

2B10 Recent Models in the Analysis of Air Traffic Flow

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Abstract

We review recent microscopic and macroscopic models of air traffic flow. The purpose of these models is to discover relationships between system variables, such as the impact of trajectory uncertainties on ATM performance or the impact of controller's strategies onto the flow patterns in far lying sectors. We also review the current status of Japanese flow management. As it is known, the major source of congestion is Tokyo International Airport and the main stream to reduce congestion includes an enhanced data flow, traffic synchronisation and internationalisation. Our main conclusion is that pioneering work is necessary in model based flow analysis. But combined with the analysis of flight data, it can provide new insight into the mechanisms of air traffic congestion. Such insight is needed to support strategic decision making in ATM.

1 Introduction

Currently, the world's major air traffic management (ATM) systems are in a process of transformation [1, 2]. One of the reasons is congestion: many airports and their surrounding airspaces are already congested today, but will cause increasing delays in the future if no actions are taken. While the fundamental problems of congestion are not new, there will be new procedures and tools to reduce congestion.

It is known since long time that competition between users of a limited resource creates congestion [3, 4]. This is true not only for air traffic, but also for telephone (or Internet) networks, road traffic, production processes and many more [5]. Congestion typically occurs during peak hours and in bottleneck situations, when the demand of the resource is higher than its limit (for example during bad weather), or when some irregularity in the flows of users exists [4, 5]. Congestion implies delays. In air transportation, delays materialise either on the ground, where aircraft have to wait before accessing a runway or during the flight, where they are deviated from their intended trajectory (vectoring, metering, holdings). Such delays lead to an increase in fuel consumption. During strategic flow planning, delays are a metric to assess the performance of the ATM system.

The common idea in future strategic flow management is to collaborate stronger with airlines [1, 2, 6]. Airline and flow managers will try to distribute the demand of the network several months in advance to avoid delays where possible. As a

main decision support, the evolution of the network will be monitored continuously and adverse network effects will be predicted. Such effects include unanticipated demand/capacity imbalances due to local capacity shortcuts or due to chain reactions in changes of airline strategies. To this end, Europe proposes the 'ATM Network Management function' and the 'User driven prioritisation process' [1]. A similar collaborative planning process will be established in the U.S., complemented by a tactical 'Flow Contingency Management', where unsolved demand/capacity imbalances will be addressed by dynamic airspace redesigns or the allocation of departure slots to runways or time-of-arrival slots in en-route airspace [2]. In Japan, the major source of congestion is the saturated airspace around the metropolitan airports, and the future efforts include an enhanced data flow and traffic synchronisation [6].

The purpose of this article is to review some recent models that allow to quantify and analyse congestion. These models are candidates to support decision making in strategic flow management. We will discuss the ideas of three model types and their underlying methodology. We do not discuss technical details, but we give references to literature where needed. This discussion serves as a basis to recommend flow models for the Japanese Airspace.

The remaining article consists of two parts. In the first part we analyse flow models we found in recent European and the U.S. literature. In the second part we analyse current flow problems in Japanese Airspace.

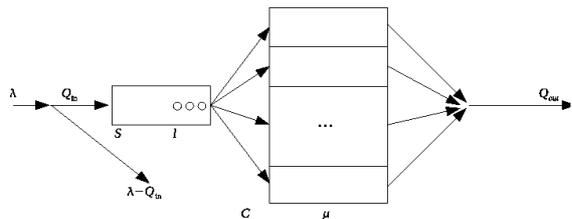


Figure 1: Schema of a single queue for an en-route sector

2 Recent Models

In this section we discuss models that were recently proposed for the analysis of air traffic flow. We focus on the methodology and the principles of the models rather than on technical details. This allows us to see the type of results and the limitations we may expect from a corresponding analysis.

2.1 Queueing Networks

As mentioned in the introduction, congestion usually appears by the combination of

- (i) a flow of customers needing service,
- (ii) some restrictions on the availability of service, and
- (iii) irregularity in the flow of customers, the servicing operation or both.

Moreover, during peak hours or bad weather conditions, where demand is higher than the available capacity, predictable congestion occurs. Systems with the above characteristics are often modelled as queueing systems [5].

In air traffic management, the flow of customers corresponds to aircraft requiring to enter a sector or a runway. The time of traversal through the sector corresponds to the service, which is limited by the sector capacities or safety constraints. Delays materialise either on the ground, where aircraft have to wait before accessing a runway or in-flight, where they are deviated from their intended trajectory. In the past, typical objectives of queueing analysis in ATM were capacity planning [7] or to perform delay predictions [8]. More recently, NASA also showed interest in the impact of 4D trajectory precision on delay [9].

An example of a single queueing system, representing an en-route sector can be seen in Figure 1. Aircraft arrive with rate λ (number of aircraft per hour) requiring entry to the sector. The capacity of the sector is μ (number of aircraft per hour). The fact that multiple aircraft are allowed to enter a sector simultaneously through different

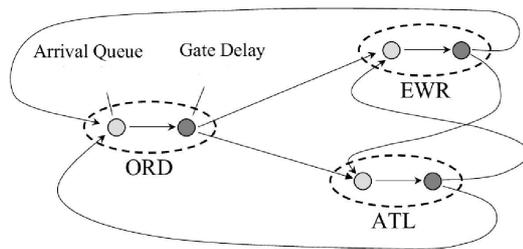


Figure 2: Schema of a queueing network for air traffic flow. Source: [11]

routes is represented by C routes. After crossing the sector, a flow with rate Q_{out} leaves it. When the capacity of the sector is attained, a queue of length l forms in front of the system. It is known that airlines cancel flights when the expected delay is too high (represented by more than S aircraft in the queue) [10]. This rate is represented by the quantity $\lambda - Q_{in}$.

In queueing networks, the output of one queue builds the input of one or a number of other ones. Figure 2 shows the schema for a network used to analyse air traffic flow, taken from Shortle et al. [11]. It consists of 3 airports. Each airport consists of two queues; one for the arrival and one for the gate. The arrival queue models the sequencing and merging of aircraft as they approach the runway. For example, one server corresponds to one runway. The gate queue may consist of several gates and models the turnaround times for aircraft. Once an aircraft leaves an airport, it enters one of its connecting airports (based on flight plan information).

A simplified mathematical analysis of a stochastic queueing network is as follows: let $\mathbf{n} = (n_1, \dots, n_J)$ denote the state of the network, meaning that there are n_i customers at queue i . The state space of the network is $\mathcal{S} = \{\mathbf{n} : n_i \geq 0, i = 1, \dots, J\}$. Now let $a(\mathbf{n}, \mathbf{m})$ be the transition rate from state \mathbf{n} to state \mathbf{m} , i.e. the number of transitions between the two states per unit time. The rule of the game is to restrict transitions to those that can be reached by the movement of exactly one customer: either by a movement from one queue to another or an arrival or a departure from the network. The question then is:

‘What is the probability that the network will be in state \mathbf{n} ?’

To obtain an answer, the joint distribution $p(\mathbf{n})$ has to be known. A main result in queueing networks is that, under some assumptions on the transition rates (for example that the arrival processes at the individual stations are independent Poisson processes), this joint distribution is of product

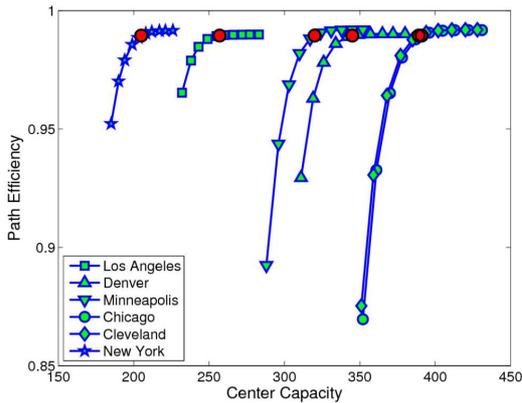


Figure 3: Variation of path efficiency with center capacity. Source: [14]

form: $p(\mathbf{n}) = p_1(n_1) \times \dots \times p_J(n_J)$. This means that the number of customers at queue i are independent random variables. Their distributions are known from single queueing theory, and these results can be used for performance analysis. Networks of this type are called (open) *Jackson Networks* and the above analysis was an example of a steady-state analysis based on a continuous-time Markov chain [5]. Other network types and analysis methods (e.g. non-markovian) can be found in literature [12].

Using a (deterministic) queueing model, Hansen [13] analyses how delays propagate in an arrival flow and concludes that cancelling certain flights would be beneficial for customers. In an ongoing project, Menon et al. [14] and Lovell et al. [15] analyse the effect of trajectory uncertainties on traffic flow efficiency. They model traffic flow networks as a function of flow densities, average velocities and sector capacities. Due to trajectory uncertainties, these quantities can vary. The idea of the paper [14] is to analyse the relationship between these variations and flow efficiency (see definition below). The authors distinguish between national, regional and local networks differing essentially in the number of nodes and level of detail. They then propose a network of national dimension, consisting of 20 air traffic control centres, connected by a number of routes. In order to analyse this network, they assume an open Jackson network with Poisson arrivals, exponential service times and multiple server nodes. The queue parameters and transition probabilities are estimated from past flight data.

As performance metric, they define for a center j the *Traffic Flow Efficiency*

$$\mathbb{E}(E_j) = 1 - \frac{\mathbb{E}(W_{qj})}{\mathbb{E}(W_j)} \quad (1)$$

where W_{qj} is the delay inside the center and $W_j = S_j + W_{qj}$ is the traversal time through the center, which is the sum of the traversal time under optimal conditions S_j and the delay. $\mathbb{E}(X)$ is the expected value of the random variable X . For a route from center i to center j , they then define the *path efficiency* as the average of the traffic flow efficiencies along the path.

A preliminary result of the analysis can be seen in Figure 3. The scenario is a flight route from an airport in the Los Angeles Center to an airport in the New York Center. It passes through four other centres on the route. The figure plots the path efficiency against center capacity for the six corresponding centres, while all other parameters were held at nominal levels. The typical shape of the relationship is a sharp increase in efficiency until a critical value, from which on stable path efficiency occurs (red points). This shape is not unexpected, since it captures the known relationship between delay and flow in a queueing system [5]. What is new is the context of the analysis: assessment of the impact of trajectory uncertainties on traffic flow efficiency. However, the results are preliminary and the authors do not (yet) validate their assumptions underlying the Jackson network.

2.2 Traffic Flow Theory

Compared to air traffic, congestion in road traffic is simpler to observe: it is a standstill of traffic flow. This phenomenon is being analysed since the 1950's, where a relationship between flow and density has been discovered. More recently, the mechanisms of 'phantom traffic jams' (jams caused without bottlenecks) or 'stop-and-go traffic' have also been explained successfully [16], [17]. The basic models in traffic flow are either microscopic, where the motion equation of individual cars is modelled, allowing for example that the acceleration of a car is proportional to the velocity difference with the leading car. Or they are a macroscopic description of the density $\rho(x, t)$ (vehicles per kilometre) or flow $J(x, t)$ (vehicles per hour) of cars along a highway. The focus in theoretical analysis is to understand the stationary solutions of the systems of equations. Historically, the first macroscopic flow model is a continuity equation, called the Lighthill-Whitham-Richards (LWR) equation

$$\frac{\partial \rho(x, t)}{\partial t} + C(\rho) \frac{\partial \rho(x, t)}{\partial x} = 0 \quad (2)$$

where $C(\rho)$ will be explained below. The solution to (2) is called a wave, describing the propagation of an initial traffic pattern $\rho(x, 0)$ with speed $C(\rho)$ (e.g. [18]). For example, if $C(\rho) = c$ were constant (> 0), the initial pattern would just move

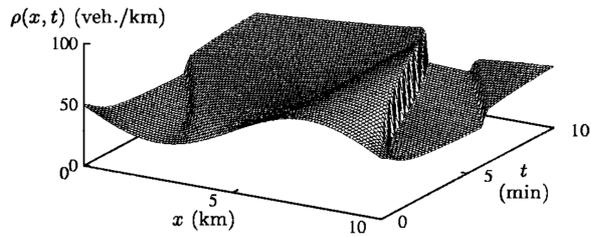


Figure 4: Transformation of initial traffic pattern and formation of shockwaves. Source: [16]

to the right such that $\rho(x, t) = \rho(x - ct, 0)$. In this case, the traffic pattern would not change its shape over time. When $C(\rho)$ depends on ρ , different densities propagate with different velocity, implying that the shape of the initial traffic pattern changes over time, but also that discontinuities in the solution $\rho(x, t)$ (so-called shock waves) appear. Figure 4 shows a spatio-temporal density plot illustrating the formation of shock waves on a circular road. The initial condition is a sinusoidal pattern (x -axis), which builds steeper and steeper gradients over time (y -axis). Moreover, in highway traffic, the vehicle speed usually decreases when the traffic density increases and it can be shown that in this case ρ propagates in opposite direction to the traffic flow. Note that here $C(\rho)$ is the speed of the traffic density and not the speed of the cars. To summarize, eq. (2) describes the propagation of an initial traffic pattern over time. This is usually solved numerically, but its accuracy is impeded by the existence of shock-waves. Moreover, recent empirical studies suggest that the relationship between density and flow is more complex [16].

In air traffic, flow is not a function of density, too. Although speed control can avoid the delivery of aircraft into a congested zone, the controllers need more information than the local density to decide their action. In this context, several authors proposed fluid dynamical models for en-route air traffic flow. Their aim is to analyse the impact of controller's actions in one sector onto the flow patterns in other sectors. This can lead to control strategies like 'Aircraft on airway 148 at FL 330 fly at 450 kn for the next hour and then have to accelerate by 10 kn per half hour'. For example Bayen et al. [19] derive a formulation in which $C(\rho(x, t))$ corresponds to the average speed of aircraft at point x and propose an optimisation where this speed is adjusted such that flow constraints along a route are satisfied. Speed control for only a fraction of aircraft inside a sector has been proposed by Menon et al. [20]. Figure 5 shows the idea. Their model is discrete in space

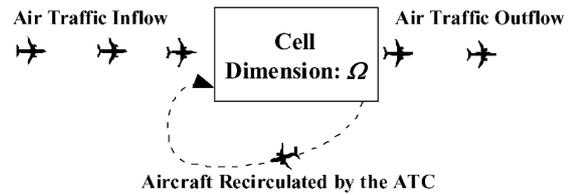


Figure 5: Control of one-dimensional traffic flow. Source: [20]

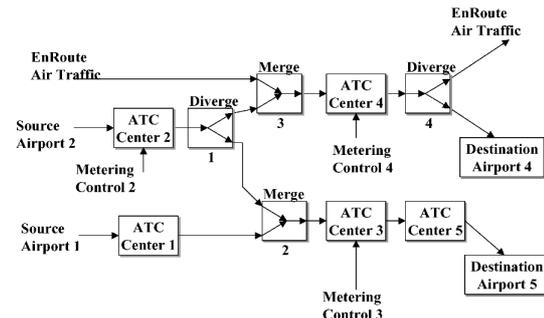


Figure 6: Flow model with merge and diverge nodes. Source: [20]

and in time. A sector x is a one-dimensional volume, with aircraft entering from the previous sector $x - 1$ with rate $J(x - 1, t)$ and leaving at its output with rate $J(x, t)$ per unit time. Air traffic controllers modulate the outflow by varying the speeds or by stretching the paths of some aircraft inside the sector. This mechanism will remove a number u_r of aircraft from the outflow. It is modelled as a loop and is called 'recirculated aircraft'. Mathematically their model can be written as

$$\rho(x, t + 1) = \rho(x, t) + J(x - 1, t) - J_r(x, t) \quad (3)$$

where $\rho(x, t)$ is the number of aircraft in sector x and time interval t and $J_r(x, t) = J(x, t) - u_r(x, t)$ represents the outflow adjusted by the number of recirculated aircraft. Note that this model also adjusts the average speed of aircraft inside a sector, but the interpretation is that u_r aircraft reduce their speed while the remaining $\rho - u_r$ keep their nominal speed.

On a network level, the authors assemble pairs of sectors with so-called merge and diverge nodes, similar to the logic of the queueing network in Figure 2. To illustrate their concept, they build a network consisting of three input flows (two departure airports, one en-route arrival flow), three metering controls, and three output flows (two arrival airports and one en-route departure stream) (see Figure 6).

Table 1: Transfer function analysis of traffic flow.
Source: [20]

Input	Destination airport 5	En-route outflow
Metering control 2	$-0.7(z^{-3} - z^{-4})$	$-0.2(z^{-2} - z^{-3})$
Metering control 3	$-(z^{-2} - z^{-3})$	Gain = 0
Metering control 4	Gain = 0	$-0.8(z^{-1} - z^{-2})$
Departure rate 1	z^{-4}	Gain = 0
Departure rate 2	$0.7z^{-4}$	$0.2z^{-3}$

Technically, their flow model takes the form of a linear, time-invariant dynamical system. This opens the way to analytical investigation, using methods from classical control theory to the system (e.g. [21]).

One example of their analysis concerns transfer functions. A transfer function describes how changes in one of the system inputs (for example the flow control in one sector) affect the changes in system output (the flow rate in another sector) given that all other inputs are at their nominal levels (e.g. [21]). Table 1 shows results for some selected sectors: here, z^{-1} represents a unit time delay (15 min in their application). For example, the transfer function between departure airport 1 and destination airport 5 indicates that any action taken at airport 1 will appear at airport 5 four time units later (z^{-4}). The entries ‘Gain = 0’ mean that the corresponding input has no impact on the output. Unfortunately, the relationship between the other flow control centres and system outputs are more complicated and not yet interpretable.

2.3 Cellular Automata

The basic idea of Cellular Automata (CA) in flow modelling is simple: divide the airspace into cells of equal size and let each cell i either be occupied by an aircraft with speed v_i or be empty. Aircraft follow their flight plan, and in each time-step they move forward v_i cells if the destination cell is free, or they adjust their speed or route by some (stochastic) rule if the cell is occupied. More generally, CA are highly idealised physical systems, in which space and time are discretized and each of the interacting units can have a finite number of discrete states. The concept was introduced in the 50’s by von Neumann and popularised in the

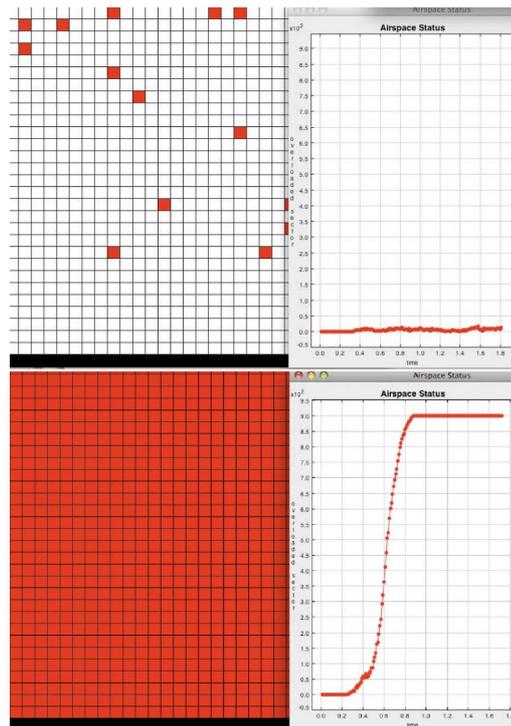


Figure 7: Transition between high and low ATM performance. Top: high performance at low density (30.000 flights). Bottom. low performance at high density (50.000 flights). Source: [22]

80’s by Wolfram [23]. Despite their simplicity, they can produce a complex behaviour, for example the standard model in vehicular traffic flow with only 4 rules (acceleration, deceleration, randomisation, position update) can reproduce the generation of ‘stop-and-go’ waves and spontaneous traffic jams [16, 23].

Since the state of a cellular automaton depends only on its previous state, the mathematical analysis of stochastic cellular automata is based on Markov Chains, similar to the queueing networks discussed above: a cell is in state n in time interval $t + \Delta t$ either when it has been in state n in interval t and no transition out of the state occurred, or if a transition from another state into state n occurred in time interval t . Considerations of this type lead to the ‘Master equation’, which can be solved exactly for very simple automata. Then, average flow rates and conditions for the creation of various jam patterns can be obtained. But the solutions get complicated as boundary conditions and details on the update rules are specified (e.g. [16]).

One feature of dynamical systems are changes in their qualitative behaviour. For example, Japanese researchers conducted an experiment in which car drivers were asked to move with con-

stant speed on a circular road. Naturally, the speed varies from time to time. The researchers found that when the average traffic density is low, such speed variations have no impact on the traffic flow. In high densities, chain reactions of velocity adaptations occur, resulting in a jam wave. This wave propagates against the velocity direction, as described in the section on traffic flow theory above. They call the density distinguishing between ‘free flow’ and ‘congestion’ the critical density [17].

In the quest for a transition between high and low performance in ATM, Ben Amor et al. [22] analysed congestion in European en-route traffic flow. Their assumption is that controllers compensate congestion by speed and route adjustments. The purpose of their analysis is to identify a critical density, from which on compensation of congestion is no more possible. Their flow model is an agent based system, which is an extension of cellular automata: a sector corresponds to a cell and an aircraft corresponds to an agent, moving from sector to sector. The rules for an aircraft at sector s_i are:

1. Availability: a sector is available if it’s capacity is not exceeded
2. Traversal time of a sector s :

$$\Delta s \quad \text{if next sector is available}$$

$$\Delta s + 1 \quad \text{else}$$
3. Re-routing: the next sector is

$$s_{i+1} \quad (\text{according to flight plan}) \text{ if available}$$

$$s_c \quad \text{with probability } p_r \text{ else}$$

where s_c is a neighbouring sector of s_i and s_{i+1} .

4. Randomisation: the traversal time Δs increases by ± 1 time unit with probabilities p_1, p_2

One can see that the model contains a number of random elements, capturing the uncertainties in ATM. For their simulations, they generate random traffic patterns (different city pairs) varying in traffic density from 10.000 to 60.000 aircraft per day. Their results are shown in Figure 7. Each panel shows the state of the sectors (white: uncongested, red: congested) and a graph of the number of congested sectors over time. For low densities < 50.000 , spontaneous congestion appears but is compensated by ATC — the number of congested sectors is constant over time (upper panel). For high densities, the number of congested sectors jumps after a short while, leading to a complete saturation of the system (lower panel). The author’s conclusion is that a transition from high to low performance at ~ 50.000 aircraft per day has been found. Note that the average traffic density in Europe is currently below 30.000. As before, the results are initial, and one can criticise that

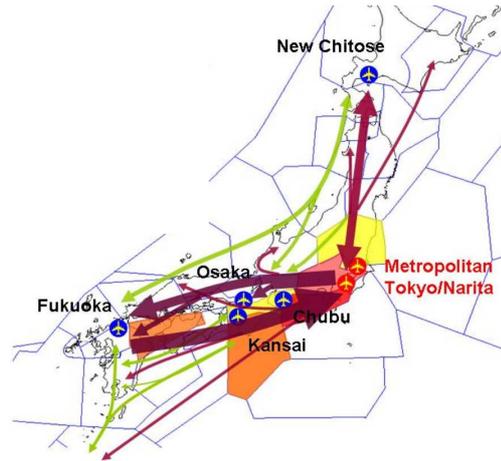


Figure 8: Major flows in Japanese Airspace. Source: [6]

their definition of ATM performance is unrealistic. Furthermore, when re-routing is not possible, the model can produce an unlimited number of capacity violations.

3 Traffic Flow in Japanese Airspace

The aim of air traffic flow management (ATFM) is to balance airspace demand with available capacity. This requires strategic decisions from experienced traffic managers, taking into account the safety constraints of the airspace but also the interests of the airlines. In Japan, flow management is a centralised service that takes place in the ATM Center in Fukuoka. The ATFM system monitors capacity and demand of the network and predicts these up to the next 6 hours. Based on this, ground delays are calculated and transmitted to the control centres and to the airlines. When necessary, pre-tactical re-routings are negotiated with the Airlines. Finally, tactical re-routings and adjustments of flow rates to the neighbouring sectors (miles-in-trail restrictions) are coordinated with the ATC centres.

Most of Japanese ATFM delays are due to saturated airspace around the metropolitan airports and on routes to Tokyo International Airport (Figure 8). In 2005, the average ATFM delay was 6 minutes, at a traffic demand of 1.24 million, out of which 2% were affected by flow management [6]. In the future, the demand is expected to grow yearly by 1-2 % especially from Asian neighbours [26]. These numbers are less alarming than in Europe and the U.S, but in order to maintain the high reliability of Japanese air transportation, Japanese

Table 2: Comparison of flow models

Model	Purpose	Structure	Comment
Queue (det.) [13]	Propagation of delays in arrival flow	Deterministic queue with randomly disturbed arrival times	Careful study
Queue (net) [14, 15]	Impact of trajectory uncertainties on flow performance	Jackson Network of 20 Control Centres. ‘Path efficiency’ as performance metric	Initial results, all critiques to queueing networks apply
Queue (sim.) [11]	Large scale simulation of queueing networks	Strategies to reduce network complexity	Useful approach when statistical analysis of simulation results is required
Fluid continuous [19]	Speed control for strategic air traffic control	Continuity equation to allow controllers decision on time of traversal	Black box approach. No insight into flow properties.
Fluid discrete [20, 24, 25]	Impact of control strategies on traffic flow	Speed control of fractions of aircraft inside sector. Continuity equation to allow controllers decision on time of traversal	Innovative idea, but linear time invariant (in this article)
Cellular Automata [22]	Phase transitions from high to low ATM performance. Criterion: number of congested sectors.	Aircraft motion in a 2D grid of sectors. Local re-routing and speed control if next sector is busy.	Unrealistic model, but brainstorm for ‘analysing a complex system’.

flow management will be improved.

Future flow management will be based on a new information flow, ranging from long-term scheduling strategies to exchange of position and intent information with en-route aircraft. In short term, new tactical ATFM tools (e.g. traffic synchronisation) will be developed and the technological basis to share data among decision makers will be improved. The long term vision is a complete system integration of aircraft, ATFM, ATC and airports and a shift towards a dynamic, trajectory based flow management [6, 27].

4 Conclusions

We reviewed models for the analysis of air traffic flow. Flow models can provide insight into the mechanisms of congestion. Such insight is needed to support strategic decision making in ATM. Table 2 summarises the results. They are all in an early stage of development. The fluid dynamical models are useful for traffic synchronization while the queueing models give more insight into the performance of the ATM network. But at the current state-of-the-art, it is fair to judge their value by their purpose and not by the results. Table 3 summarises the methodologies that were discussed in this review. We added the category ‘fast time simulation’ as a baseline comparison. One can con-

clude that each methodology has its own strengths and weaknesses.

To conclude, traffic patterns propagate in a predictable way until the trajectories foreseen in the flight plans change. What makes the analysis challenging are the reasons for such changes. In the case of vehicular traffic flow, they are often assumed to be local interactions with other cars; for example the acceleration of one car is proportional to the speed difference with its leading car. But in the case of ATM, interactions between aircraft are the result of complex decision making, taking into account the strategies of controllers and airlines, predictions of weather and traffic, and other operational uncertainties. Pioneering work is necessary to understand these interactions.

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Table 3: Comparison of methodologies

	Simulation (fast time)	Queue	Fluid	Cellular Automata
Strengths	Realistic.	Description of behaviour close to capacity limit. Long history in ATM. Mature discipline.	Propagation of traffic patterns. Theoretical tools (e.g. stability analysis) available. Extensions exist (e.g. for particle interactions). Computational efficient control procedures	Identification of minimal requirement for congestion dynamics. High speed (very large scale simulations possible). Analytical results.
Weaknesses	Statistical properties difficult to obtain.	No good description of dynamic behaviour (only long-term behaviour well known). In networks, transitions from one node to another are governed by schedules, not random probabilities. Schedules may dynamically change due to weather, crew delays, ...	Basic models do not provide enough fidelity to model conflicts and other interactions. Analytical properties of extended models unknown. Flight plan data has to be interpolated to validate continuous models	Unrealistic.
Application area	All over ATM.	Delay analysis. ATM performance.	Tactical ATFM. Fundamental research.	Fundamental research.

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