2.3.1: Concurrency

Wednesday the 1st of March 2017
Duration: 2h

Exercise 1:

\texttt{sem 1 a}
\texttt{proc p = P(a);V(a)}
\texttt{init: 3p}

1) Draw the geometric model $X$ of the program above.

2) Let $p$ and $q$ be the lower and upper corners of the geometric model $X$. How many elements does the set $\pi_1 X(p, q)$ (the set of directed homotopy classes of directed paths from $p$ to $q$) contain?

3) Prove that two directed paths on that model are weakly dihomotopic if and only if their underlying paths are homotopic in the usual “undirected” sense.

4) Make a comment about the following program.

\texttt{sem 1 a}
\texttt{sem 2 b}
\texttt{proc p = P(b);P(a);V(a);V(b)}
\texttt{init: 3p}

Exercise 2:

Given $n \geq 2$, the $n$-philosophers diner is the following program.

\texttt{sem 1 a_1 \ldots a_n}
\texttt{proc p_1 = P(a_1);P(a_2);V(a_1);V(a_2)}
\vdots
\texttt{proc p_k = P(a_k);P(a_{k+1});V(a_k);V(a_{k+1})}
\vdots
\texttt{proc p_n = P(a_n);P(a_1);V(a_n);V(a_1)}
\texttt{init: p_1 \ldots p_n}

1) Draw the geometric model for $n = 2$ and its category of components.

An execution trace is said to be maximal when it cannot be extended to a longer execution trace. To every execution trace one associates an admissible sequence of multi-instructions.

2) For $n = 2$, give all the sequences of multi-instructions coming from maximal execution traces of the program.

3) For all $n$, give all the sequences of multi-instructions (coming from an execution trace) that end up in a deadlock.

4a) For all $n$, give the maximal block covering of the forbidden region.

4b) Compute the prime decomposition of the geometric model of the program.

Let $\mu_0, \ldots, \mu_q$ be the sequence of multi-instructions corresponding to some partial execution trace $\gamma$ of the $n$-philosophers diner. Considering all the execution traces extending $\gamma$, we obtain the finite set $S$ of all the multi-instructions $\mu_{q+1}$ that could be executed after the sequence $\mu_0, \ldots, \mu_q$.

Suppose that, at each step of the execution, $\mu_{q+1}$ is chosen at random in $S$ (all
the element of $S$ occurring with the same probability).

5) Compute the limit of the probability that a deadlock occurs when the number of philosophers (i.e. $n$) goes to infinity.

Exercise 3:
1) Prove there exists infinitely many precubical sets whose realizations in $\mathcal{T}_{\mathcal{P}}$ are homeomorphic to the compact unit segment $[0,1]$ though their realizations in $\mathfrak{d}_{\mathcal{T}_{\mathcal{P}}}$ are pairwise non isomorphic.

2) Given a finite precubical set $K$, prove there exists infinitely many precubical sets whose realizations in $\mathcal{T}_{\mathcal{P}}$ are homeomorphic to that of $K$ though their realizations in $\mathfrak{d}_{\mathcal{T}_{\mathcal{P}}}$ are pairwise non isomorphic.

Exercise 4:
1) Find a program whose geometric model (as defined in the course) is not connected.

2) Comment the claim: “we can suppose that the geometric model of a program is connected”.

Exercise 5:
We denote by the running processes of a program by $G_1, \ldots, G_n$.

1) Explain why the set of execution traces of a program is open in the space of directed paths on its geometric model.

2) Suppose the set of execution traces is arc-connected, what can be said about the program?

3) Find a program such that the set of admissible paths on $|G_1| \times \cdots \times |G_n|$ is neither open nor closed in the space of directed paths on $|G_1| \times \cdots \times |G_n|$.

Exercise 6:
The initial object of a category $\mathcal{C}$ is (when it exists) an object $I$ such that $\mathcal{C}(I, X)$ is a singleton for all objects $X$ of $\mathcal{C}$.

1) Prove that if the cubical set $K$ is not the initial object of $\mathcal{CS}$, then $K_n \neq \emptyset$ for all $n \in \mathbb{N}$. Prove that it does not hold for precubical sets.

The terminal object of a category $\mathcal{C}$ is (when it exists) an object $T$ such that $\mathcal{C}(X, T)$ is a singleton for all objects $X$ of $\mathcal{C}$.

2) Compute the realization in $\mathcal{T}_{\mathcal{P}}$ of the terminal cubical set.

3) Give a simple description of the underlying set of the realization in $\mathcal{T}_{\mathcal{P}}$ of the terminal precubical set.

Exercise 7:
1) Find a prime isothetic region whose category of component is not prime.

2) Find infinitely many non-isomorphic prime isothetic regions whose categories of components are not prime.