Probabilistic Methods in Concurrency

Lecture 3
The pi-calculus hierarchy: separation results

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The $\pi$-calculus hierarchy

- $\pi_a$: asynchronous $\pi$
- $\pi_{ic}$: asynchronous $\pi$ + input-guarded choice
- $\pi_{op}$: asynchronous $\pi$ + output prefix
- $\pi_s$: asynchronous $\pi$ + separate choice
- $\pi_I$: $\pi$ with internal choice (Sangiorgi)
- $ccs_{vp}$: value-passing $ccs$

Diagram:

- $\pi_a$ to $\pi_{ic}$
- $\pi_{ic}$ to $\pi_{op}$
- $\pi_{op}$ to $\pi_s$
- $\pi_s$ to $\pi_I$
- $\pi_I$ to $ccs_{vp}$

Arrows:
- $\rightarrow$: Language inclusion
- $\rightarrow$: Encoding
- $\leftarrow$: Non-encoding
The separation between $\pi$ and $\pi_s$

This separation result is based on the fact that it is not possible to solve the symmetric leader election problem in $\pi_s$, while it is possible in $\pi$.

- Some definitions:

  - **Leader Election Problem (LEP):** All the nodes of a distributed system must agree on who is the leader. This means that in every possible computation, all the nodes must eventually output the name of the leader on a special channel. Out:
    - No deadlock
    - No livelock
    - No conflict (only one leader must be elected, every process outputs its name and only its name)

  - **Symmetric LEP:** the LEP on a symmetric network
    - Hypergraphs and hypergraph associated to a network
    - Hypergraph automorphism
    - Orbits, well-balanced automorphism
    - Examples
    - Symmetry
The separation between $\pi$ and $\pi_s$

- **Theorem**: If a network with at least two nodes has an automorphism $\sigma \neq \text{id}$ with only one orbit, then it is not possible to write in $\pi_s$ a symmetric solution to the LEP.

- **Corollary**: The same holds if the automorphism is well-balanced.

- **Proof (sketch)**. We prove that in $\pi_s$ every system trying to solve the electoral problem has at least one diverging computation.

1. If the system is symmetric, then the first action cannot be $\overline{\text{out}} k$.

2. As soon as a process performs an action, let all the other processes in the same orbit perform the same action as well. At the end of the round in the orbit, the system is again symmetric.

Note that the system can change communication structure dynamically.
The separation between $\pi$ and $\pi_s$

• Crucial point: if the action performed by $P_i$ is a communication with $P_j$ in the same orbit, we need to ensure that $P_j$ can do the same action afterwards.

• This property holds in fact, due to the following:

• Lemma: Diamond lemma for $\pi_s$

• Note that in $\pi$ (in $\pi$ with mixed choice) the diamond lemma does not hold
The separation between $\pi$ and $\pi_s$

- Remark: In $\pi$ (in $\pi$ with mixed choice) we can easily write a symmetric solution for the LEP in a network of two nodes:

$$P_0 = x.out\ 0 + \bar{y}.out\ 1$$
$$P_1 = y.out\ 1 + \bar{x}.out\ 0$$
The separation between $\pi$ and $\pi_s$

- **Corollary:** there does not exist an encoding of $\pi$ ($\pi$ with mixed choice) in $\pi_s$ which is homomorphic wrt $|$ and renaming, and preserves the observables on every computation.

- **Proof (sketch):** An encoding homomorphic wrt $|$ and renaming transforms a symmetric solution to the LEP in the source language into a symmetric solution to the LEP in the target language.
The separation between $\pi$ and $\pi_I$, $ccs_{vp}$

- Theorem: If a network with at least two nodes has a well-balanced automorphism $\sigma \neq \text{id}$ such that
  - $\forall i$ and $\forall$ node $P$, if $\sigma^i \neq \text{id}$ then there is no arc between $P$ and $\sigma^i(P)$,
  then in $\pi_I$ and $ccs_{vp}$ there is no symmetric solution to the LEP.

- Example: a network which satisfies the above condition
The separation between $\pi$ and $\pi_I$, $ccs_{vp}$

- A solution to the leader election problem for the same network in $\pi$
The separation between $\pi$ and $\pi_I$, $ccs_{vp}$

- **Corollary:** there does not exist an encoding of $\pi$ ($\pi$ with mixed choice) in $\pi_s$ which is homomorphic wrt $|$ and renaming, does not increase the connectivity, and preserves the observables on every computation.