a simple programming language.

Computer science project

HUMANITIES AND SOCIAL SCIENCES (2 ECTS)

Masterpieces of French and Western Literature

This course offers students the opportunity to engage in study and discussion of some of the most significant texts of Western literature. Its purpose is to give an understanding of the works and of their relation to the values of the time and the region in which they occur. Some of the historical and critical issues commonly addressed in this course are the representation of reality in literature, changing views of human moral problems, and the building of Western culture including historical and national differences. In order to introduce to French culture, a significant part of the studied works will be French.

- **Semester 6**

10 weeks of classes, 8 weeks for the Bachelor Thesis

Scientific courses:
12 hours of mathematics, 120 hours in total, 6 ECTS
12 hours of computer science, 120 hours in total, 6 ECTS
4 hours of optional scientific elective: biology, 40 hours in total, 2 extra ECTS

Other courses:
5 hours of general culture, 1 ECTS
2 hours of foreign languages, 20 hours in total, 1 ECTS
2 hours of innovation management and entrepreneurship, 20 hours in total, 1 ECTS
2 hours of sports, 20 hours in total, 1 ECTS

MATHEMATICS (6 ECTS)

Algebra & Geometry

The topics covered in this course will be the following:

Basics of algebraic structures, Groups of isometries, Linear groups on real and complex numbers; Basics of linear representations; Basics of differential geometry.

Signals & Systems

The topics covered in this course will be the following:

Signals and Systems, frequencies, spectral representation, scale Filters and transfer functions
Periodic signals and Fourier series, the discrete Fourier series, the Fast Fourier transform, periodic convolution with the DFT.

Fourier transform of integrable functions, the inverse Fourier transform, the Schwartz class, Convolution of functions, convolution, derivation and regularization; Fourier transform in $2(\mathbb{R})$ (Plancherel) convolution and the Fourier transform of $LL^2(\mathbb{R})$. Continuous-time systems, generalized solutions of differential equations, stability, realizability, gain and response time, examples.

Sampling and discrete time filters; Sampling and Poisson's formula; Shannon sampling theorem; Discrete filters and convolution: discrete signals and systems, the convolution of discrete signals, causality and stability of a discrete filter; zz-transform and discrete filters; Time-frequency analysis: the windowed Fourier transform, Gabor's formula; Wavelet analysis: basic construction, the wavelet transform, orthogonal wavelets, multiresolution analysis.

Introduction to discrete time random signals: strict-sense and weak-sense stationarity, autocorrelation function, linear filtering of stationary processes, spectral density and harmonizability. Linear prediction

Applications: speech processing; image restoration; coding; computational mathematics. Simulation; statistics; introduction to PDEs; numerical optimization.

Computational mathematics

COMPUTER SCIENCE (6 ECTS)

Concurrent and distributed computing

Today's programs and calculations operate not on one computer at a time, but rather on groups of processors or machines working together in concert. But ensuring efficiency and cooperation among the threads of a program is a deeply subtle, and fascinating, problem. This course aims to provide the techniques required to master efficient distributed programming, avoiding the many pitfalls that arise when computations share their resources.

Complexity

Theoretical Computer Science has shown that computational problems can be classified according to how difficult they are to solve. We now know that some problems are intrinsically impossible to solve in a reasonable amount of time, or with a reasonable amount of resources. This course describes the rigorous model of computation required to compare and classify computational problems and their difficulty, giving an introduction to the theory of computational complexity and the standard complexity classes.

Computer graphics

This course explores fundamental concepts in 2D and 3D computer graphics, including digital images, 2- and 3-dimensional geometry, curves and surfaces, perspective, ray tracing, filtering and antialiasing, the graphics pipeline, and human visual perception.