Precubical and Continuous Control Flow

Topology workshop
Paris 2014

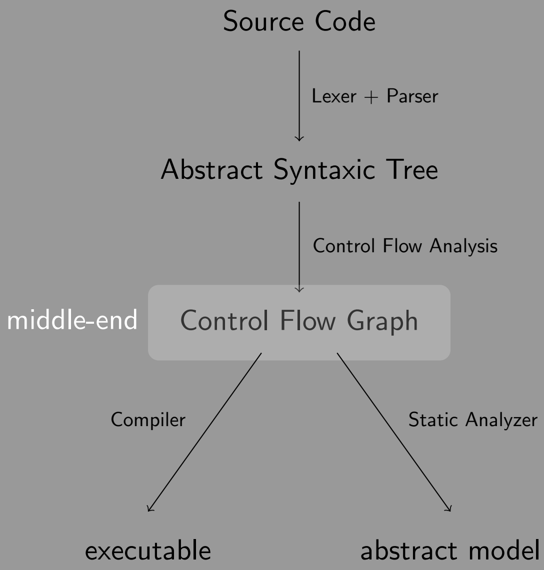
Emmanuel Haucourt

CEA-Tech, NanoInnov

The 15th of July

Control Flow Graphs of Sequential Processes

Control Flow Analysis, Frances E. Allen, SIGPLAN Notices 1970



The overall idea

of Static Analysis

The model of a program should be a finite representation of an overapproximation of the collection of all its execution traces.

Precubical sets

as presheaves over $\square^+$

K_0

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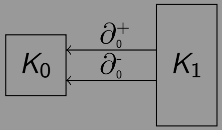

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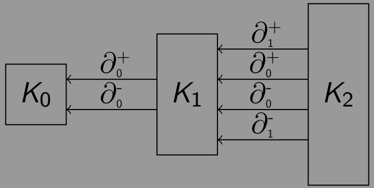

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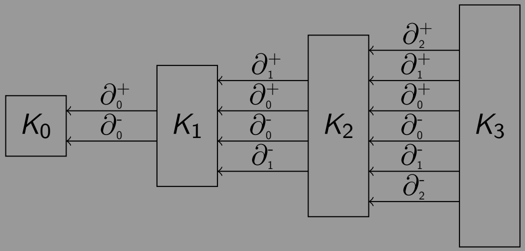

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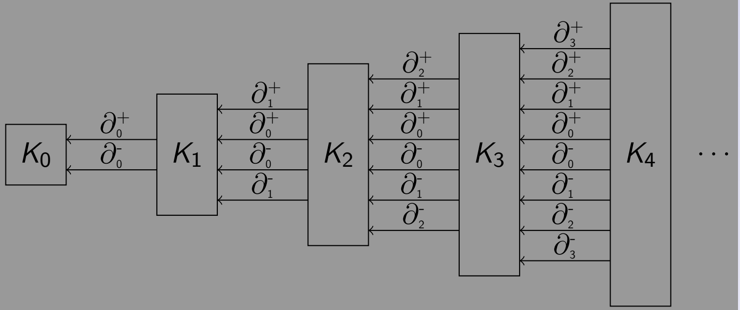


Tensor product
of precubical sets

Given precubical sets K and K' of dimension p and q, the set of n-cubes for $0 \leqslant n \leqslant p+q$

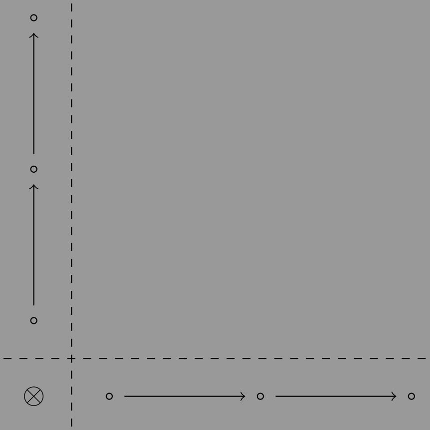
$$(K \otimes K')_n = \bigsqcup_{i+j=n} K_i \times K_j$$

For $x \otimes y \in K_i \times K'_j$ with $i + j = n$ the k^{th} face map, with $0 \leqslant k < n$, is given by

$$\partial_k^{\pm}(x \otimes y) = \begin{cases} \partial_k^{\pm}(x) \otimes y & \text{if } 0 \leqslant k < i \\ x \otimes \partial_{k-p}^{\pm} y) & \text{if } i \leqslant k < n \end{cases}$$

Example of tensor product

of precubical sets



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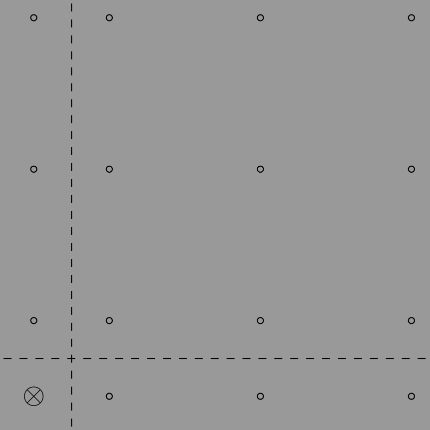

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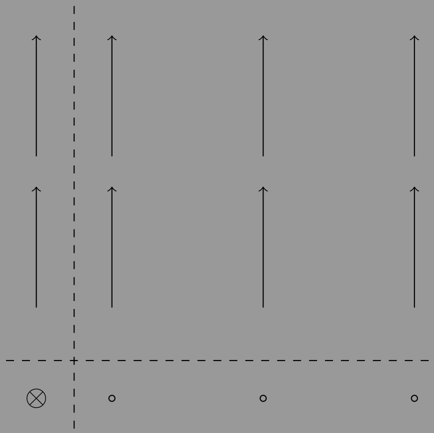

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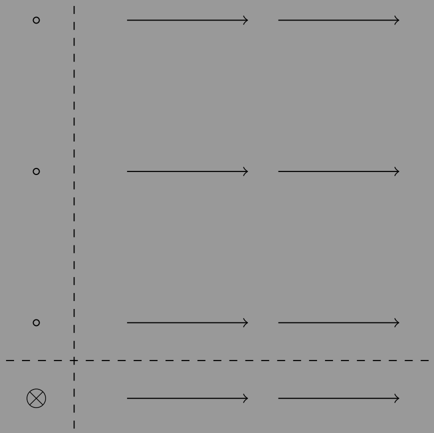

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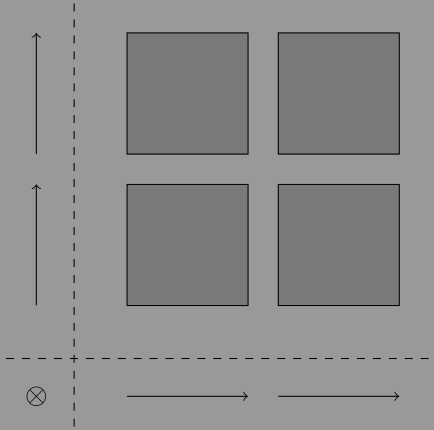

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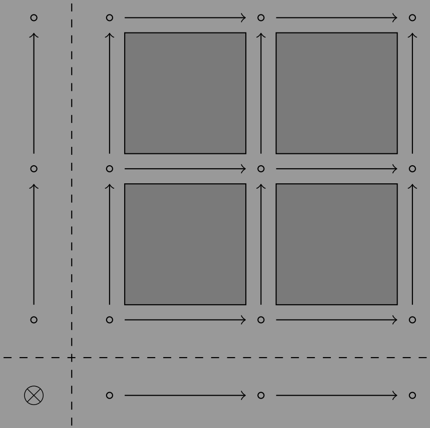


The PV language

Dijkstra 68 - Input language for ALCOOL in an extended form

- *Sem*: set of semaphores with arity in $\mathbb{N} \setminus \{0, 1\}$
- *Mtx*: set of mutex, an alias for a semaphore of arity 2
- A semaphore `x` of arity n is a resource offering $n - 1$ tokens, each process can hold one token or more
- A process acquire a token executing the instruction `P(x)` and release it executing the instruction `V(x)`
- A mutex can be held by only one process at the time
- Trying to perform `P(x)` though `x` is not available blocks the execution unless `x` is a mutex already held by the process
- The instruction `V(x)` is not blocking
- *Wait*: set of synchronization bareers with arity in $\mathbb{N} \setminus \{0, 1\}$
- Instruction `W(x)` blocks the execution of the process until n (arity of `x`) processes are blocked by `x` then all the execution are resumed at the same time

Extending the middle-end representation

Conservative process

A process is said to be conservative when for all paths γ, the amount of resources available at the arrival of γ only depends on the amounts of resources that were available at the origin of γ.

$$\partial^{-}\gamma = \partial^{-}\gamma' \text{ and } \partial^{+}\gamma = \partial^{+}\gamma' \;\Rightarrow\; [\![\gamma]\!] \cdot \delta(x) = [\![\gamma']\!] \cdot \delta(x)$$

Being conservative is decidable and induces a potential function.

The potential function

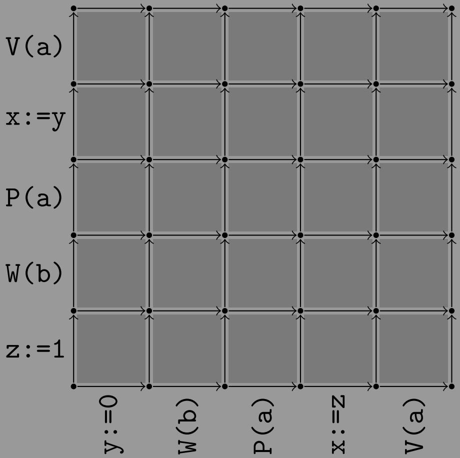
of a PV program $P_1|\cdots|P_d$

- assume each P_k is conservative
  and F_k the associated potential function
- let $K_0 = V_1 \times \cdots \times V_d$ the 0-dimensional cubes of the
  tensor product of the cfgs
- The potential function $F : K_0 \times \mathcal{R} \to \mathbb{N}$ is

$$F(v_1, \ldots, v_d, x) = \sum_{k=1}^{d} F_k(v_k, x)$$

Control Flow Precubical Set: an example

`y:=0.W(b).P(a).x:=z.V(a)|z:=0.W(b).P(a).x:=y.V(a)`



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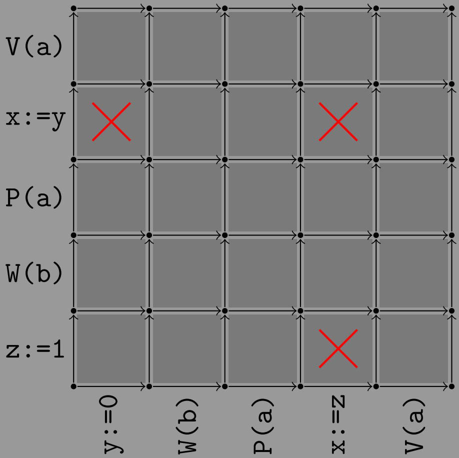

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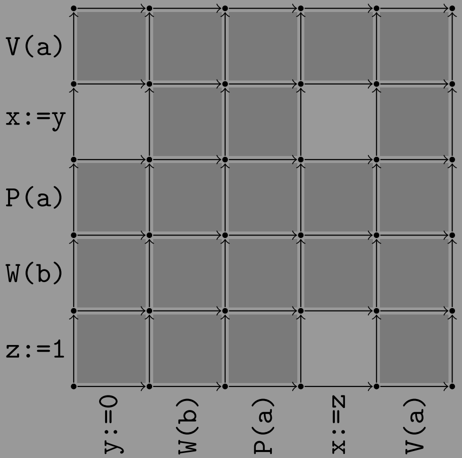

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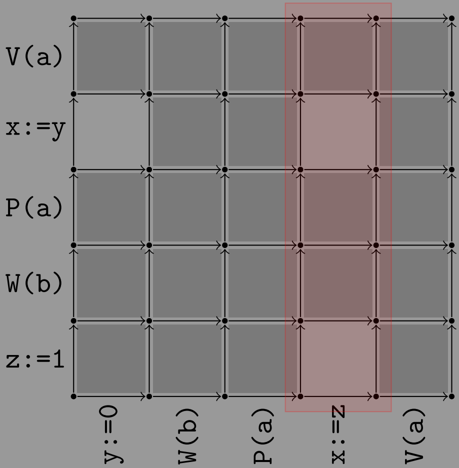

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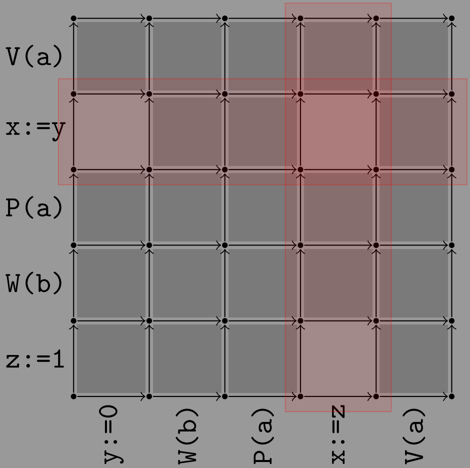

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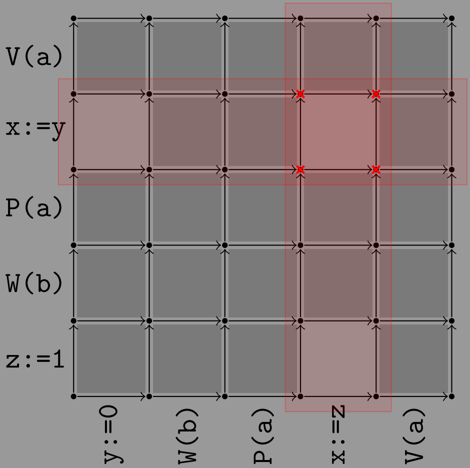

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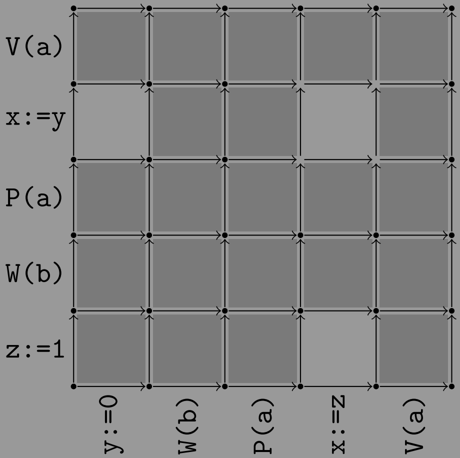

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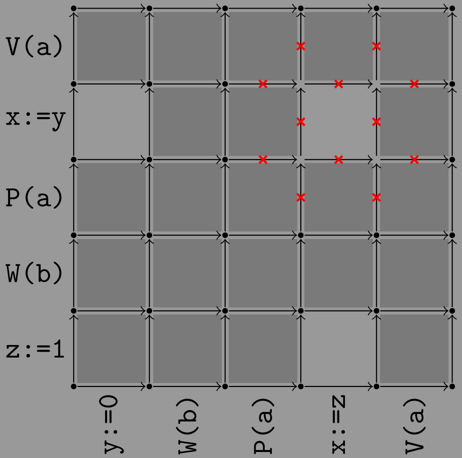

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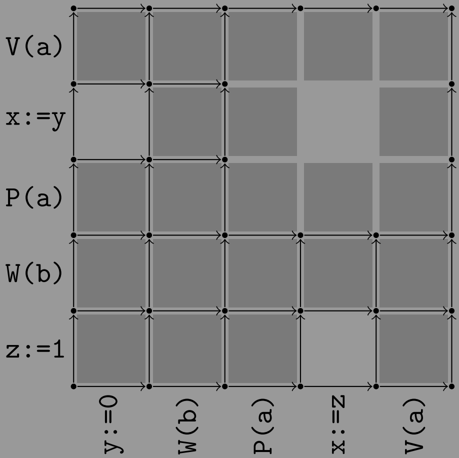

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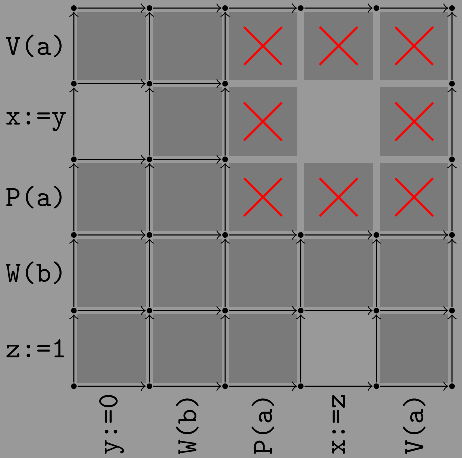

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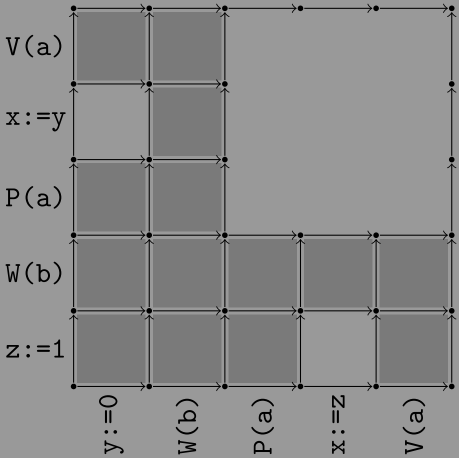

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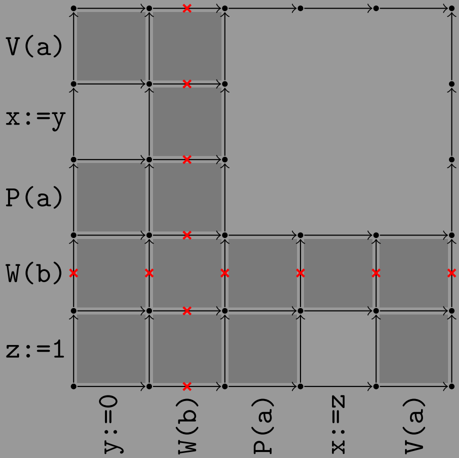

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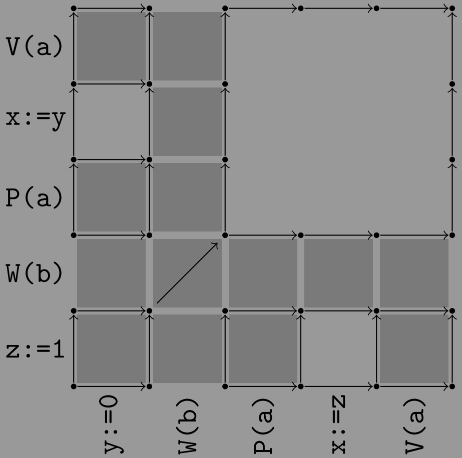

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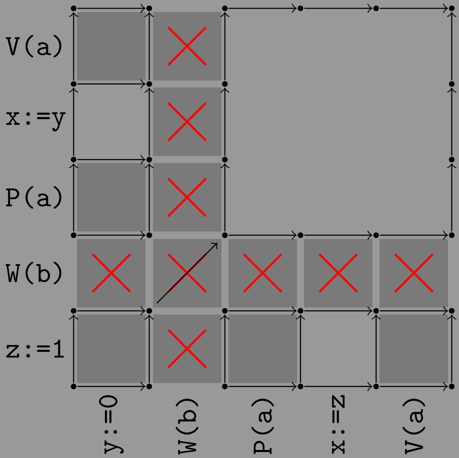

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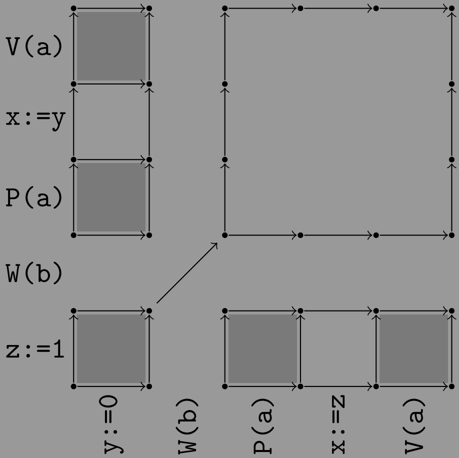

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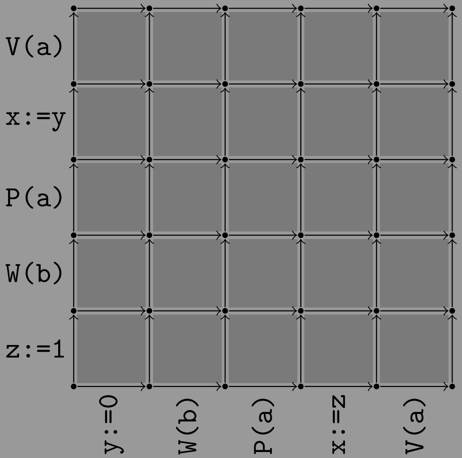

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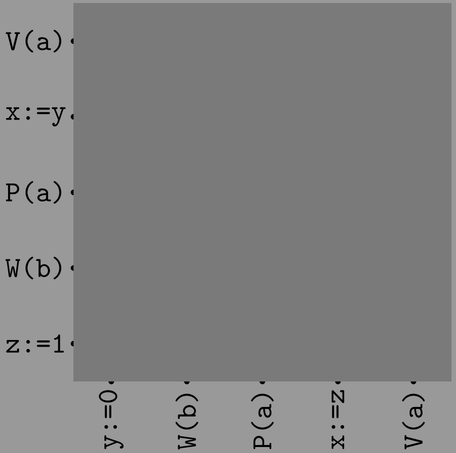

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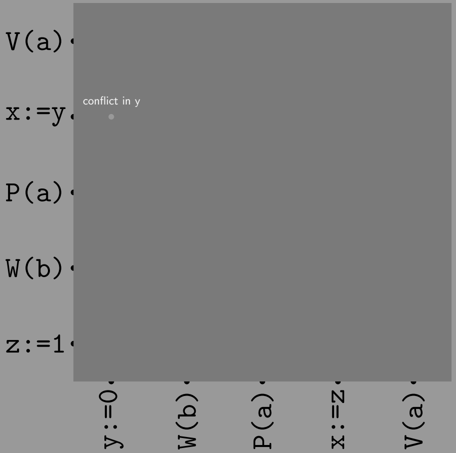

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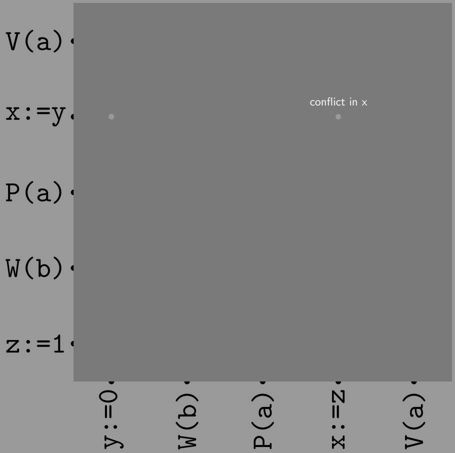

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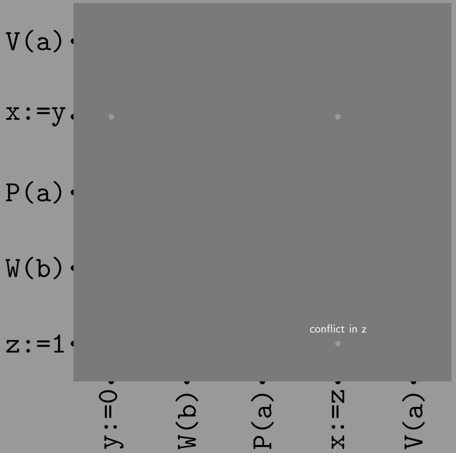

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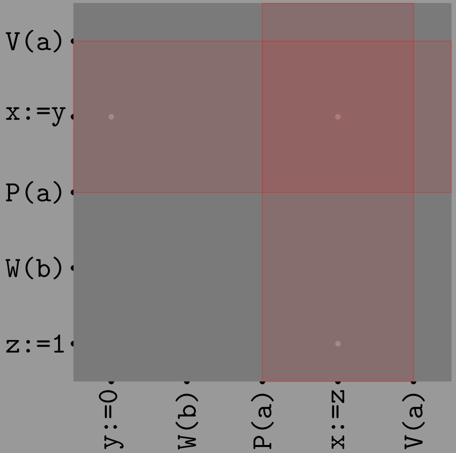

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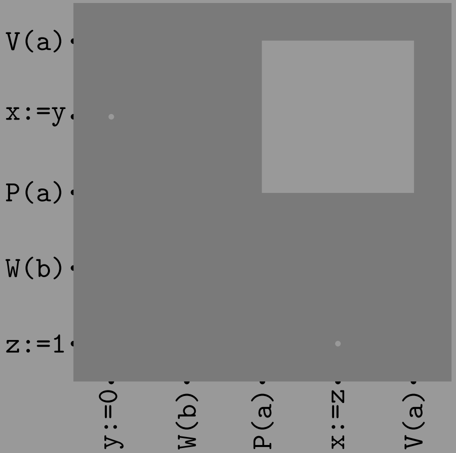

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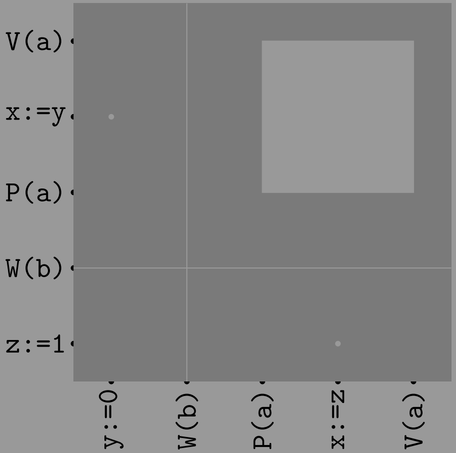

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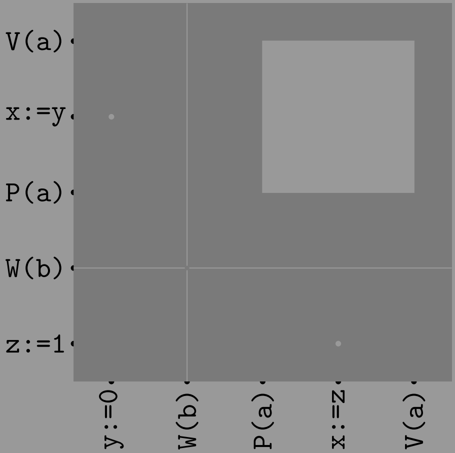
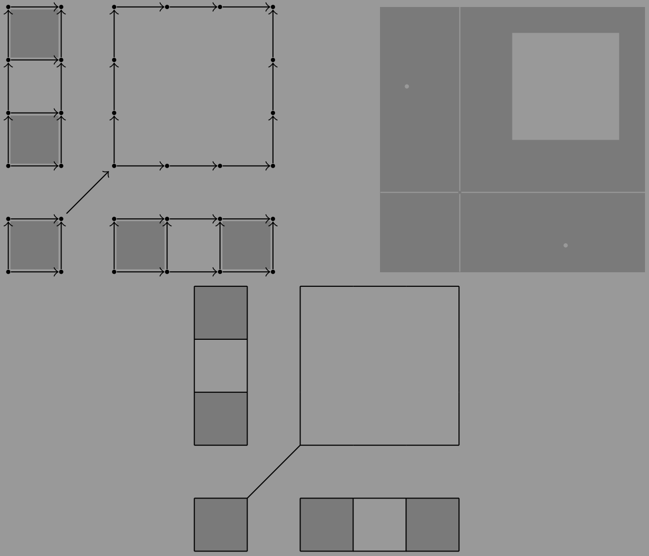


Comparing

Discrete vs Continuous



Cubical areas

- A pospace is a topological space with a closed partial order
- The Morphisms of pospace are the continuous increasing maps
- A *n*-cube is the product of a *n*-uple of intervals of $\mathbb{R}$
- A *n*-cubical area is a finite union of *n*-cubes
- A *n*-cubical area inherits a pospace structure from $\mathbb{R}^n$

Prime decomposition theorem for areas

- areas form a commutative monoid with cartesian product
- this commutative monoid is free
- prime decomposition of $[\![P]\!]$ provide information about parallel decomposition of P.

Dipath

on a cubical area X

- Dipath are continuous increasing maps $\gamma : [0, r] \to X$ with $r \geqslant 0$, $\partial^- \gamma = \gamma(0)$ and $\partial^+ \gamma = \gamma(r)$
- Concatenation $\gamma \cdot \delta : [0, r + r'] \to X$ when $\partial^- \gamma = \partial^+ \delta$;

$$\gamma \cdot \delta(t) = \begin{cases} \delta(t) & \text{if } t \leqslant r \\ \gamma(t) & \text{if } r \leqslant t \end{cases}$$

- If X is the model of a program then the dipaths on X is an overapproximation of the execution traces
- Infinitely many paths between two points

Elementary homotopy

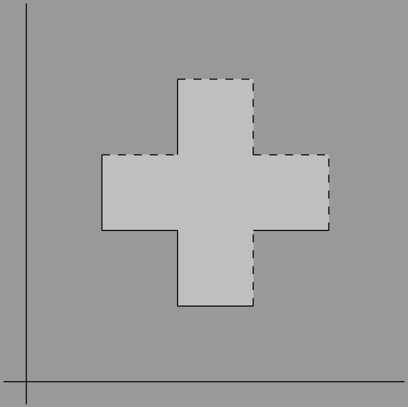
- dihomotopy $h:[0,r]\times[0,\rho]\to X$ a morphism s.t.
  $h(0,\_)$ and $h(r,\_)$ are both constant
- anti-dihomotopy $h:[0,r]\times[0,\rho]\to X$ such that
$(t,x)\mapsto h(t,-x)$ is a dihomotopy
- elementary homotopy $h_n*\cdots*h_1$ where each h_k
  is either a dihomotopy or an antidihomotopy
- $\gamma\sim\delta$ when there exist an elementary homotopy between
$\gamma\theta$ and $\delta\psi$ for some θ and ψ both increasing and surjective,
and sharing their domain of definition.

Characterizing dihomotopy classes through areas

- X cubical area, for all dipath γ there exist a cubical area s.t. $\delta \sim \gamma$ iff img(δ)$\subseteq A_\gamma$
- in fact $\gamma \sim \delta$ iff $A_\gamma = A_\delta$
- further there is a finite collection $\mathcal{K}$ of subareas of X such that for all γ and δ sharing their extremities,

$\gamma \sim \delta$ iff for all $K \in \mathcal{K}$, img(γ)$\subseteq K \Leftrightarrow$ img(δ)$\subseteq K$

Dihomotopy classes as cubical areas



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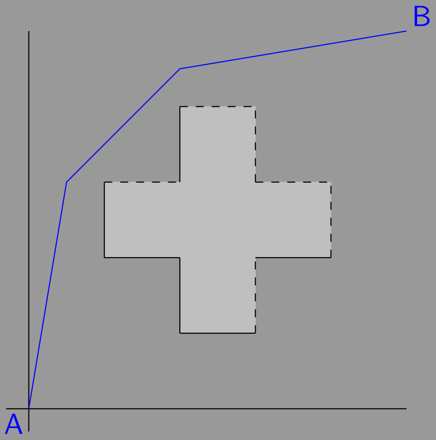

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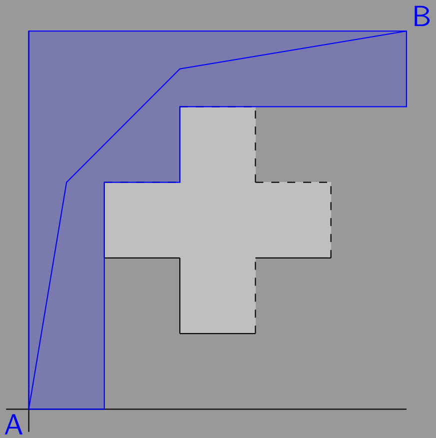

Thank you