Decomposition of finite almost free categories

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A presentation of a category C is a graph G together with a congruence $\sim$ over the category freely generated by G, namely FG, such that

$$C \cong FG/\sim$$

Any (small) category admits a "biggest" presentation provided by the left adjoint to the forgetful functor $Cat \to Grph$. In that case, the arrows of G are all the morphisms of C, the congruence being given by all the pairs of paths on G whose composite are equal.

A morphism is said to be irreducible when it cannot be written as a composite of (at least two) nonidentity morphisms. Let us define the notion of least presentation. A generating family for a category C is a collection G of morphisms of C such that any nonidentity morphism of C can be written as a composite of elements of G. If the irreducible elements of C form a generating family, then it is necessarily the least one.

More formally, a category C has a least defintion when it is generated by its irreducible elements which form a graph G, and when there is a least equivalence relation $\sim$ on the morphisms of FG such that $C \cong FG/\sim^*$, with $\sim^*$ being the congruence generated by $\sim$.

One remarkable feature of free categories if that they enjoy a least presentation: the graph G being the one that satisfies $FG \cong C$. There are however nonfree categories that admit a least presentation. Among them one finds all the finite loop-free categories.

We ask the following questions:

- Do nonempty finite almost free categories admit a unique decomposition theorem ?
- Can they be embeded into fundamental categories of local pospaces ?
- Can we find a faithful linear representation for each of them ?

References

Decomposition of finite loop-free categories, master thesis, Thibault Balabonski. 2006.

Algebraic topology and concurrency, L. Fajstrup, M. Raussen, É. Goubault. TCS. 2006