Consequences of Independence Assumptions in Flow Planning

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Abstract

We analyze the impact of random disturbances (delays, cancellations, etc) on flight schedules. We first show that disturbances that occur completely at random cause systematic gaps between the number of planned and of observed flights. These gaps can be expressed by a linear function when the arrival process is Poisson. They follow a non-linear function for the empirical arrival process. We then analyze past flight data and show that dependencies between the disturbances of successive flights exist in reality. We identify similarities with two linear time-series models. The result is empirical and we give two ideas for further analysis before interpretation. We validate our results on randomly chosen sectors from the European Airspace. We conclude that even if all controllable uncertainties are eliminated, systematic gaps between the number of planned and observed flights have to be expected. This analysis is a step to understand the impact of uncertainties on air-traffic flow planning. Based on this one can identify flight schedules that lead to a minimum of gaps.

1 Introduction

The European airspace is a network of sectors and routes. *Sectors* are geographical regions and *routes* connect sectors. Currently more than 600 sectors and more than 4000 routes build the transportation network, serving more than 22.000 daily flights to take place. Every aircraft has An *initial schedule*. This is a sequence $(S_1, t_1), ..., (S_n, t_n)$ of sectors S_i and entry times t_i in the sector. Prior to departure, a flow planning procedure assigns departure slots to aircraft to avoid congestion. This results in a *planned schedule* $(S_1, t_1'), ..., (S_n, t_n'), t_1' \geq t_1$ with possibly delayed departure times. Each sector is supervised by two controllers, one in charge of conflict detection, and the other in charge of conflict resolution. This is achieved by re-routing and speed adjustments to keep minimum distances between aircraft. Slot allocation takes place at least two hours before the intended take-off time. Air-traffic controllers can thus anticipate the amount of traffic in their sec-

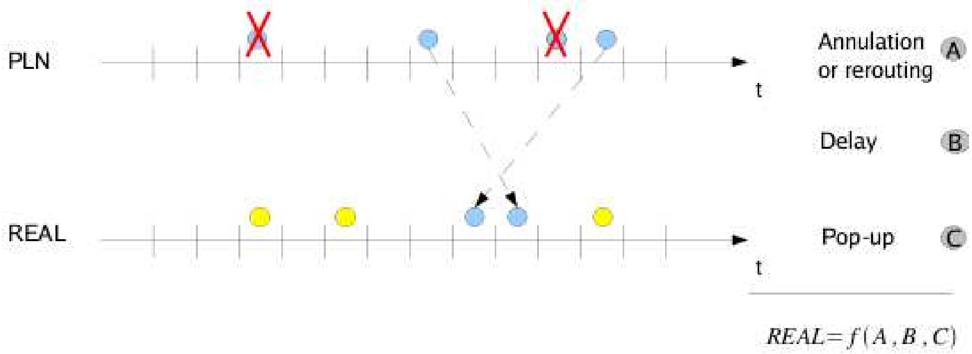

Figure 1: Events at a sector entry.

tors.

Figure 1 shows three events that disturb the flow planning.

(a) Cancellation and rerouting: some of the scheduled flights are canceled or rerouted to other sectors.

(b) Delay: the arrival times of aircraft can be delayed. Delay occurs either at the departure airport or during flight.

(c) Pop-up: aircraft that are rerouted from other sectors or that have not submitted their schedules arrive.

Disturbances transform a regulated schedule into an *observed schedule* $(S'_1, t''_1), ..., (S'_m, t''_m)$. Thus, aircraft may arrive at different sectors and at different time-points than planned. For example in the year 2004, 17.7 % of the flights departed- and 18.5 % arrived more than 15 min behind their schedule [Commission, 2006].

It is natural to think that random events deviate flights from their schedules. For example, weather conditions or unpredictable events (e.g. passenger delay or technical failure) constantly affect the system. One likes to assume that such events do not disturb the flow planning systematically. In average, their effects should be canceled out.

On the other hand there are mechanisms that systematically disturb the flow planning. For example, airlines use the cancellation of flights as a strategy to avoid high departure delays [Mukherjee et al., 2005]. Next, departure delays are often caused by delayed connecting flights, which can propagate over the whole network [Eurocontrol, 2002]. Moreover, pilots have different departure strategies; five minutes before or ten minutes after the scheduled departure slot [Eurocontrol, 2002].

Deviations from schedules lead to *gaps* between the number of planned and observed aircraft entering sectors. This causes safety problems and sub-optimally used capacity. In this paper we analyze conditions for the *systematical occurrence* of such gaps. To do so, we analyze the following hypothesis:

> 'Gaps between planned and observed traffic are exclusively due to random fluctuations in the sector entry times.'

If this is the case, they are a natural, and unavoidable characteristic of the flow system. If not, non-random forces apply to the system and the occurrence of gaps might be controllable.

Related Work [Wanke et al., 2004] decompose sector demand uncertainties into the same three categories 'cancellation, delay and pop-up' as above. They further subdivide the 'delay' component into 'routing/altitude, departure time and flight progress'. They analyze the distributions of all components, conditionally to external, discrete variables. For example the probability of a pop-up depends on the time-of-day, the day-of-week and of the prediction horizon. More complex distribution models, for example for re-routing, are developed, too. They identify that pop-ups can be described by geometric distribution functions. The other components do not allow for substantial conclusions.

[Mukherjee et al., 2005] propose an analytic model for the probability of cancellation at congested airports. Their assumption is that airlines cancel flights when the expected delays exceed a threshold. This is modeled as a maximum flow problem, where the flows are constrained by the initial demand and the threshold. The probability of cancellation is then the ratio between the maximum flow and the initial demand.

In the same paper, [Mukherjee et al., 2005] describe an analytic model for the en-route delay that is caused by capacity limitations at the arrival airport. Such delay is materialized by air-holdings. They assume a non-homogeneous Poisson process as arrival process and an Erlang distributed runway usage time. Given the hourly demand and capacity schedules, they derive the probability distribution $P_t(n)$ of having n aircraft in the system

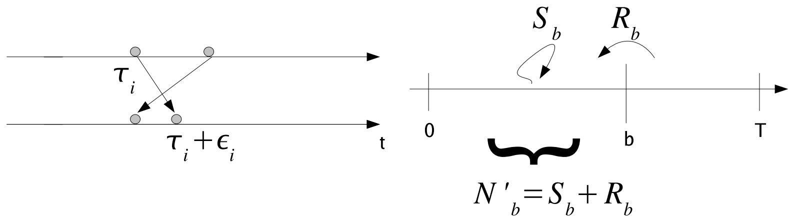

Figure 2: Relationship between randomly disturbed Point processes. Left: Point process. Right: Counting process.

at time t. They validate their models in a simulation game with decision makers from the airline industry and conclude that it can be used as a decision tool in the strategic slot allocation procedure.

None of them, however, conclude that gaps between planned and observed traffic occur *systematically*.

Outline The paper consists of two parts: In the first part, we analyze the effect of random disturbances on flight schedules. We build a probabilistic model of traffic flow and on disturbances of it. We derive analytically and experimentally the relationship between classes of flight schedules and their disturbed versions. From this we conclude that gaps between the number of planned and observed traffic are to be expected even when disturbances occur completely at random. In the second part we analyze past flight data to see how these disturbances look like in reality. We discover patterns in the sample autocorrelation function that are similar to those of classical and long-memory time-series models. We conclude that dependencies between the disturbances of successive flights exist. We then interpret and discuss the results. Ideas for future work finish the article.

2 Formalization

We analyze the effect of random disturbances on the arrival times of aircraft in a flight sector.

More formally, we analyze the model:

$$\tau_i' = \tau_i + \epsilon_i \quad , i \in \mathbb{N} \tag{1}$$

with

$$\tau_i' : \quad \text{observed arrival time of aircraft } i \ \ (\in \mathbb{R}) \tag{2}$$

$$\tau_i : \quad \text{planned arrival time of aircraft } i \ \ (\in \mathbb{R}) \tag{3}$$

$$\epsilon_i : \quad \text{disturbance of aircraft } i \ \ (\text{a continuous random variable}) \tag{4}$$

to answer two questions: (i) what is the relationship between the processes $\{\tau_i'\}$ and $\{\tau_i\}$ when the disturbances (4) occur completely at random ? And (ii) how do these disturbances look like in reality?

We consider $\{\tau_i'\}$ and $\{\tau_i\}$ as sequences of events, also called Point processes [Cox and Isham, 1980]. Point processes can be characterized by the arrival times τ_i or by the number of arrivals $N(0,b)$ in the interval $(0,b]$, called *counting process*. Figure 2 shows the main ideas. A planned arrival at time τ_i is disturbed randomly and occurs at $\tau_i + \epsilon_i$ (left panel). The observed counting process $N'(0,b)$ is the number of aircraft $S(0,b)$ that remain in the interval $(0,b]$ plus the number of arrivals that enter the interval from the right $R(b,T)$ (right panel).

Operations on point processes in equation 1 are called 'random translation' [Cox and Isham, 1980] or more generally 'Point Process systems' [Brillinger et al., 2002]. Random translation is a classical operation on Point processes in time [Cox and Isham, 1980, Snyder and Miller, 1991]. For example, it is known that the rate of a stationary Point process does not change by random translation. Under certain conditions, the resulting process is asymptotically Poisson [Cox and Isham, 1980].

3 Results

We have two results. The first one is about the impact of pure random disturbances on flight schedules. It shows that the relationship between the planned and observed counting process is linear when the planned process is Poisson. When no hypothesis about the planned process is made, this relationship becomes non-linear. The second result shows that dependencies between successive disturbances exist in reality. Taken both results together one can say that systematic gaps between planned and observed traffic have to be expected even when all controllable disturbances are eliminated. This raises the question whether there are schedules that lead to a minimum of gaps.

3.1 Consequences of Independence Assumption

In this part we analyze the impact of pure random disturbances on planned flight schedules. For this we consider $\{\epsilon_1, \epsilon_2, ..., \epsilon_n\}$ as independently and identically distributed random variables with mean $\mu_i = \mu$ and variance $V_i = \sigma^2$. We have a theoretical and an experimental result about the corresponding counting processes.

Definition A *point process* is a random process whose realizations consist of times $P = \{\tau_i\}, \tau_i \in \mathbf{R}, i \in \mathbb{Z}$

Definition Let P be a Point process. A *random translation* $\{\tau_1, ..., \tau_n\} \rightarrow \{\tau_1 + \epsilon_1, ..., \tau_n + \epsilon_n\}$
where ϵ_i are random variables, results in a Point process P' in which points in P are shifted to new locations.

Definition Let $N_{ab} = card\{\tau_i : a < \tau_i \leq b\}$ be the number of arrivals in $(a, b]$ in a Point process P. Let P and P' be two Point processes.

$$\mathbf{E}(N'_{ab} \mid N_{ab} = k) = \sum_{l=0}^{\infty} l \cdot Pr(N'_{ab} = l \mid N_{ab} = k)$$

is called the *conditional expectation of process P' given P.*

Theoretical Result Let $P = \{\tau_i\}$ be a Poisson process with rate λ. Let $P' = \{\tau_i + \epsilon_i\}$ be a random translation of P with disturbances ϵ_i that are (i) independently and (ii) identically distributed with mean μ and variance σ^2.
Then the conditional expectation of P' given P is linear.

Proof: (sketch) We derive the conditional distribution of the number of arrivals in the disturbed process given the number of arrivals in the planned process and take its expectation:

$$\mathbf{E}(N(\cdot)'|N(\cdot) = k) = kp_S + \lambda p_R$$

where p_S is the probability of points remaining in the interval $(a, b]$ and p_R is the probability of points entering the interval (see Figure 2 for the idea and the appendix for the whole proof).

Bounds We give bounds for the probabilities p_S and p_R. Let $U_{ab} \sim Unif(a, b)$ and ϵ with $\mathbf{E}(\epsilon) = 0, V(\epsilon) = \sigma^2$ be two independent random variables.

1. From Tchebycheff's inequality follows:

$$p_S = Pr(U_{ab} + \epsilon \in [a, b]) \geq \frac{2}{3} - \frac{4\sigma^2}{(b-a)^2}$$

   For example $Pr(U_{a,a+5} + \epsilon \in [a, a+5]) \geq 0.1$ for $\sigma^2 > 3$ and $Pr(U_{a,a+10} + \epsilon \in [a, a+10]) \geq 0.1$ for $\sigma^2 > 15$.

2. From Cramer's inequality follows:

$$p_R = Pr(U_{bT} + \epsilon \leq b) \leq \frac{1}{4} \frac{(T-b)^2 + 12\sigma^2}{(T-b)^2 + 3\sigma^2}$$

   For example $Pr(U_{b,b+60} + \epsilon \leq b) \leq 0.26$ for $\sigma^2 \in \{0, .., 20\}$

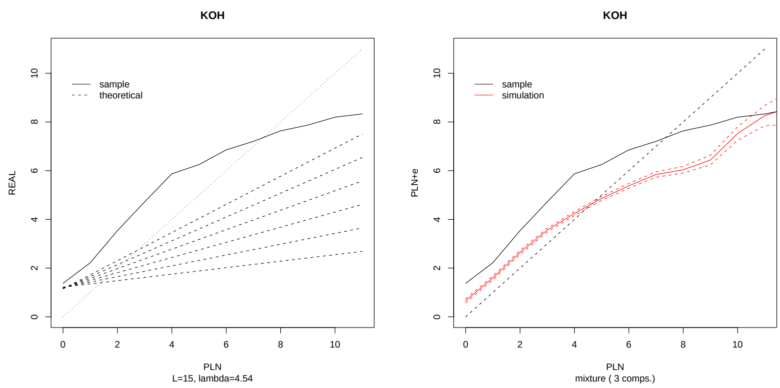

Figure 3: Conditional expectations of randomly translated Point processes. Left: Poisson process. Right: empirical arrival process.

□

The theoretical result states that systematic gaps between the planned and the observed counting process exist when the only force on the flight schedules is that the arrival times are randomly disturbed. When the arrival process is Poisson, such gaps are linear in the planned counting process. This is illustrated in the left part of Figure 3. It shows the conditional expectation of the disturbed process given the planned process for different values of σ^2, the standard deviation of the disturbance of an aircraft (dotted lines). The sample means are plotted in bold. Clearly, such linear functions do not correspond to the sample data. Either the Poisson assumption is wrong, the disturbances ϵ_i are not identically and independently distributed in reality or other forces than random translation apply to the system.

Experimental Result When the disturbances ϵ_i are independently and identically Gaussian distributed and no hypothesis about the arrival process $\{\tau_i\}$ is made, the conditional expectation of the disturbed process given the planned process is non-linear.

To obtain the experimental result, we simulate the model $\tau_i' = \tau_i + \epsilon_i$, where $\{\tau_i\}$ is the planned arrival process and ϵ_i are independently and identically distributed Gaussian random variables. The right part of Figure 3 shows a typical result. It displays the conditional expectation of the simulated process (y-axis) given the planned process (x-axis). The red curve is the simulation result, where the ϵ_i are drawn from a Gaussian distribution with mean=5 and variance=10, which are realistic values. The black curve shows the sample means. The simulation result has a similar shape than the sample means. It lies throughout below the sample means. The disagreement is larger for high traffic densities (PLN ≥ 4) Agreement between the two schedules is tested with the null-hypothesis $H0: \mu_{s_i} = \mu_i, \forall i$, where μ_{s_i} is the simulated mean value of the number of arrivals under the condition that i have been planned. μ_i is the corresponding sample mean. In the case of Figure 3, the hypothesis is rejected in 9 of the 10 cases (t-test, 5 % level, two-sided).

Simulations of Gaussian random translations are run on twelve randomly selected sectors. The results are summarized in the table below:

Noise	$\geq 90\%$	$\leq 30\%$	**no hd**
Gauss	17 %	25 %	74 %

The results of the t-tests are classified by two criteria: agreement $\geq 90\%$ and agreement $\leq 30\%$. 17 % of the sectors have good agreement and 25 % have strong disagreement with the sample data (first and second column). Moreover, the disagreement occurs mainly (74%) in situations with high traffic densities (third column).

We conclude firstly that gaps between planned and observed counting processes have to be expected, even when disturbances occur completely at random. And secondly, either the disturbances are not independently and identically distributed in reality or other forces than random translation apply to the system.

3.2 Data Analysis of disturbances

In this section we analyze the possibility that dependencies exist in the sequence $\{\epsilon_1, \epsilon_2, ..., \epsilon_n\}$.

Dependencies between successive terms Figure 4 shows the time plot of the disturbances ϵ_i. The mean seems to increase and decrease in periods of 30 units. No global time trend is visible. The variance is constant over time.

The left part of Figure 5 shows sample autocorrelation and partial autocorrelation of the disturbances ϵ_i. Autocorrelations start at ≈ 0.1 and decay slowly until lag 40. The partial autocorrelations decay until lag 15. Interpretation is difficult, non-stationarity in the mean may cause spurious coefficients.

The right part of Figure 5 shows sample autocorrelation and partial autocorrelation of the first difference of the disturbances $\nabla\epsilon_i = \epsilon_i - \epsilon_{i-1}$. A single peak of -0.49 at lag 1 of the acf and an exponentially decaying pattern in the pacf can be seen. This is the characteristics of an IMA(1,1) process:

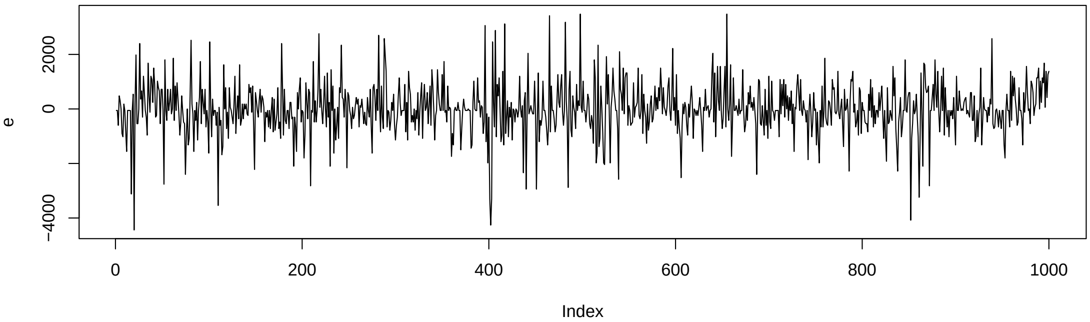

Figure 4: Time plot of the disturbances ϵ_i.

$\epsilon_i = a_i + (\epsilon_{i-1} - \theta a_{i-1})$ with a_i i.i.d. random variables. An interpretation is that random events (e.g. weather conditions at departure airport) cause a delay a_i of aircraft i and (very weakly) of its successor $i-1$.

But there is no justification to difference the data because no linear trend and no random walk can be assumed a priori. A possible explanation is an autoregressive dependency close to 1. To analyze this we pose the ARMA(1,1) model:

$$\epsilon_i = \phi\epsilon_{i-1} + \theta a_{i-1} + a_i \qquad (5)$$

with a_i i.i.d. random variables. For $\phi = 1$, the special case IMA(1,1)

$$\nabla\epsilon_i = \theta a_{i-1} + a_i \qquad (6)$$

arises.

The autocovariance function of the ARMA(1,1) is

$$\gamma(k) = \phi\gamma(k-1), \quad k \geq 2$$

It behaves like that of an AR(1) scheme after the first lag. The IMA(1,1) is non-stationary; the autocovariance is not a function of the lag k. ARMA(1,1) and IMA(1,1) models are applied in industrial control, where the impact of random disturbances on the production scheme is studied [Box and F.Jenkins, 1976]. Our analysis is exploratory; we do not have a priori arguments for neither of the models. However, a direct interpretation for model 5 is that a delayed aircraft is followed by a delayed aircraft. Moreover, external events disturb i and its successor. On the other hand, due to duality results, an

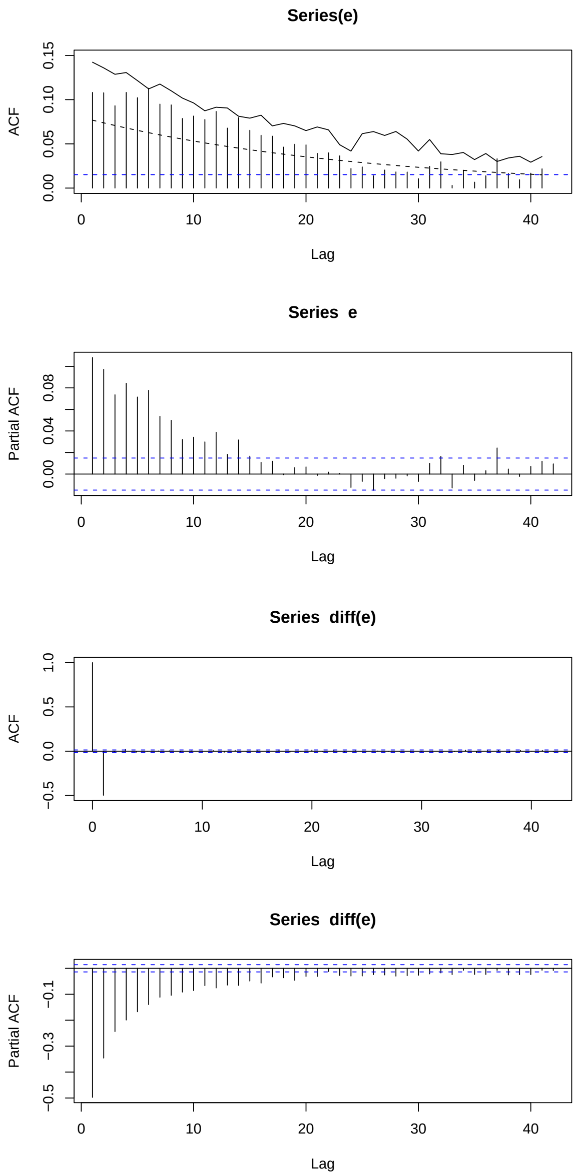

Figure 5: Sample autocorrelation and partial autocorrelation plots. Left: ϵ_i. Right: $\nabla\epsilon_i = \epsilon_i - \epsilon_{i-1}$

Name	Model	ϕ	θ	σ^2	AIC
ARMA	$\epsilon_i = \phi\epsilon_{i-1} + \theta a_{i-1} + a_i$	0.96 (0.005)	-0.88 (0.007)	18.11	291884.4
IMA	$\epsilon_i = \epsilon_{i-1} + \theta a_{i-1} + a_i$	1	-0.94 (0.004)	18.24	292110.1

Table 1: Comparison ARMA(1,1) and IMA(1,1).

ARMA(1,1) model represents a large class of processes. Interpretation can become ambiguous [Kendall, 1989].

Table 1 compares the fit of an IMA(1,1) model with an ARMA(1,1) model to our data. The parameters are obtained by exact maximum likelihood estimation. In the ARMA model, the AR parameter ϕ is close to 1 but significantly different from it. The MA-parameter θ is -0.88. There is also a mean value μ estimated (not shown). The variance of the unexplained part a_i is 18.1^2 min. For the IMA model, the MA parameter is -0.94. No mean is included in the model. The estimated variance of the unexplained series a_i is 18.24^2 min. The AIC of the ARMA model is lower than for the IMA model, despite the larger number of parameters. But the main argument against the IMA model is that differencing does not make sense physically. Figures 8 and 7 show diagnostic plots for the ARMA model. The residuals contain no trend and no autocorrelation. The normality assumption, however, cannot be justified. Table 2 contains the parameter estimations for eight randomly selected sectors of the European airspace. The autoregressive parameter is always close to 1 and the moving-average parameter is always negative.

To better understand this result we look at sequences of correlated deviations. A simple pattern would be that long sequences of correlated disturbances exist. This could be interpreted by events at departure airports that affect several aircraft (e.g. runway congestion). Figure 6 shows the distribution of the lengths of correlated sequences. There are rarely more than two successively correlated arrivals and never more than 6. Thus the arrival patterns are heterogeneous. This is the same for sectors with only one arrival route and for more complex sectors. This has to be analyzed in more detail.

We conclude that the disturbances ϵ_i are not independent in reality. Their dependencies show similar characteristics than IMA(1,1) and ARMA(1,1) models. However, these findings are empirical and both models cannot be justified a priori.

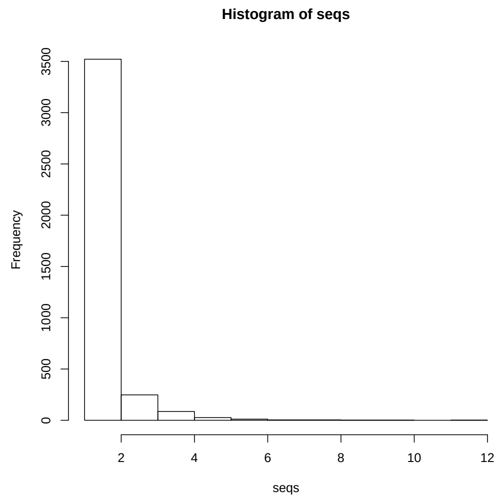

Figure 6: Distribution of the lengths of sequences with correlated successive arrivals.

4 Interpretation

The analysis of consequences of independence assumptions (section 3.1) shows that systematic gaps between planned and observed counting processes are to be expected, when disturbances occur completely at random. When the planned process is Poisson, these gaps can be described by a linear function of the counting process, no matter how the distributions of the random disturbances look like. This is intuitive since the two processes become independent for large σ^2, and the gaps between them are then described by a horizontal line through $p_R\lambda$. The linearity in this model can be explained by the fact that all terms enter the distribution in additive and constant ways. When the planned process is the empirical flight schedule, the gap function becomes non-linear. Thus the non-linearity of the observed gaps is partly due to the structure of the arrival process.

The data analysis (section 3.2) shows that dependencies between the disturbances of successive flights exist in reality. We identified two linear time-series models with similar dependency patterns. However, an arrival flow consists of aircraft from different origins and of different types. Moreover, the correlation structure shows that the correlation last up to $\approx 2h$. A possible interpretation is that delayed aircraft keep their delay on their way back. All this suggests that the data consists of superpositions of different processes. This has to be analyzed in more detail before justifying a data generating mechanism.

5 Conclusion and Future Work

We analyzed the impact of random disturbances on flight schedules. We modeled the planned arrival process into a flight sector as a Point process and the observed arrival process as a random translation of the planned process. We analyzed two classes of arrival processes; a Poisson process and the empirical arrival process obtained from schedule data. Our results are (i) when the disturbances are purely random, systematic gaps exist between the number of planned and observed traffic entering flight sectors. And (ii) that the disturbances are not independently distributed in reality.

From this we conclude that even if the current dependencies were eliminated, systematic gaps in flight planning would remain. For future work we propose therefore (i) the identification of classes of flight schedules that are robust to random disturbances. This leads to new constraints in the flow planning algorithms. And (ii) a better understanding why the current disturbances are correlated. For this we suggest to disaggregate the data to identify homogeneous groups of aircraft with similar characteristics. One important such group is the one that generates long-term autocorrelations, because aircraft generally fly the same route several times a day. There are similar phenomena in the analysis of data from telecommunication networks [Cappé et al., 2002].

This analysis is a step to understand the impact of uncertainties on air-traffic flow planning. This builds the basis for a reasonable optimization of available resources.

6 Acknowledgements

The author likes to thank Philippe Baptiste and Pierre Collet for constructive discussions.

A Proof of the theorem

Figure 2 shows the idea. The planned arrival time τ_i of an aircraft is disturbed by some ϵ_i, translating the process P into the process P' (left). For the proof, we are interested in the counting process N_b' of the randomly translated process P', dependent on the initial process P. For this we need to know the number S_b of aircraft remaining in (0,b] and those R_b that arrive additionally (right).

Let:

1. $P = \{\tau_i\}$ a Poisson process with rate λ (the planned schedule)

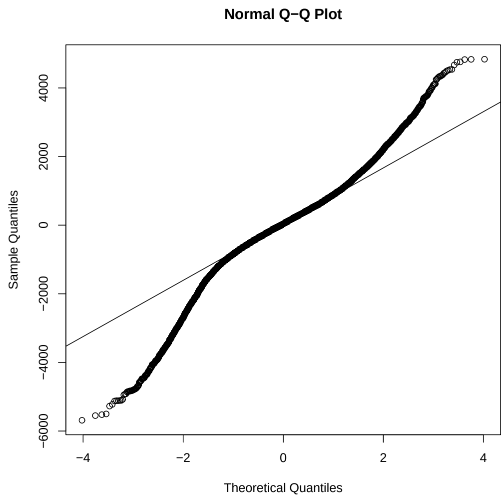

Figure 7: QQ plot of the residuals of the ARMA model.

2. $P' = \{\tau_i + \epsilon_i\}$ a random translation of P (ϵ_i are i.i.d. random variables with mean μ and variance σ^2, independent of τ_i).

3. N_{ab}, N'_{ab}: number of arrivals in P, P' in $(a, b]$.

4. $S_{ab} = card(\{\tau_i + \epsilon_i \in (a, b] \mid \tau_i \in (a, b]\})$: number of arrivals remaining in $(a, b]$

5. $R_{ab} = card(\{\tau_i + \epsilon_i \in (a, b] \mid \tau_i \geq b\})$: number of arrivals entering $(a, b]$ from the right.

6. U_{ab}: a uniformly distributed random variable in $(a, b]$ (the planned arrival time of an aircraft).

Probability distribution We derive $\mathbf{E}(N'_{ab}|N_{ab} = k)$ as a function of k and σ^2. Since $N'_{ab} = N'_{0b} - N'_{0a}$ we only need to derive $Pr(N'_b \mid N_b)$.

$$\begin{aligned} Pr(N'_b = n|N_b = k) &= Pr(S_b + R_b = n|N_b = k) \\ &= \sum_{l=0}^{n} Pr(S_b = l, R_b = n - l|N_b = k) \end{aligned}$$

since S_b and R_b are independent:

$$= \sum_{l=0}^{n} Pr(S_b = l|N_b = k) Pr(R_b = n - l|N_b = k)$$

and R_b is independent of N_b:

$$= \sum_{l=0}^{n} Pr(S_b = l | N_b = k) Pr(R_b = n - l)$$
$$= \sum_{l=0}^{n} Pr(S_b = l | N_b = k) \sum_{j=0}^{\infty} Pr(R_b = n - l | N_{bT} = j) Pr(N_{bT} = j)$$

Let $p_1 = Pr(U_b + \epsilon \in [0, b])$, $p_2 = Pr(U_{bT} + \epsilon \in [0, b])$. Then

$$= \sum_{l=0}^{min(n,k)} \binom{k}{l} p_1^l (1 - p_1)^{k-l} \sum_{j=n-l}^{\infty} \binom{j}{n-l} p_2^{n-l} (1 - p_2)^{j-n+l} \frac{\lambda^j e^{-\lambda}}{j!} \tag{7}$$

$$= \sum_{l=0}^{min(n,k)} \binom{k}{l} p_1^l (1 - p_1)^{k-l} \frac{(\lambda p_2)^{n-l} e^{-\lambda p_2}}{(n-l)!} \tag{8}$$

This is the sum of a $binomial(p_1, k)$ and a $Poisson(\lambda p_2)$ variable. It follows:

$$\mathbf{E}(N'_b | N_b = k) = kp_1 + \lambda p_2 \tag{9}$$

Notes

- There is no dedicated point 0 in the aircraft application. It is possible, that points 'leave' the interval $(0, b]$ to the left or 'arrive' from the left (before midnight). This would lead to $2\lambda p_2$ in (8), when the intervals are of equal length (and to $\lambda p_2 + \mu p_3$, when the processes and intervals are different). In both cases, the linearity in k of the conditional expectation (9) remains.
- The variance of the distribution (8) is $kp_1q_1 + \lambda p_2$ (the variables are independent). Thus, the distribution is not Poisson. For high k and low p_1 it can also be approximated by a Poisson distribution.
- The impact from random rerouting is a simple extension of the results. It corresponds to thinning (cancellations, rerouting) and superposition (arrival of rerouted aircraft), which are linear operations.

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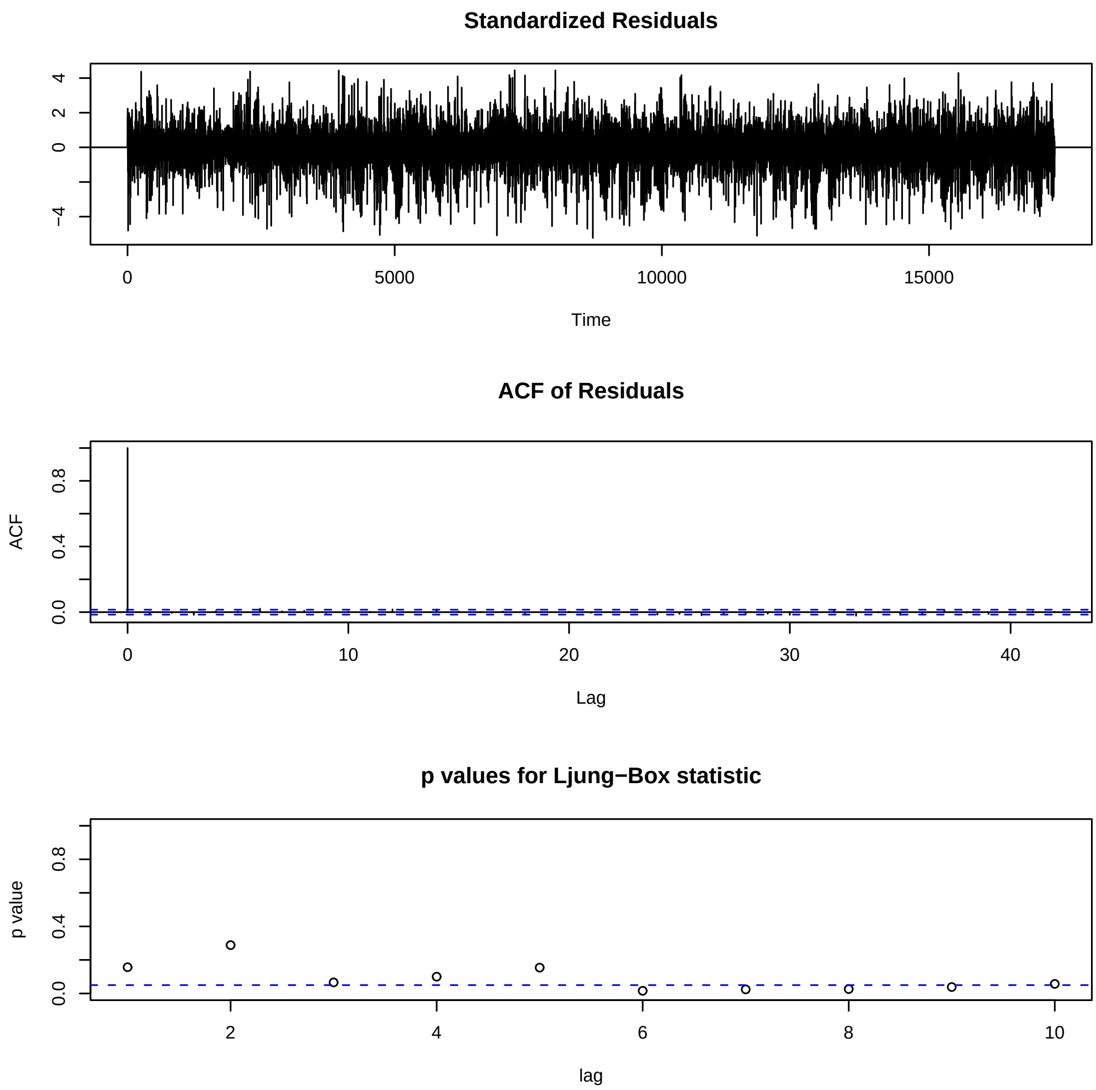

Figure 8: Diagnostics for the ARMA(1,1) model. $\epsilon_i = 0.96\epsilon_{i-1} + -0.88a_{i-1} + a_i$. Top: residual time plot. Middle: acf of the residuals. Bottom: p-values for the portmanteau test.

Sector	ϕ	θ	μ	σ^2	loglik	AIC
LUE	0.98 (0.003)	-0.92 (0.005)	0.9 (29.4)	1359967	-171041.9	342091.7
CLW	0.95 (0.005)	-0.88 (0.007)	62.2 (14.6)	813595	-154553.4	309114.9
EXN	0.97 (0.003)	-0.92 (0.006)	196.3 (21.8)	766710	-141918.0	283844.0
EUY	0.98 (0.002)	-0.93 (0.004)	204.0 (21.1)	781522	-176817.1	353642.2
LUW	0.97 (0.003)	-0.91 (0.005)	150.7 (27.6)	151341	-166914.3	333836.5
KOH	0.96 (0.005)	-0.88 (0.007)	89.5 (21.2)	1181206	-145938.2	291884.4
NIH	0.97 (0.003)	-0.90 (0.005)	102.1 (21.0)	1034263	-188931.2	377870.5
LNH	0.96 (0.004)	-0.88 (0.006)	69.8 (22.2)	1061602	-150761.3	301530.5

Table 2: Validation ARMA(1,1) for eight randomly selected sectors.

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