Directed Algebraic Topology and Concurrency

Emmanuel Haucourt

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MPRI : Concurrency (2.3)

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Paradigm

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- `proc` <identifier> = <basic block>
- `init` <multiset of identifiers>
  e.g. `init a 2b 3c`

Expressions and values

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$\wedge$	minimum	$\vee$	maximum
$+$	addition	$-$	substraction
$*$	multiplication	$/$	division
$\leqslant$	less or equal	$\geqslant$	greater of equal
$<$	strictly less	$>$	strictly greater
$=$	equal	$\neq$	not equal
$\neg$	complement	$\%$	modulo
$\perp$	bottom		

nullary	unary
$\perp$, $x \in \mathbb{R}$, $v \in \mathcal{V}$	$\neg$
binary	
$\wedge, \vee, +, -, *, /, <, >, \leqslant, \geqslant, =, \neq, \%$	

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- `(`*L*`)` enclose a list of instructions between parenthesis to make it a single instruction

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- If all the expressions evaluate to zero, then L_{n+1} is triggered.

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Due to the branchings, basic blocks are actually trees.

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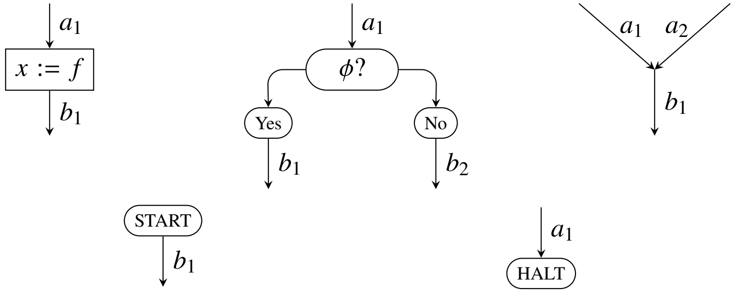
- No such structure exist for parallel programs.

Generators



The Hasse-Syracuse algorithm in PAML

```
var x = 7
```

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proc p = ()+[x=1]+J(q)

proc q = (x:=x/2) + [x%2=0] + (x:=3*x+1) ; J(p)
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init p
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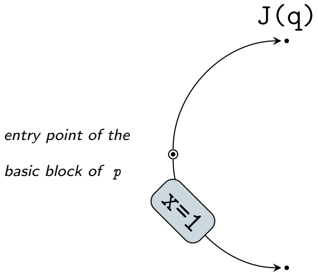

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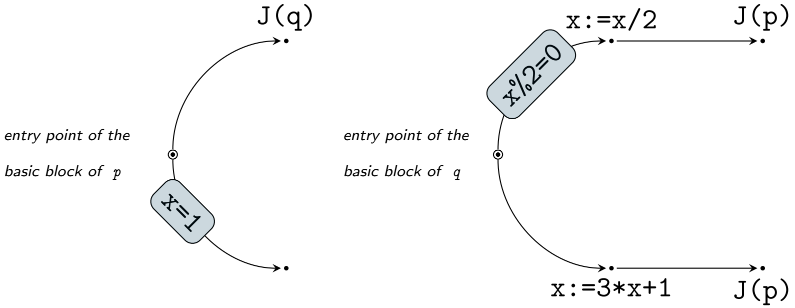

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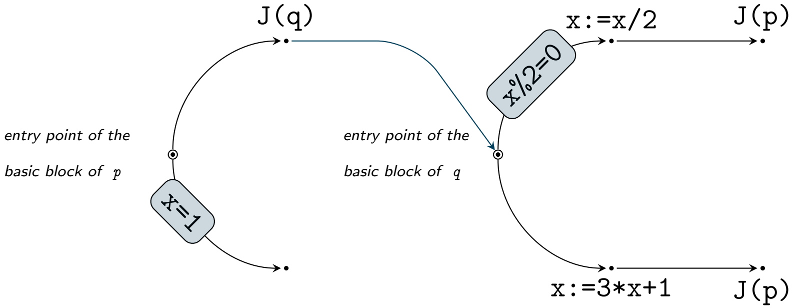

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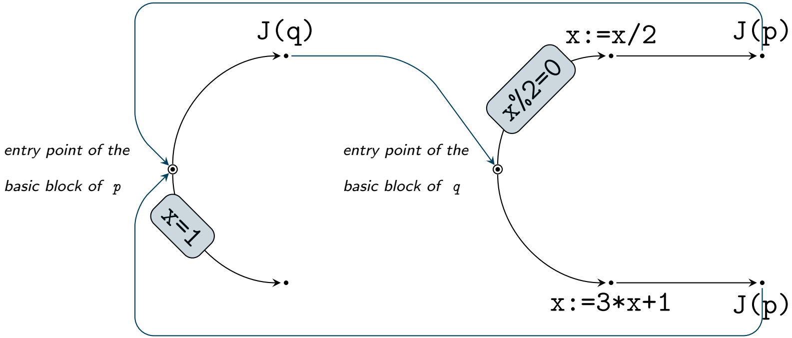

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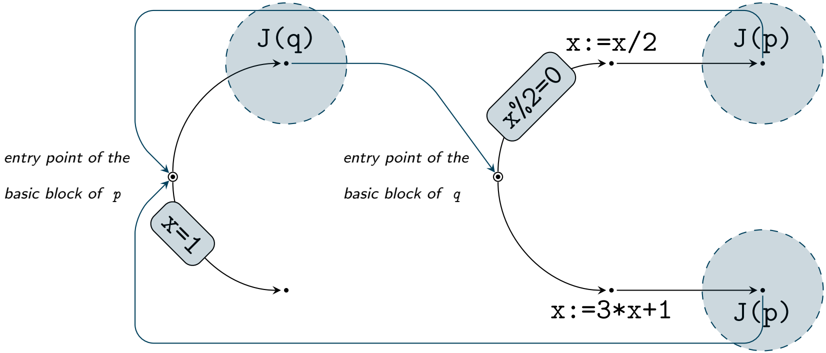

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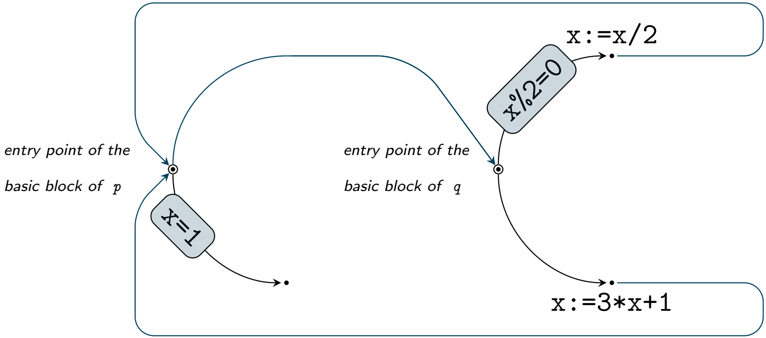

Reducing the Control Flow Graph

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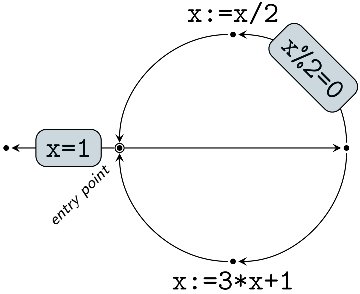

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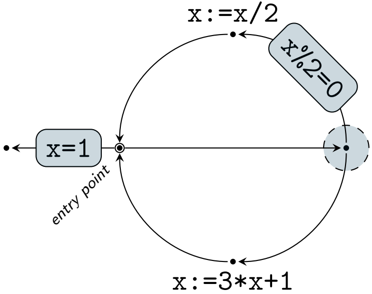

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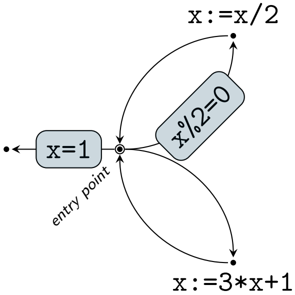

An execution trace on a control flow graph

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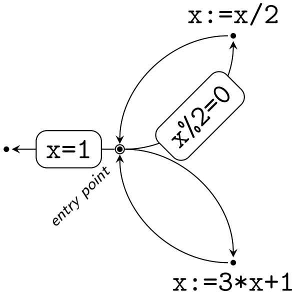

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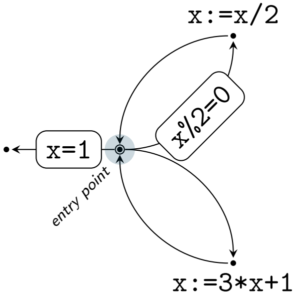

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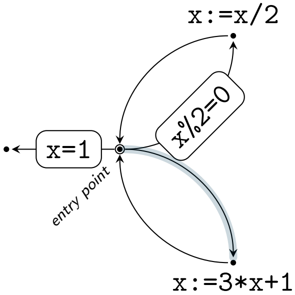

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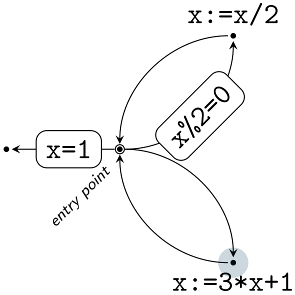

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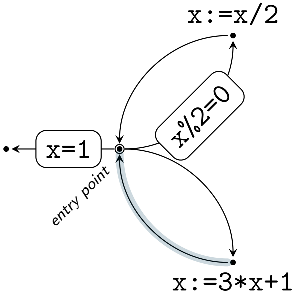

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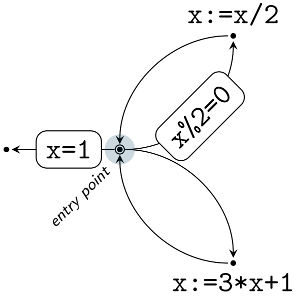

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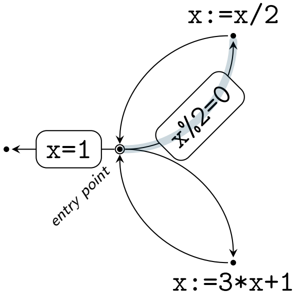

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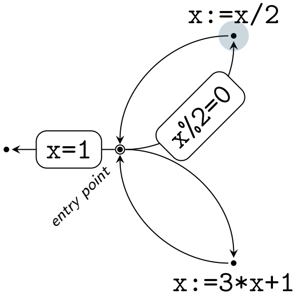

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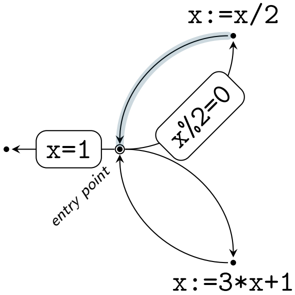

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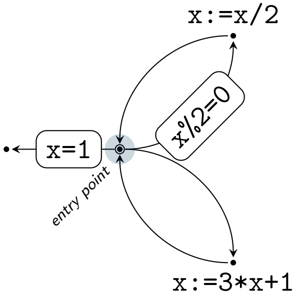

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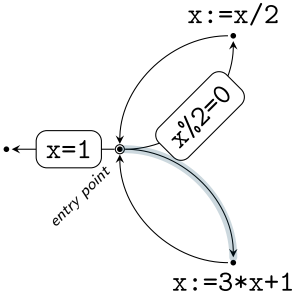
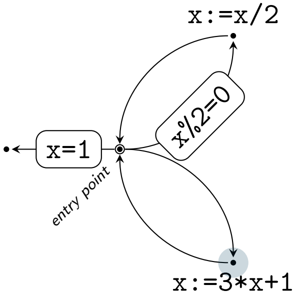


the current value of `x` is 11

An execution trace on a control flow graph

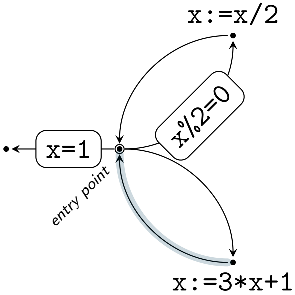
of the Hasse-Syracuse algorithm



the current value of x is 34

An execution trace on a control flow graph

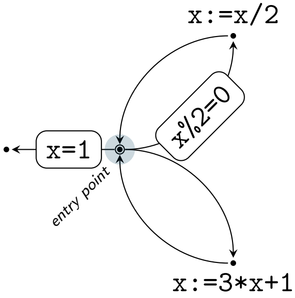
of the Hasse-Syracuse algorithm



the current value of x is 34

An execution trace on a control flow graph

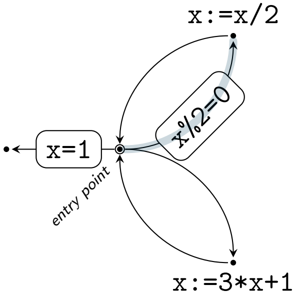
of the Hasse-Syracuse algorithm



the current value of `x` is 34

An execution trace on a control flow graph

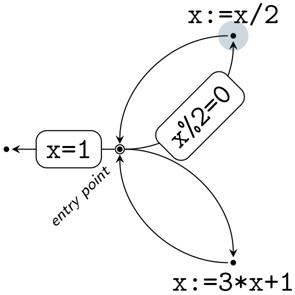
of the Hasse-Syracuse algorithm



the current value of `x` is 34

An execution trace on a control flow graph

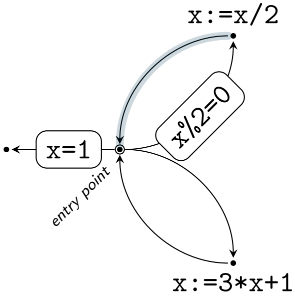
of the Hasse-Syracuse algorithm



the current value of x is 17

An execution trace on a control flow graph

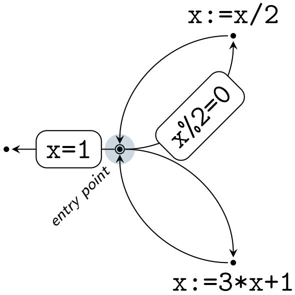
of the Hasse-Syracuse algorithm



the current value of x is 17

An execution trace on a control flow graph

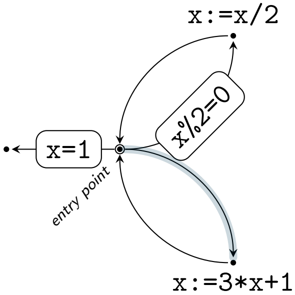
of the Hasse-Syracuse algorithm



the current value of x is 17

An execution trace on a control flow graph

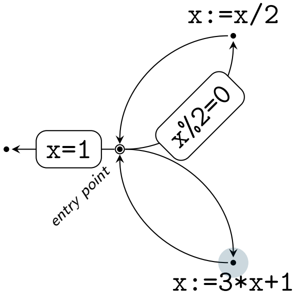
of the Hasse-Syracuse algorithm



the current value of x is 17

An execution trace on a control flow graph

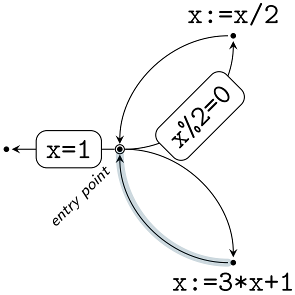
of the Hasse-Syracuse algorithm



the current value of x is 52

An execution trace on a control flow graph

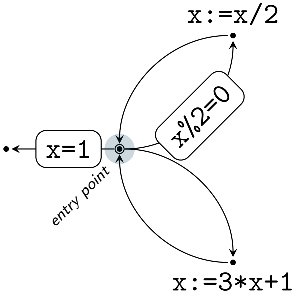
of the Hasse-Syracuse algorithm



the current value of `x` is 52

An execution trace on a control flow graph

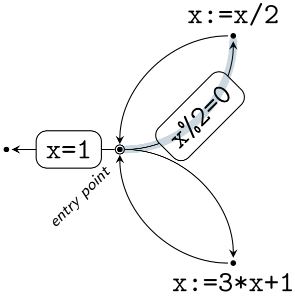
of the Hasse-Syracuse algorithm



the current value of x is 52

An execution trace on a control flow graph

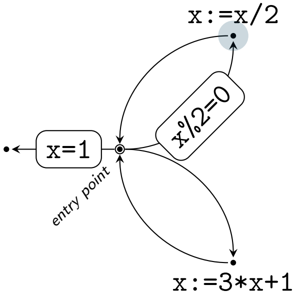
of the Hasse-Syracuse algorithm



the current value of x is 52

An execution trace on a control flow graph

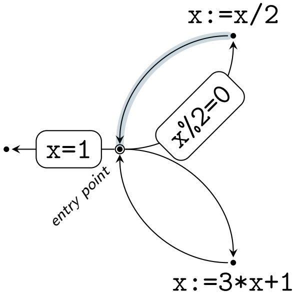
of the Hasse-Syracuse algorithm



the current value of x is 26

An execution trace on a control flow graph

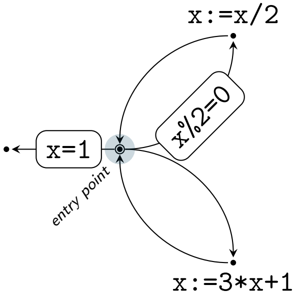
of the Hasse-Syracuse algorithm



the current value of `x` is 26

An execution trace on a control flow graph

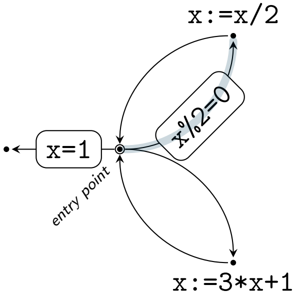
of the Hasse-Syracuse algorithm



the current value of `x` is 26

An execution trace on a control flow graph

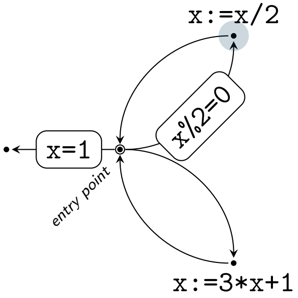
of the Hasse-Syracuse algorithm



the current value of `x` is 26

An execution trace on a control flow graph

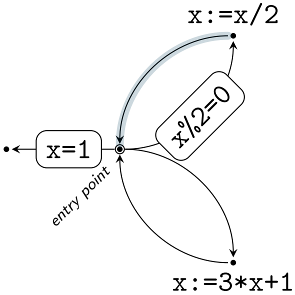
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

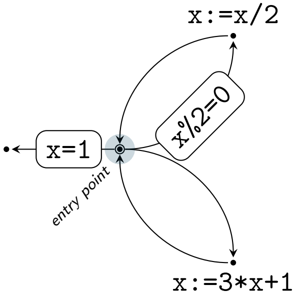
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

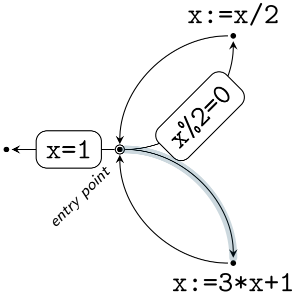
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

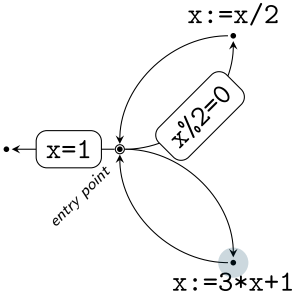
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

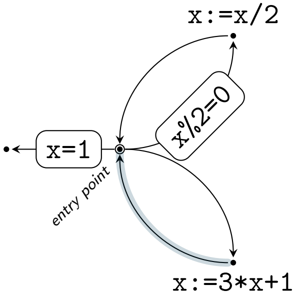
of the Hasse-Syracuse algorithm



the current value of x is 40

An execution trace on a control flow graph

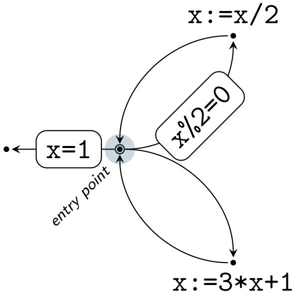
of the Hasse-Syracuse algorithm



the current value of `x` is 40

An execution trace on a control flow graph

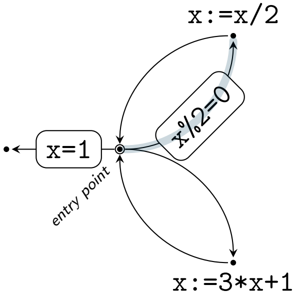
of the Hasse-Syracuse algorithm



the current value of x is 40

An execution trace on a control flow graph

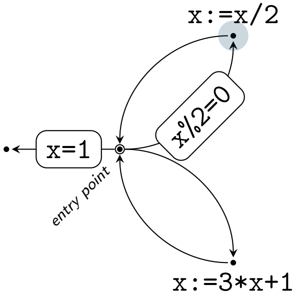
of the Hasse-Syracuse algorithm



the current value of x is 40

An execution trace on a control flow graph

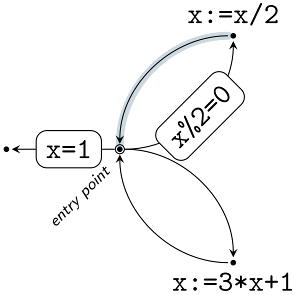
of the Hasse-Syracuse algorithm



the current value of x is 20

An execution trace on a control flow graph

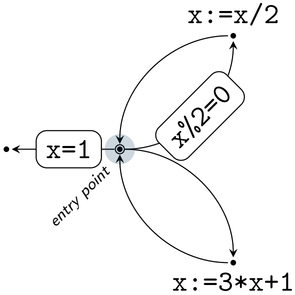
of the Hasse-Syracuse algorithm



the current value of x is 20

An execution trace on a control flow graph

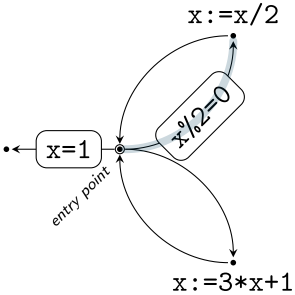
of the Hasse-Syracuse algorithm



the current value of x is 20

An execution trace on a control flow graph

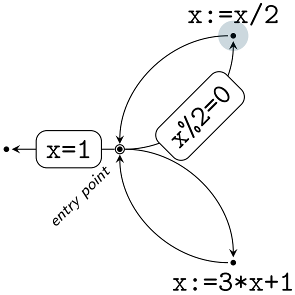
of the Hasse-Syracuse algorithm



the current value of `x` is 20

An execution trace on a control flow graph

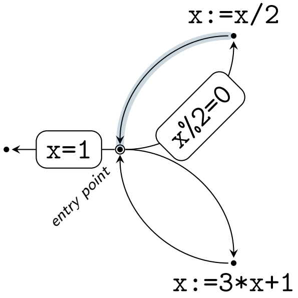
of the Hasse-Syracuse algorithm



the current value of x is 10

An execution trace on a control flow graph

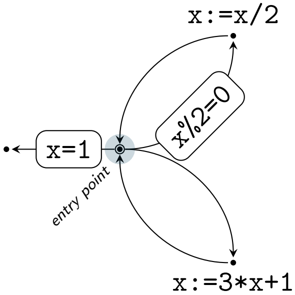
of the Hasse-Syracuse algorithm



the current value of `x` is 10

An execution trace on a control flow graph

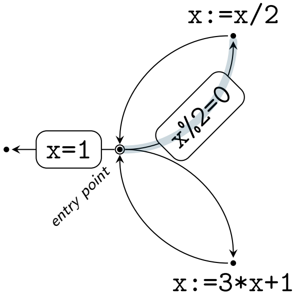
of the Hasse-Syracuse algorithm



the current value of x is 10

An execution trace on a control flow graph

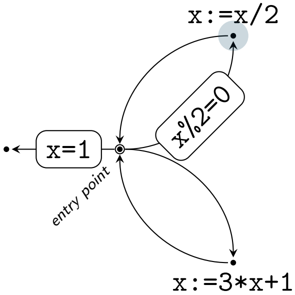
of the Hasse-Syracuse algorithm



the current value of x is 10

An execution trace on a control flow graph

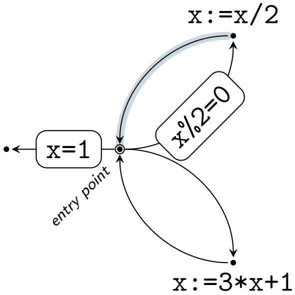
of the Hasse-Syracuse algorithm



the current value of `x` is 5

An execution trace on a control flow graph

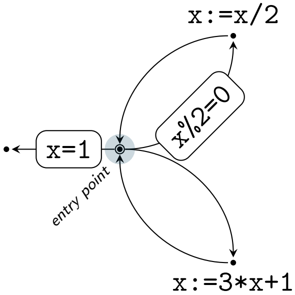
of the Hasse-Syracuse algorithm



the current value of `x` is 5

An execution trace on a control flow graph

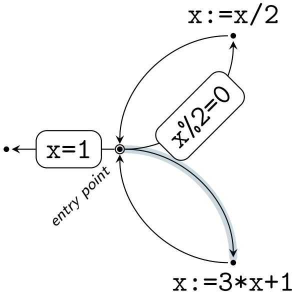
of the Hasse-Syracuse algorithm



the current value of x is 5

An execution trace on a control flow graph

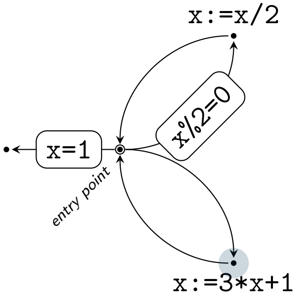
of the Hasse-Syracuse algorithm



the current value of x is 5

An execution trace on a control flow graph

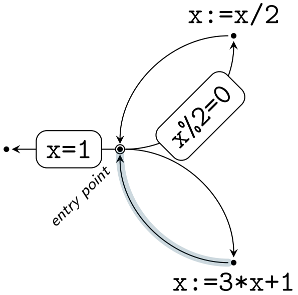
of the Hasse-Syracuse algorithm



the current value of x is 16

An execution trace on a control flow graph

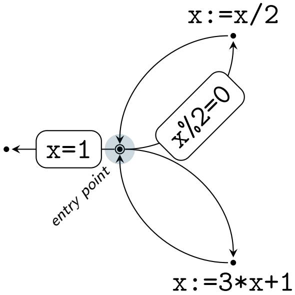
of the Hasse-Syracuse algorithm



the current value of `x` is 16

An execution trace on a control flow graph

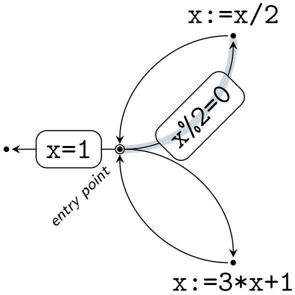
of the Hasse-Syracuse algorithm



the current value of `x` is 16

An execution trace on a control flow graph

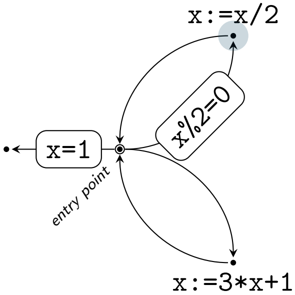
of the Hasse-Syracuse algorithm



the current value of `x` is 16

An execution trace on a control flow graph

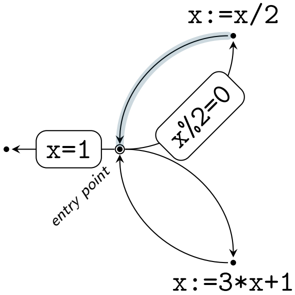
of the Hasse-Syracuse algorithm



the current value of `x` is 8

An execution trace on a control flow graph

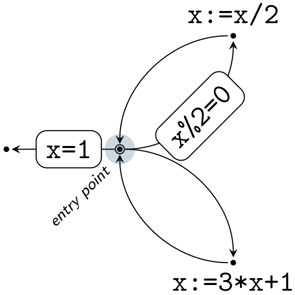
of the Hasse-Syracuse algorithm



the current value of x is 8

An execution trace on a control flow graph

of the Hasse-Syracuse algorithm



the current value of x is 8

An execution trace on a control flow graph

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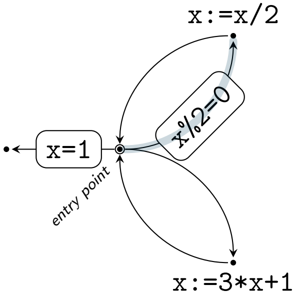
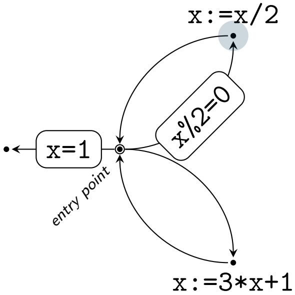


the current value of x is 8

An execution trace on a control flow graph

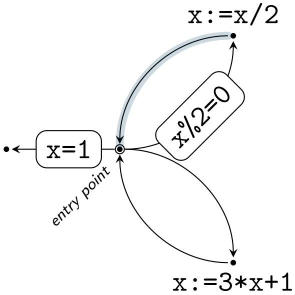
of the Hasse-Syracuse algorithm



the current value of `x` is 4

An execution trace on a control flow graph

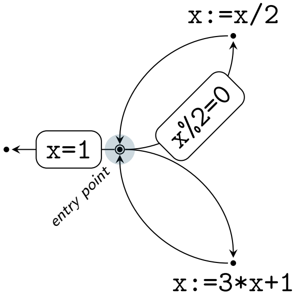
of the Hasse-Syracuse algorithm



the current value of `x` is 4

An execution trace on a control flow graph

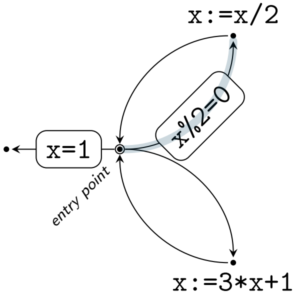
of the Hasse-Syracuse algorithm



the current value of x is 4

An execution trace on a control flow graph

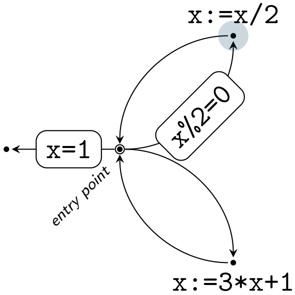
of the Hasse-Syracuse algorithm



the current value of x is 4

An execution trace on a control flow graph

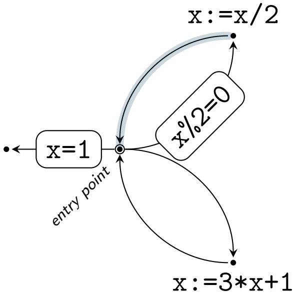
of the Hasse-Syracuse algorithm



the current value of x is 2

An execution trace on a control flow graph

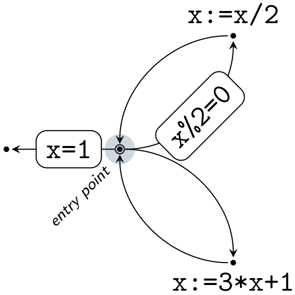
of the Hasse-Syracuse algorithm



the current value of `x` is 2

An execution trace on a control flow graph

of the Hasse-Syracuse algorithm



the current value of `x` is 2

An execution trace on a control flow graph

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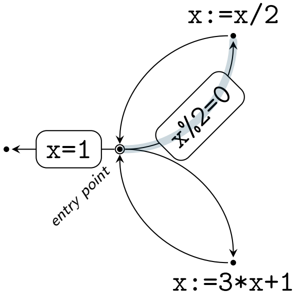
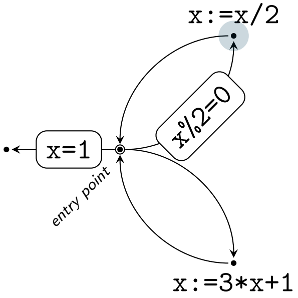


the current value of x is 2

An execution trace on a control flow graph

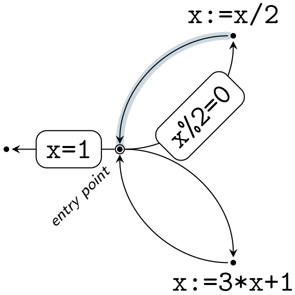
of the Hasse-Syracuse algorithm



the current value of x is 1

An execution trace on a control flow graph

of the Hasse-Syracuse algorithm



the current value of x is 1

An execution trace on a control flow graph

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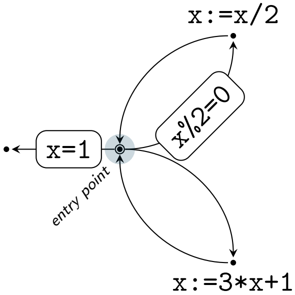
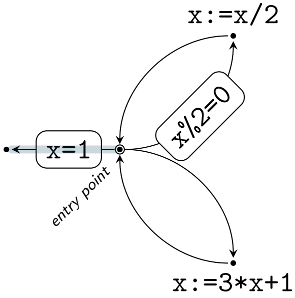


the current value of `x` is 1

An execution trace on a control flow graph

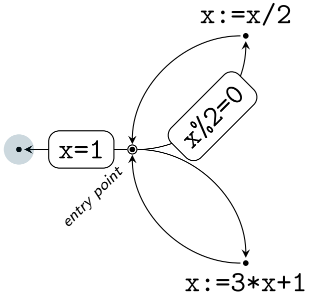
of the Hasse-Syracuse algorithm



the current value of x is 1

An execution trace on a control flow graph

of the Hasse-Syracuse algorithm



the current value of x is 1

Execution traces as paths over a control flow graph

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- Any execution trace induces a path

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- Any execution trace induces a path
- Some paths do not come from an execution trace
- Therefore the collection of path provides a (strict) overapproximation of the collection of execution traces
- The (infinite) collection of paths is entirely determined by the (finite) control flow graph

The overall idea of static analysis

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Any model of a program should contain a finite representation of an overapproximation of the collection of all its execution traces.

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Any model of a program should contain a finite representation of an overapproximation of the collection of all its execution traces.

One the goal of the course it to provide such a structure for a large class of PAML programs.

Restrictions from the PAML syntax

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By construction the PAML language enforces the following restrictions

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Restrictions from the PAML syntax

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- the other operators are interpreted as expected

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The arrows are interpreted as intermediate positions of the instruction pointer so a point on a control flow graph is either a vertex or an arrow.

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A sequence $\mu_0, \dots, \mu_{q-1}$ of multi-intructions is said to be admissible at state σ when for all $k \in \{0, \dots, q-1\}$ the multi-instruction μ_k is admissible at state $\sigma \cdot \mu_0 \cdots \mu_{k-1}$.

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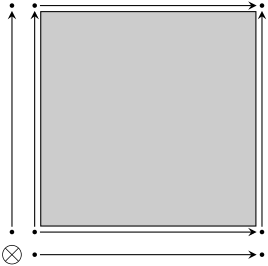
Discrete paths are “continuous”



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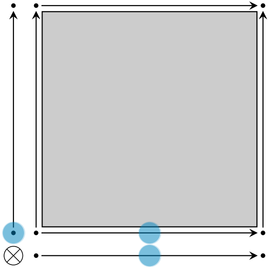

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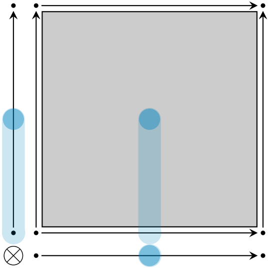

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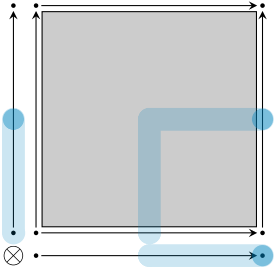

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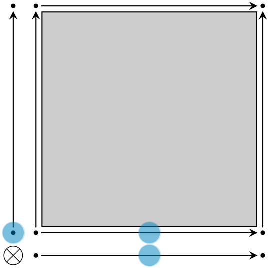

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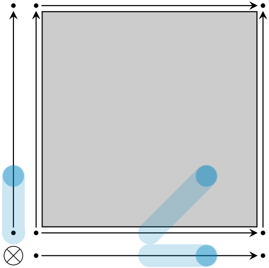

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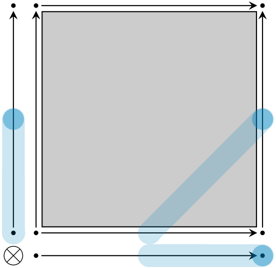

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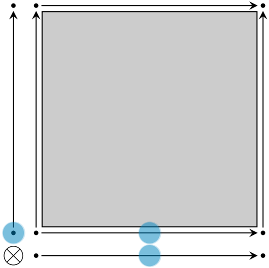

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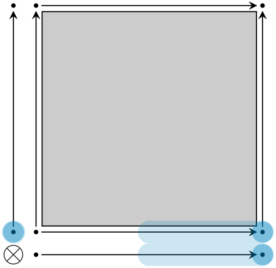

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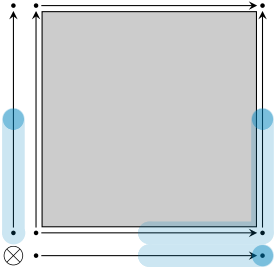

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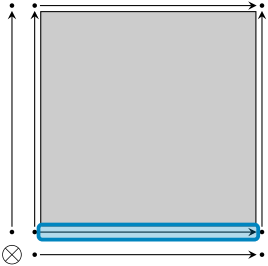

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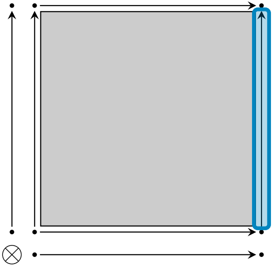

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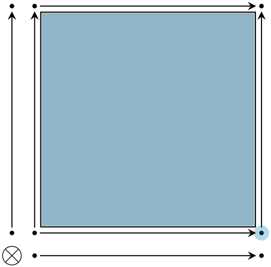

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An admissible path is an execution trace when all the conditional branchings met along the way are respected: for all $k \in \{0, \ldots, q-2\}$ and all $i \in \{1, \ldots, n\}$ such that $\mu_k(i)$ is a branching.

$$\big(\mu_k(i)\big)(\sigma \cdot \mu_0 \cdots \mu_{k-1}) \quad = \quad \gamma_i(k+2)$$

Concurrent access

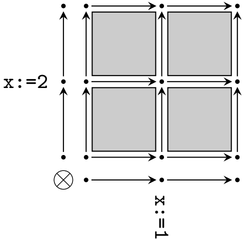
```
var x = 0
```

```
proc p = x:=1

proc q = x:=2
```

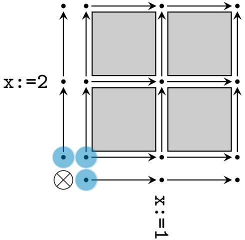
```
init p q
```

Admissible execution trace



the value of `x` is 0

Admissible execution trace



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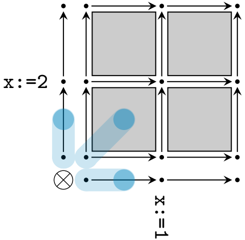
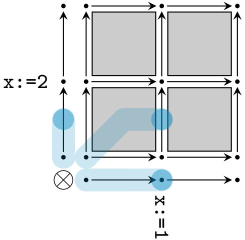
Admissible execution trace



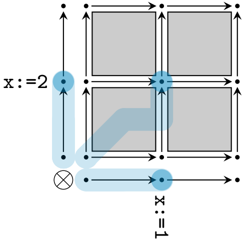
the value of `x` is 0

Admissible execution trace



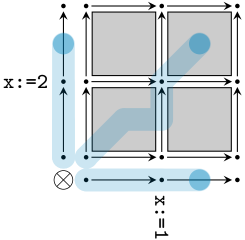
the value of x is 1

Admissible execution trace



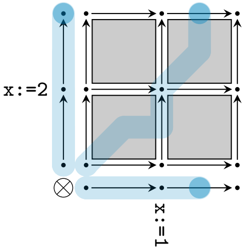
the value of x is 2

Admissible execution trace



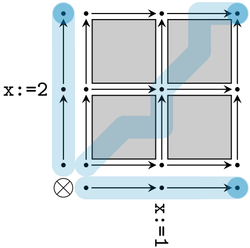
the value of `x` is 2

Admissible execution trace



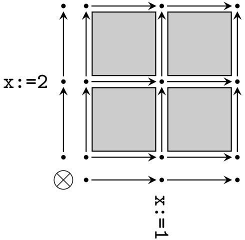
the value of `x` is 2

Admissible execution trace



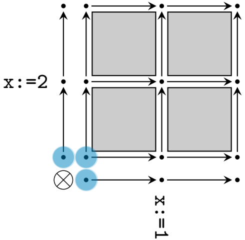
the value of x is 2

Not admissible execution trace



the value of `x` is 0

Not admissible execution trace



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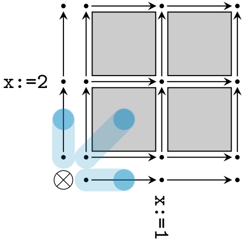
Not admissible execution trace



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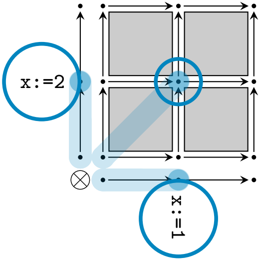


the value of x is ?

Lack of resources

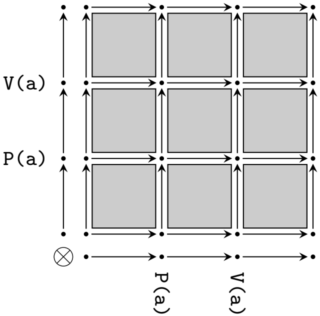
```
sem 1 a
```

```
proc p = P(a);V(a)
```

```
init 2p
```

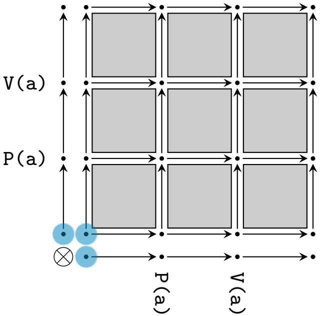
Admissible concurrent execution trace

`sem 1 a`



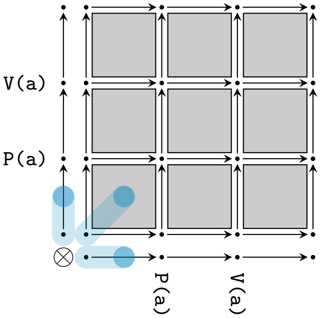
Admissible concurrent execution trace

`sem 1 a`



Admissible concurrent execution trace

`sem 1 a`



Admissible concurrent execution trace

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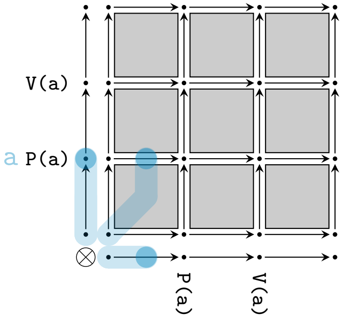
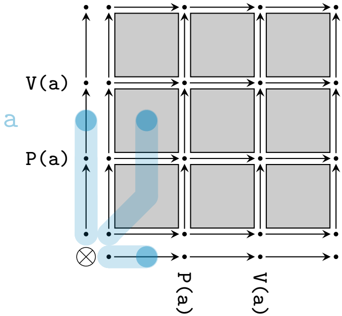


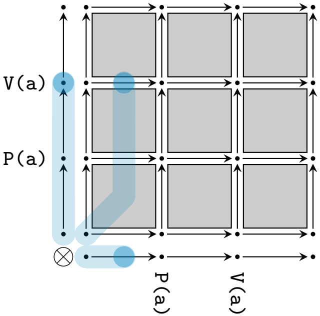
Admissible concurrent execution trace

`sem 1 a`



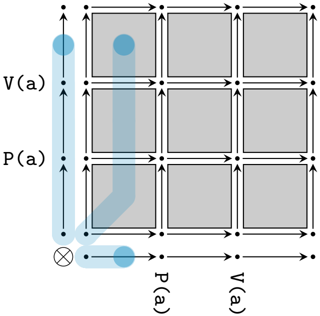
Admissible concurrent execution trace

`sem 1 a`



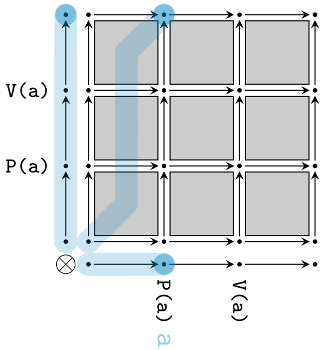
Admissible concurrent execution trace

`sem 1 a`



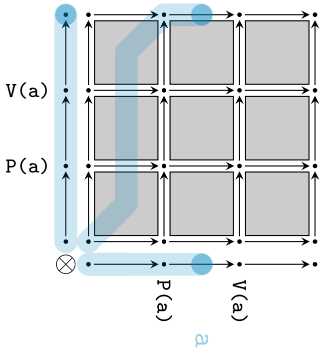
Admissible concurrent execution trace

`sem 1 a`



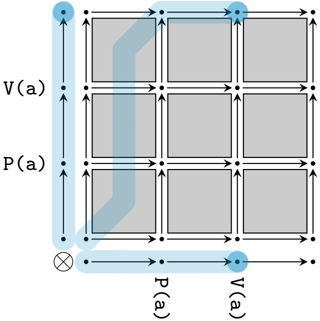
Admissible concurrent execution trace

`sem 1 a`



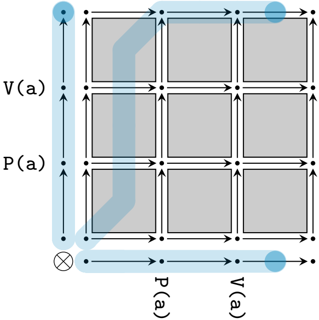
Admissible concurrent execution trace

`sem 1 a`



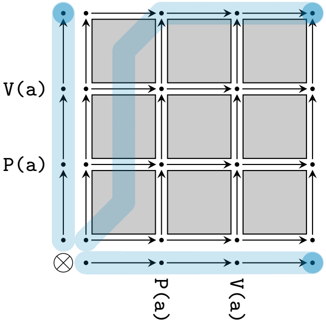
Admissible concurrent execution trace

`sem 1 a`



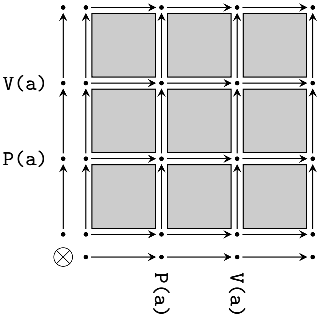
Admissible concurrent execution trace

`sem 1 a`



Not admissible concurrent execution trace

`sem 1 a`



Not admissible concurrent execution trace

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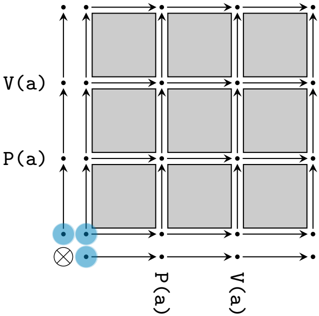
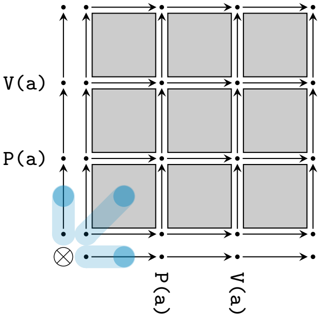


Not admissible concurrent execution trace

sem 1 a



Not admissible concurrent execution trace

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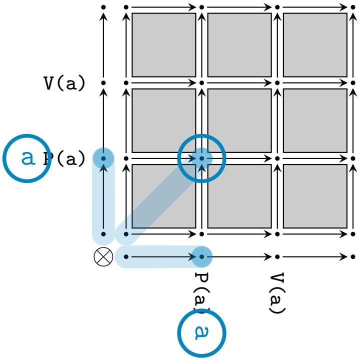


Synchronisation

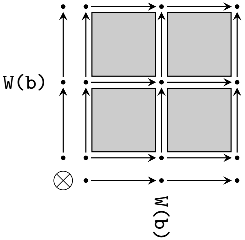
```
sync 1 b
```

```
proc p = W(b)
```

```
init 2p
```

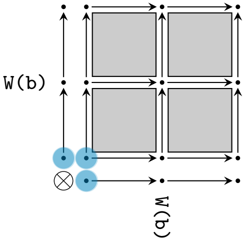
Concurrent execution trace

`sync 1 b`



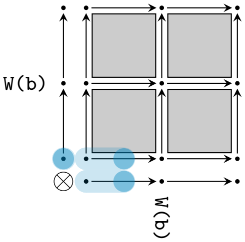
Concurrent execution trace

`sync 1 b`



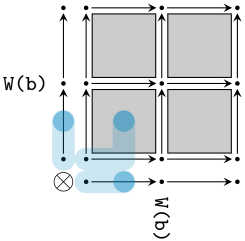
Concurrent execution trace

`sync 1 b`



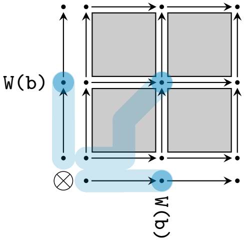
Concurrent execution trace

`sync 1 b`



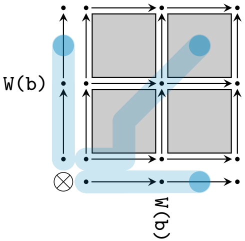
Concurrent execution trace

`sync 1 b`



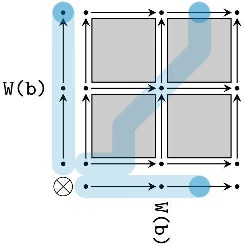
Concurrent execution trace

`sync 1 b`



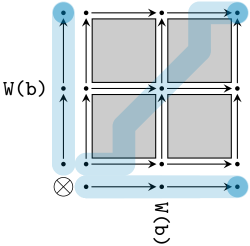
Concurrent execution trace

`sync 1 b`



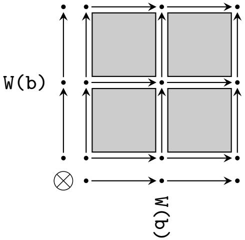
Concurrent execution trace

`sync 1 b`



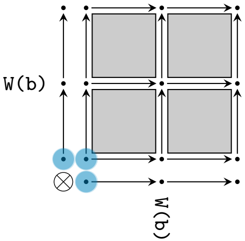
Not admissible concurrent execution trace

`sync 1 b`



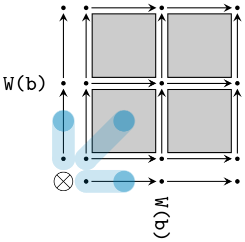
Not admissible concurrent execution trace

`sync 1 b`



Not admissible concurrent execution trace

`sync 1 b`



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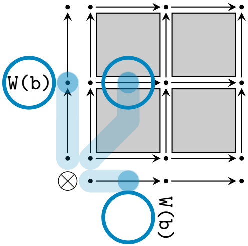

Next goal

Encode admissibility into a model.

The potential functions of processes and programs

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A process G is conservative when for all directed paths starting at the origin, the amount of semaphores held by the process at the end of the path only depends on its arrival point.

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In that case the process G comes with a potential function F_G

$$\{\text{semaphores}\} \times \{\text{points}\} \to \mathbb{N} \quad \cong \quad \{\text{points}\} \to \{\text{multisets over } \mathcal{S}\}$$

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A program Π is conservative when so are its processes $G_1, \ldots, G_n$ and its potential function is given by

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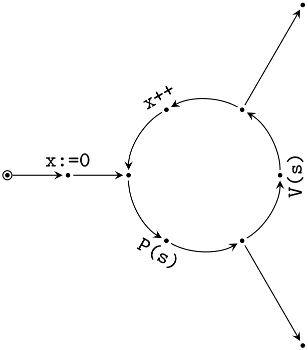
$$F_\Pi(p_1, \ldots, p_d) = \sum_{k=1}^{d} F_{G_k}(p_k)$$

If $F_\Pi(s, p) > \text{arity}(s)$ for some semaphore s, then p is forbidden.

Conservative process

example



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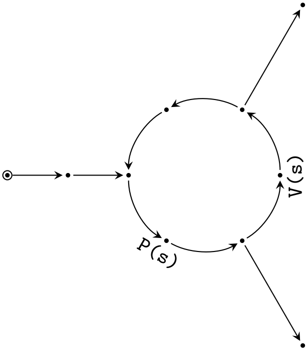
example



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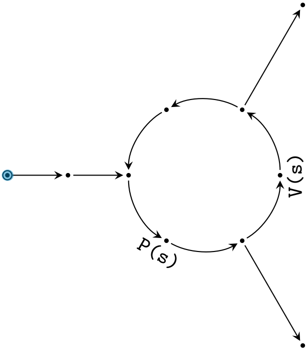

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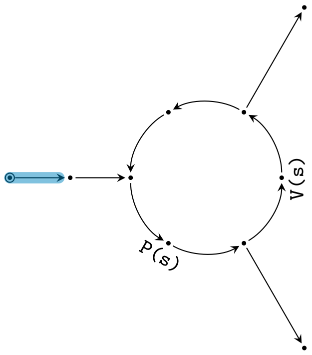

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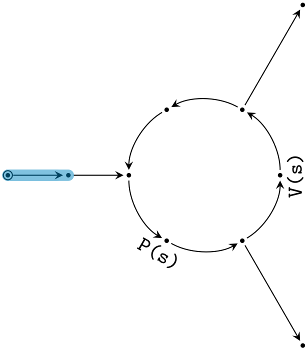

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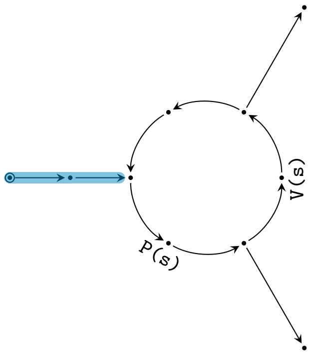

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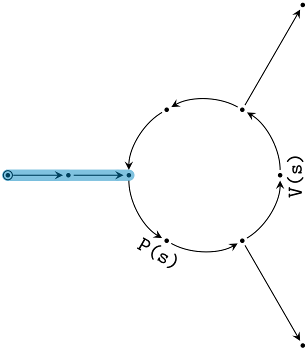

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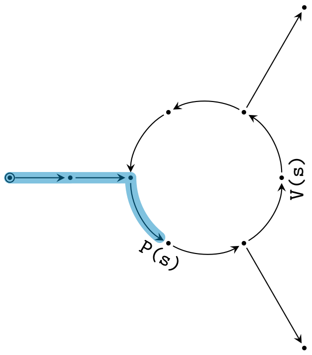

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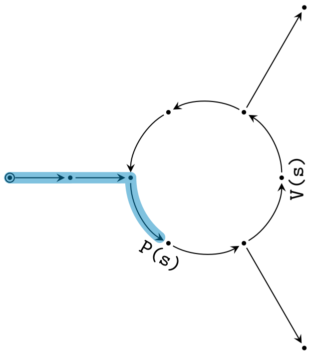

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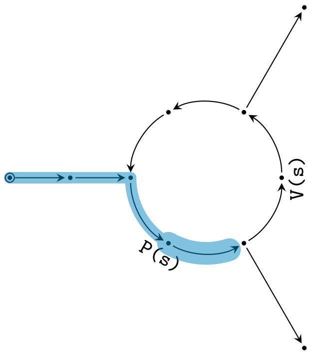
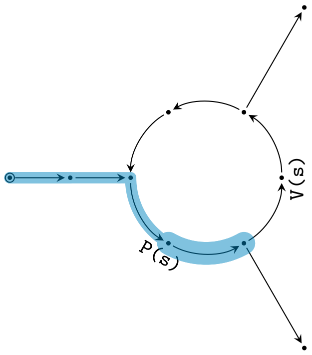


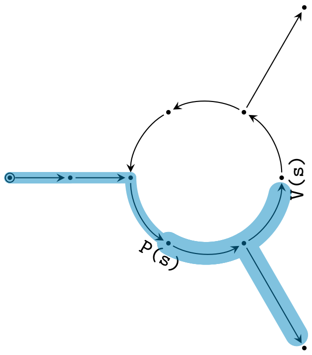
Conservative process

example



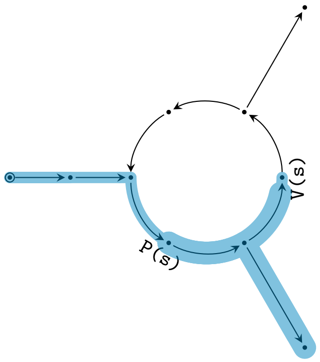
Conservative process

example



Conservative process

example



Conservative process

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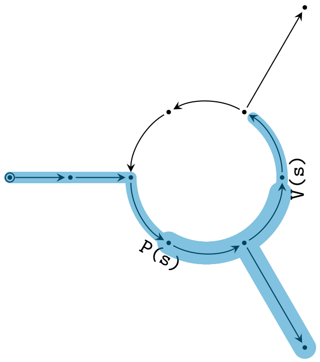

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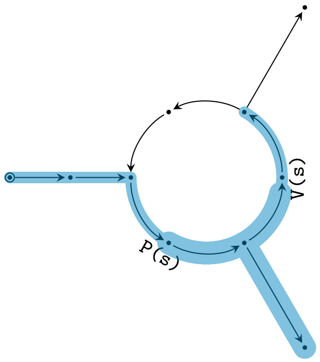

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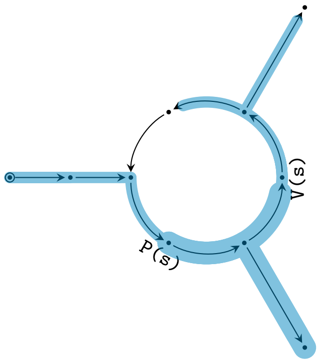

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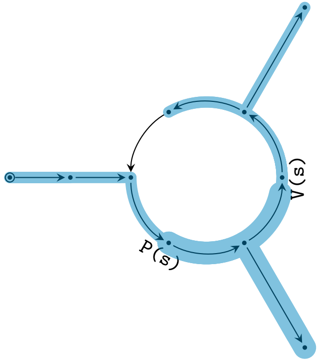
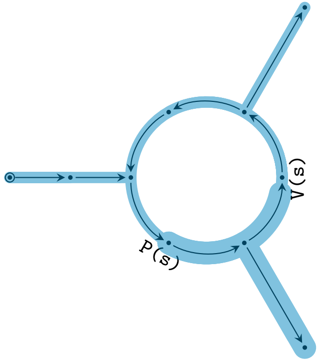


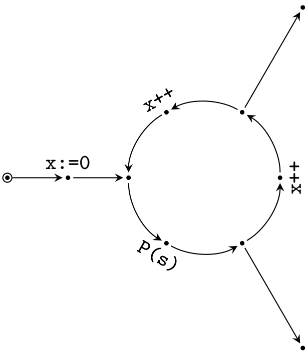
Conservative process

example



Not conservative process

example



Not conservative process

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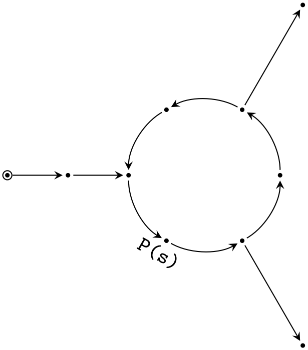

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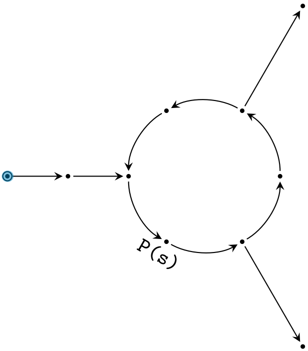

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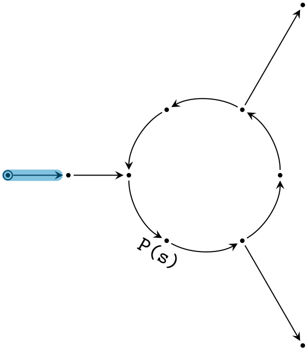

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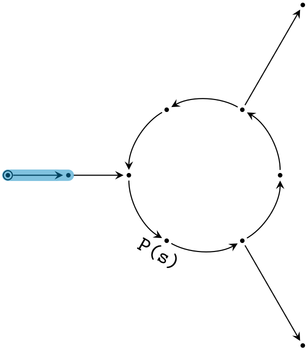

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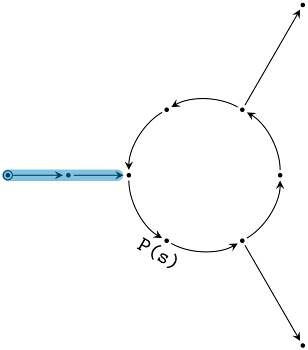

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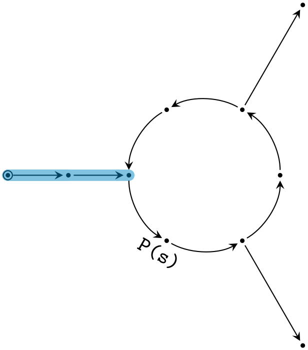

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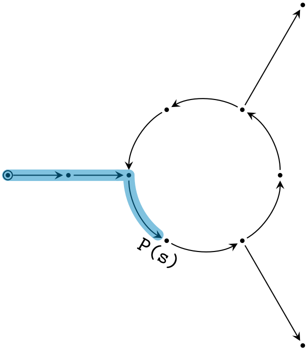

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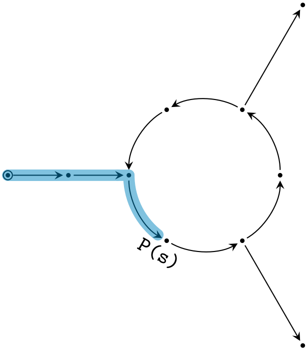

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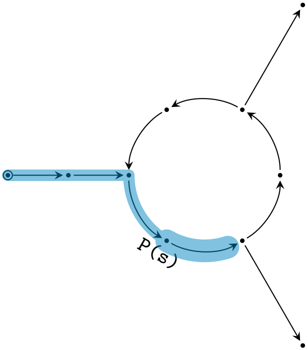

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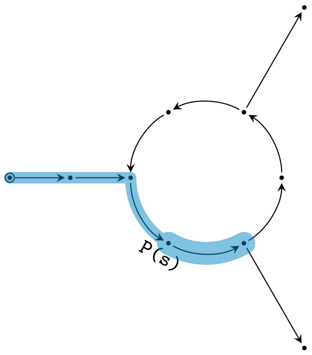

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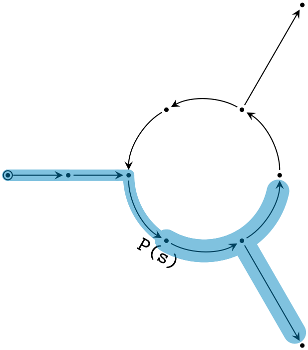

Not conservative process

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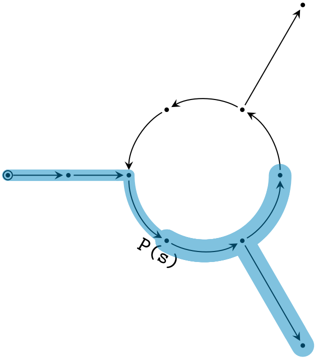

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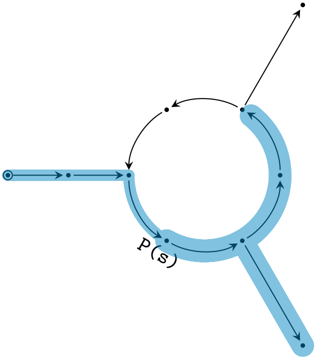

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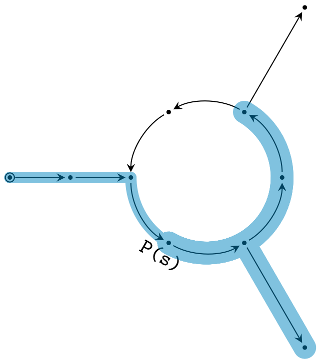

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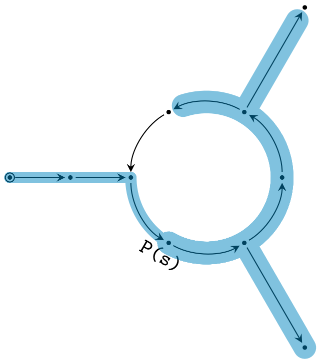

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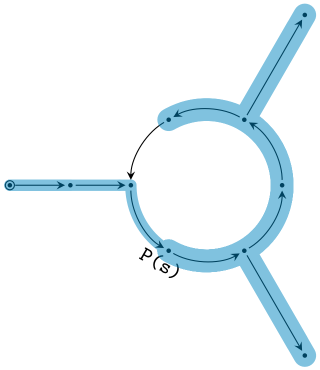

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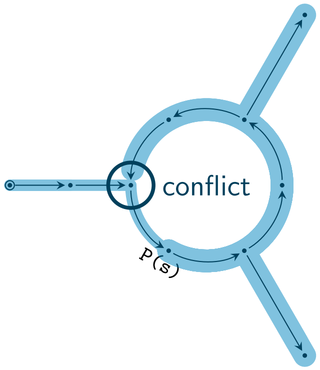

Conservativity is decidable

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We inductively define a sequence of partial functions $\pi_n : \{\text{points}\} \to \mathbb{N}^{\mathcal{S}}$.

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- Assuming that π_n is defined, for all pairs of points (p, p') such that:
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  we define a strict extension of π_n, by setting:

$$p' \quad \mapsto \quad \begin{cases} \pi_n(p) & \text{if } \partial^- p' = p \\ \pi_n(p) \cdot \lambda(p') & \text{if } p' = \partial^+ p \end{cases}$$

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- If the following holds for all ordered pairs of points (p, p') such that $\partial^- p' = p$ or $p' = \partial^+ p$, then G is conservative, otherwise it is not.

$$\pi(p') \quad = \quad \begin{cases} \pi(p) & \text{if } \partial^- p' = p \\ \pi(p) \cdot \lambda(p') & \text{if } p' = \partial^+ p \end{cases}$$

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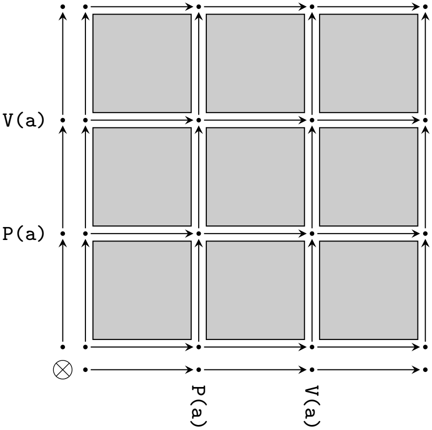
The discrete model is the complement of its forbidden set.

$$\{\text{points of the program}\} \setminus \{\text{forbidden points}\}$$

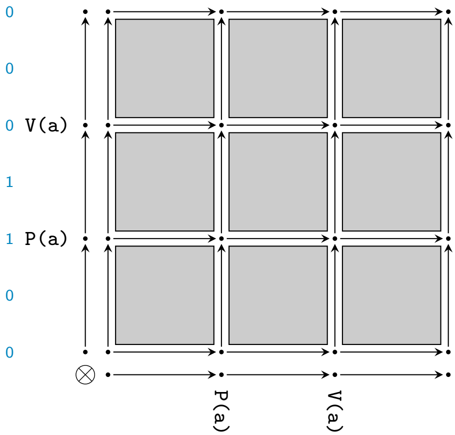
Discrete model

sem 1 a



Discrete model

`sem 1 a`



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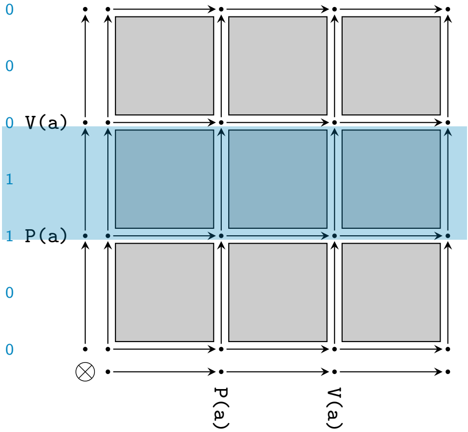

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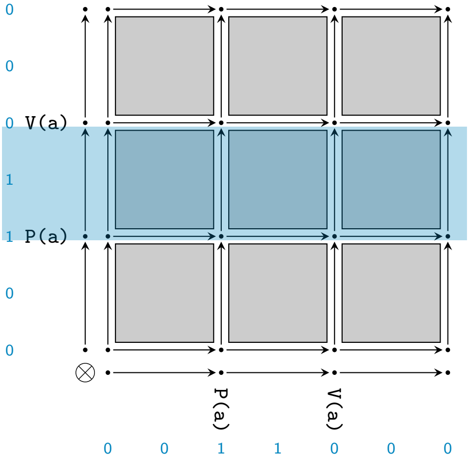

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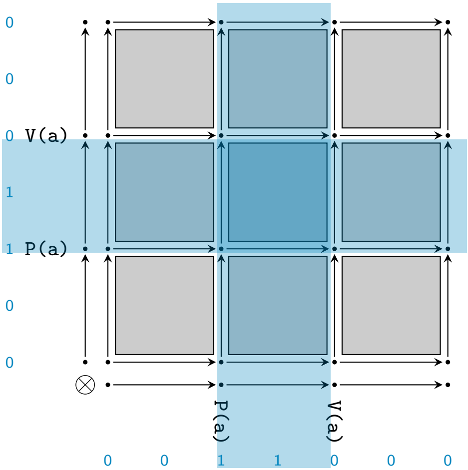

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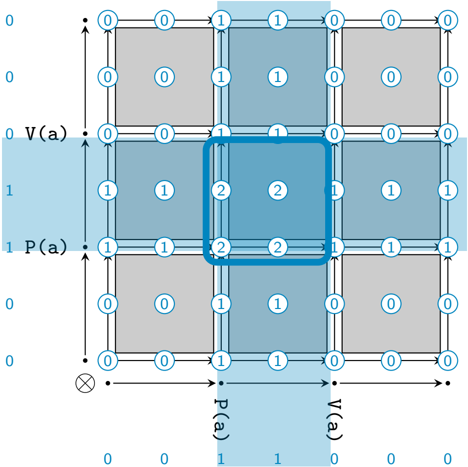

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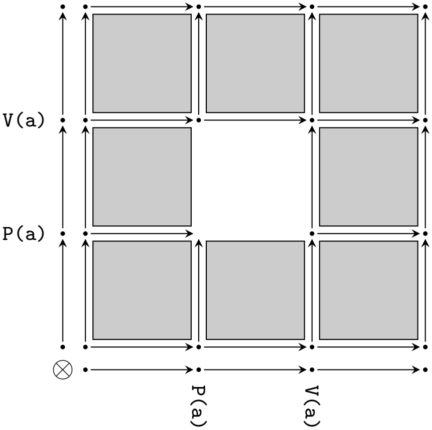

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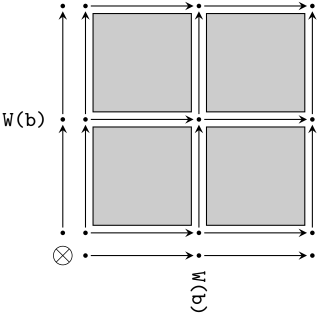

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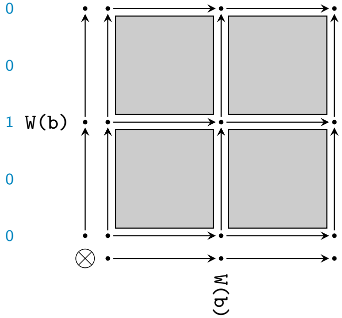

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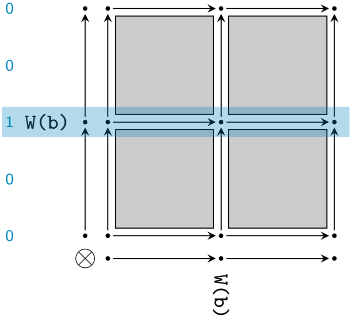

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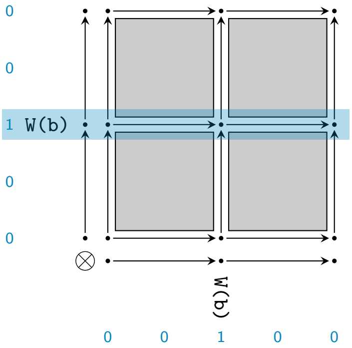

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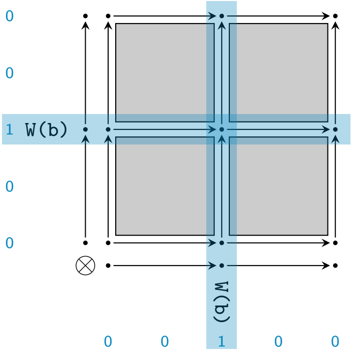

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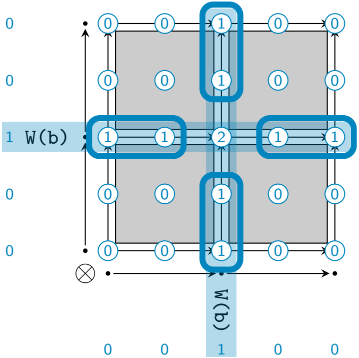

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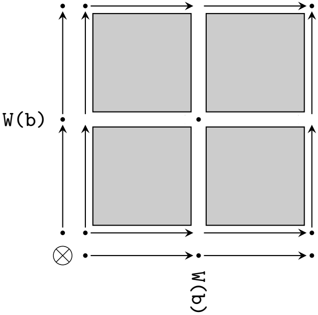

Main theorem of discrete models

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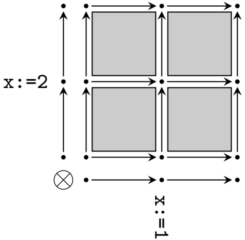
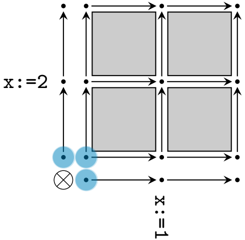
- Any directed path on a discrete model (i.e. which does not meet any forbidden point) is admissible.
- Conversely, for each admissible path which meets a forbidden point there exists a directed path which avoids them and such that both directed paths induce the same sequence of multi-instructions.

Admissible execution trace



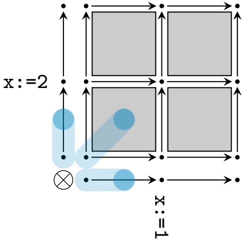
the value of x is 0

Admissible execution trace



the value of `x` is 0

Admissible execution trace



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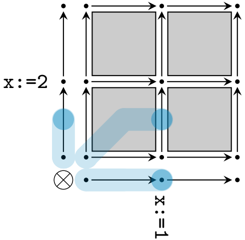
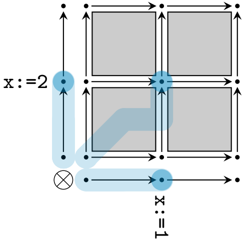


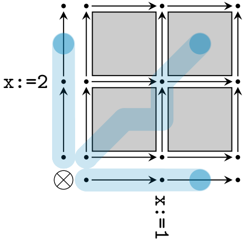
the value of x is 1

Admissible execution trace



the value of x is 2

Admissible execution trace



the value of $\mathtt{x}$ is 2

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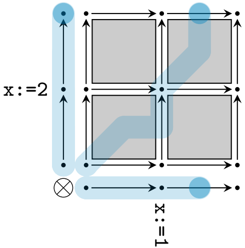

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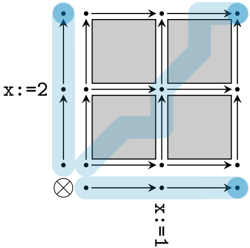

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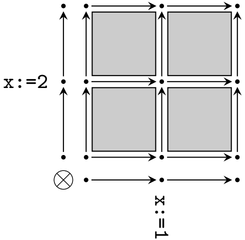
Admissible execution trace avoiding forbidden points



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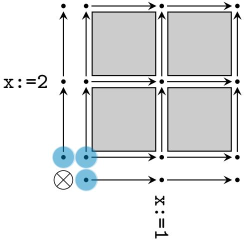

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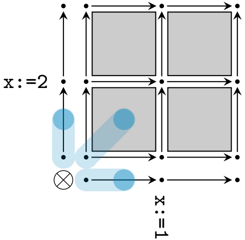

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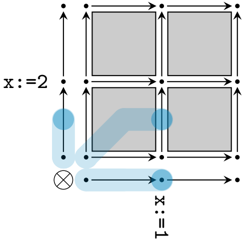

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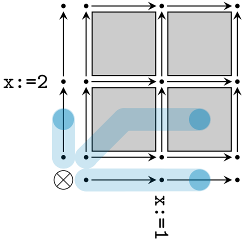

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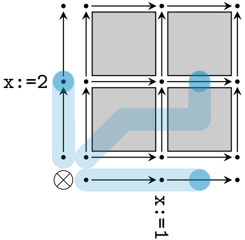

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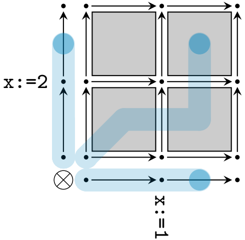
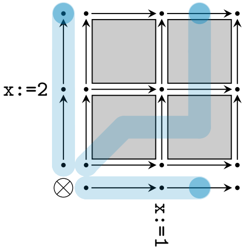


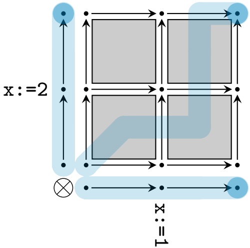
the value of x is 2

Admissible execution trace avoiding forbidden points



the value of x is 2

Admissible execution trace avoiding forbidden points



the value of `x` is 2

Replacement