FLATNESS BASED OPEN LOOP CONTROL FOR THE TWIN ROLL STRIP CASTING PROCESS

Ch. Fleck [*,1] **Th. Paulus** [**,2] **A. Schönbohm** [**,3]
D. Abel [**] **F. Ollivier** [*]

[*] *Laboratoire STIX FRE CNRS 2341, École Polytechnique, 91128 Palaiseau CEDEX, France*
[**] *Aachen University of Technology, Institute of Automatic Control, Steinbachstr. 54, D-52074 Aachen, Germany*

Abstract: Strip casting technology is the most recent innovative steel casting technology that integrates casting and rolling into a single production step. The strip thickness is supposed to be changed independently of the contact time of the molten steel with the rollers, which determines the quality of the produced steel strip. This leads to a decoupling problem which is difficult to tackle with classical control approaches due to the nonlinearity and the varying dead time occuring in the manipulated variables of the system. In order to steer the system between two operating points and for the startup procedure, a flatness based open loop control scheme is presented. Starting from the process model, the flatness of the system outputs is shown and the trajectory generation for the manipulated variables is described which ensure that the outputs follow the desired trajectories.

Keywords: flatness, twin roll strip casting, steel, open-loop control, double roller

1. INTRODUCTION

Twin-roll strip casting combines casting and rolling into a single operation, thus reduces the number of operations compared to conventional continuous casting to produce thin strips (1-60 mm thickness). Among lower investment and production costs, thin strip casting considerably reduces the energy consumption (Luiten, 2001). Furthermore, due to high cooling rates, it can increase the mechanical properties of the metal. Although the concept goes back to the 19th century with the patent of Bessemer in 1866 and is very simple, its application has proven to be extremely difficult due to, e.g., extremely strained thermal stresses of the rolls, supplying the liquid steel to the melt pool homogeneously to avoid unsymmetrical solidification of the metal (Ohler *et al.*, 2002), or avoiding pre-solidification at the edges between the rolls and the dams (Luiten, 2001). If the past two past decades of research and development have solved most of the technical problems, very rare results have been published concerning the control involved in this process (Simon, 2000), specially not in the motion planning joining two steady states (Fleck and Abel, 2002).

The paper is organized as follows. Initially an adequate model (Simon, 2000) of the process is presented and the constraints related to the setpoint changing during the production are formulated. Then the three inputs, three outputs systems is shown to be flat, a property which will be used to calculate the motion planning between the set-

[1] Christian.Fleck@stix.polytechnique.fr
[2] Th.Paulus@irt.rwth-aachen.de
[3] A.Schoenbohm@irt.rwth-aachen.de

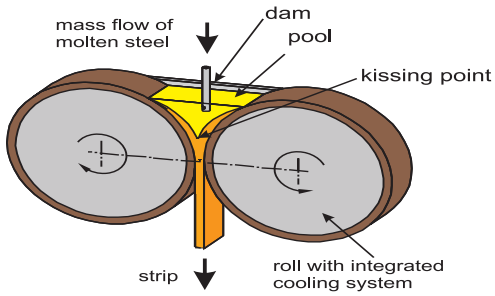

Fig. 1. plant overview

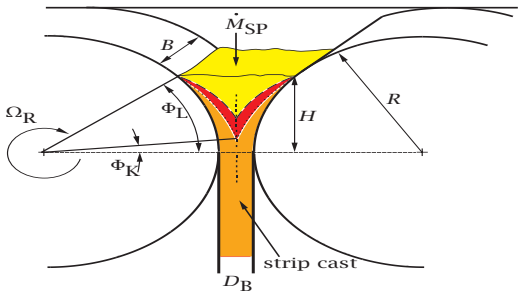

Fig. 2. system variables

points under the given constrains. The relevance of our approach is illustrated by some simulation results and finally the paper is completed with some concluding remarks.

2. MODELLING

2.1 Process Overview

The double roller method of continuous casting involves pouring the liquid steel directly into the gap of two horizontal rolls, which are rotating in opposite direction, figure 1 shows the basic setup. The molten steel begins to solidify at contact with the surface of the rolls since these are cooled to a temperature below the solidus temperature of the steel alloy. The two layer of solidified steel must concur at a point above the narrowest point of the gap between the two rolls in order to ensure a solid strip leaving the gap. The process control must guarantee that this condition is well met during production. On the other hand the joining point should be as low as possible in order to reduce the forming force acting upon the plant. Dams at each side of the rolls prevent the molten steel from leaving the pool.
The continuous casting machine consists of a steel frame in which the two rolls are installed. The gap between the rolls can be controlled by means of an electro-hydraulic positioning system which acts upon one roll mounted in low-friction linear bearings. Electric drives rotate the rolls and a cooling system absorbs the heat form the molten steel in order to initiate the solidification process.

2.2 Process Modell

The strip casting process consists of a solidification and a forming process. The manipulated variables of the system are the distance between the roll mountings $D_E(t)$, the angular velocity of the rolls $\Omega_R(t)$ and the mass flow of the molten steel $\dot{M}_{SP}(t)$. The output variables are the force $F(t)$, which is acting upon the rolls due to the joining of the two solidified layers, and the actual strip thickness $D_B(t)$. Regarding quality aspects the contact time $\Delta T(t)$, which will be defined later, and the pools level of molten steel $H(t)$ are also of interest. The dynamics of the electrical drives of the rolls and the hydraulic positioning system for the roll gap are neglected since they are much faster than the process dynamics. Figure 2 shows the rolling system with the associated variables. The melt pool is considered to be a mass storage which is feeded by in- and outgoing mass flows. Under the condition of non-slip between the strip and the roll surface and that the density of the molten and solid steel is the same, the continuity equation leads to the following formula

$$\dot{H}(t) = \frac{\dot{M}_{SP}(t) - \rho \cdot B \cdot R \cdot D_B(t) \cdot \Omega_R(t)}{\left(D_B(t) + 2 \cdot \left(R - \sqrt{R^2 - H^2(t)}\right)\right) \cdot \rho \cdot B} \quad (1)$$

with

$\dot{M}_{SP}$ - molten mass flow entering system
Ω_R - angular velocity of the rolls
D_B - thickness of steel strip
H - level of the pool of molten steel
ρ - density of the strip material
R, B - radius, width of roll.

The contact angle $\Phi_L(t)$ describes the area of the roll surface which is in contact with the melt. It is connected to the pool height $H(t)$ via the simple geometric formula

$$\Phi_L(t) = \arcsin \frac{H(t)}{R} \quad (2)$$

and its time derivative is given by

$$\dot{\Phi}_L(t) = \frac{1}{\sqrt{1 - \left(\frac{H(t)}{R}\right)^2}} \cdot \frac{\dot{H}(t)}{R}. \quad (3)$$

The time a particle is in contact with a roll surface while travelling from the surface of the pool to the narrowest point of the gap is called the contact time $\Delta T(t)$. This means that the contact time is dependant on the angular velocity $\Omega_R(t)$ of the rolls and the pool height $H(t)$ resp. the contact angle $\Phi_L(t) - \Phi_K(t)$, which represents the travelled distance. The solidification process begins on contact of a particle with the cooled roll surface and continues to build a layer of solidified steel around the rolls until the joining of the two layers. This means if the contact time is known, it is possible to calculate the thickness of the solidified layers via a layer growth law.
The integral of the angular velocity over the

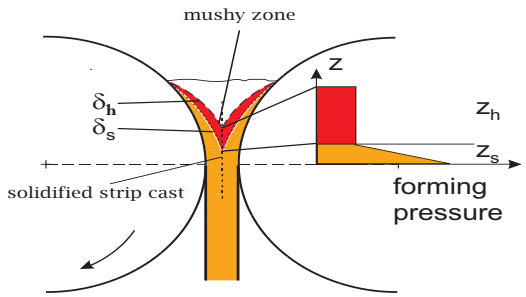

Fig. 3. Pressure profile for solid and mushy zone

contact time is equal to the difference between the contact angle $\Phi_L(t)$ delayed for the contact time and $\Phi_K(t)$:

$$\int_{t-\Delta T(t)}^{t} \Omega_R(\tau)d\tau = \Phi_L(t-\Delta T(t)) - \Phi_K(t) \qquad (4)$$

$\Phi_K(t)$ describes the angle at which the two solidified layers meet each other and are jointed. The joining point of the two layers is also called the kissing point. Derivation of equation (4) with respect to t and transformation lead to an equation for $\Delta\dot{T}(t)$

$$\Delta\dot{T}(t) = 1 - \frac{\Omega_R(t) + \dot{\Phi}_K(t)}{\Omega_R(t-\Delta T(t)) + \dot{\Phi}_L(t-\Delta T(t))} \qquad (5)$$

The growth of the solidified layer is governed by a partial differential equation which in this case (Fleck, 2004) can be modelled by the following relation:

$$\delta_h(t) = C \cdot \Delta T(t)^{\beta} \quad \text{with} \quad C, \beta = const. \qquad (6)$$

Condition for the kissing point:

$$(R + x_s \cdot \delta_h(t)) \cdot \cos(\Phi_K(t)) = R + \frac{D_B(t)}{2} \qquad (7)$$

Since an steel alloy is used for the strip casting process there is not only a solid and liquid phase but also a mushy zone where only part of the melt is already solid. Equation (6) describes the thickness of the heterogeneous layer which consists of a solid part and a mushy part (Simon, 2000). The thickness of the completely solidified layer is connected to equation (6) via a form factor x_s, which leads to the following equation for the thickness of the solid layer:

$$\delta_s(t) = x_s \cdot C \cdot \Delta T(t)^{\beta} = x_s \cdot \delta_h(t) \qquad (8)$$

The force $F(t)$ which is acting upon the rolls at the merging point of the two solidified layers is dependant on the pressure profile at the rolls, as shown in figure 3. The pressure profile consists of two parts, one for the mushy zone and a second one for the solidified area. The forming at the heterogeneous area takes place at a constant forming resistance. The increasing yield stress in the solid area is caused by the restricted material flow in the direction of the strip (Simon, 2000). For the calculation of the force the joining points $z_h(t)$ and $z_s(t)$ must be determined. Simple geometric considerations lead to

$$z_h(t) = \sqrt{(R+\delta_h(t))^2 - \left(R + \frac{D_B(t)}{2}\right)^2} \qquad (9)$$

$$z_s(t) = \sqrt{(R + x_s \cdot \delta_h(t))^2 - \left(R + \frac{D_B(t)}{2}\right)^2}. \qquad (10)$$

Integration of the pressure profile (figure 3) yields for the reacting force $F(t)$:

$$F(t) = B \cdot \left(z_h(t) \cdot k_{f0} + a_f \cdot \frac{z_s^2(t)}{2} \right) \qquad (11)$$

with $\quad k_{f0}, a_f = const.$

The resulting thickness $D_B(t)$ of the produced steel strip is mainly determined by the rolling gap $D_E(t)$. Additionally it is influenced by the reacting force $F(t)$ due to the resilience R_{fr} of rolls, bearings and frame,

$$D_B(t) = D_E(t) + R_{fr} \cdot F(t) \qquad (12)$$

With this set of nonlinear time varying equation the behaviour of the system is adequately described. The basic system behaviour is shown in figure 4 and 5 for changes of the angular velocity $\Omega_R(t)$ and mass supply $\dot{M}_{SP}(t)$. The contact time $\Delta T(t)$ acts upon the system as a variable dead time, since changes of the mass flow $\dot{M}_{SP}(t)$ are delayed by the contact time before they act upon the contact time itself. As it can easily be seen

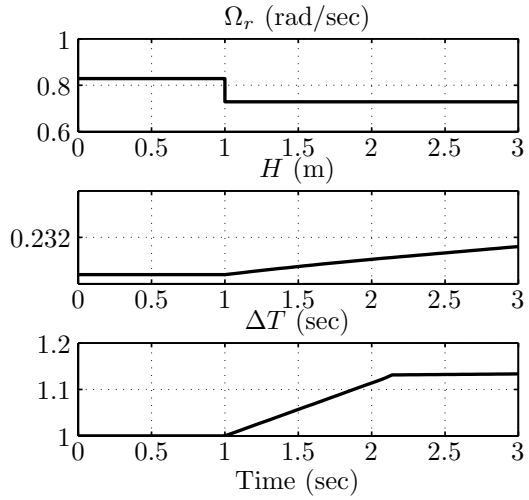

Fig. 4. Step response for $\Omega_R(t)$, $\dot{M}_{SP} = const.$

the system exhibits integral behaviour both if the angular velocity or the mass flow are changed while keeping the other variable constant.

3. DETERMINATION OF A FLAT OUTPUT

3.1 Changing of operation points

There are, depending on the objective of the production, two principal reasons for the changing

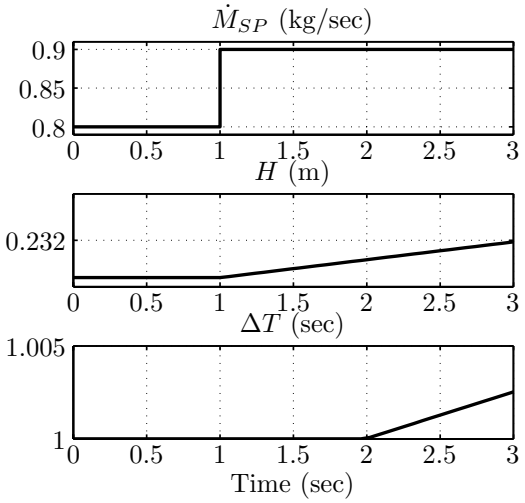

Fig. 5. Step response for $\dot{M}_{SP}(t)$, $\Omega_R(t) = const$,

of the operation point. By keeping the force constant, which is a gage of constant quality, the demand can be to change the thickness $D_B(t)$ of the strip or to change the amount of produced steel strip per hour. This decoupling problem is formulated in (Simon, 2000) as follows. Is it possible to change the operating point by keeping the force $F(t)$ constant such that

(1) changing the strip thickness $D_B(t)$ while keeping the pool height $H(t)$ constant
(2) or vice-versa ?

Using the flatness of the system, these questions can be easily answered.

3.2 Flatness

There are a lot of applications of flatness to systems with concentrated and distributed parameters (Martin *et al.*, 1997; Fleck and Abel, 2003) and also to delay systems (Fliess and Mounier, 1998).
Because of the contact time $\Delta T(t)$, the described system contains a varying delay time like the examples considered in (J.Y.Dieulot *et al.*, 2003),(Rudolph and Winkler, 2003) and concerning a chemical reactor with partial recycle. In this references, through a substitution of the independant variable, namely the time, the system statisfies the classical (J.Y.Dieulot *et al.*, 2003) or a generalized $\delta - \pi$ (Rudolph and Winkler, 2003) flatness defintion. This is not the case for the system under consideration and therefore flatness is here defined in a less formal manner (Fleck, 2004):

Definition 1. A system is flat, if there exists a vector $\boldsymbol{y}_f$ such that

- the components of $\boldsymbol{y}_f$ are differentially independent $\Rightarrow dim(\boldsymbol{y}_f) = dim(\boldsymbol{u})$ with $\boldsymbol{u}$ as the input of the system.
- all states and input variables starting from a stationary point at $t = 0$ are determined over the time intervall $[0 \ldots \infty[$ by the time trend of $\boldsymbol{y}_f$ and its derivatives.

It will be shown, in the next section, that the twin roll strip casting process holds this definition.

3.3 Flatness of the twin-roll strip casting process

Since the twin-roll strip casting process has as many inputs as outputs, it seems natural to verify if the outputs are the components of a flat output. In order to check whether the strip thickness $D_B(t)$, the reacting force $F(t)$ and the pool height $H(t)$ represent a flat output, it is sufficient to verify that all variables described in the considered process can be written as functions (in the following section denoted by f_i) of this variables and their derivatives (cp. Def. 1).
Rearranging equation (12) yields an expression for the gap distance $D_E(t)$

$$D_E(t) = D_B(t) - R_{fr} \cdot F(t) = f_1(D_B(t), F(t)). \quad (13)$$

which is obviously a function of $D_B(t)$ and $F(t)$
The six equations (6,7,8,9,10,11) relating the eight variables $z_h(t), z_s(t), \delta_h(t), \delta_s(t), \Delta T(t), \Phi_K(t), F(t), D_B(t)$ constitute a set of purely algebraic equations. Therefore all the variables are determined if $F(t)$ and $D_B(t)$ are known.

$$\left.\begin{matrix} z_h(t), z_s(t), \delta_h(t) \\ \delta_s(t), \Delta T(t), \Phi_K(t) \end{matrix}\right\} = f_{2,3,4,5,6,7}(D_B(t), F(t)) \quad (14)$$

The contact angle $\Phi_L(t)$ is directly dependant on the component $H(t)$, see equation (2).

$$\Phi_L(t) = f_8(H(t)) \quad (15)$$

From the equation (5), the input variable $\Omega_R(t)$ can be expressed as

$$\Omega_R(t) = -\dot{\Phi}_K(t) + \left(1 - \Delta\dot{T}(t)\right) \cdot \left(\Omega_R(t - \Delta T(t)) + \dot{\Phi}_L(t - \Delta T(t))\right) \quad (16)$$

From equations (14) and (15) we obtain

$$\Delta\dot{T}(t), \dot{\Phi}_K(t) = \frac{df_{6,7}}{dt}(D_B(t), F(t)) = f_{9,10}\left(D_B(t), \dot{D}_B(t), F(t), \dot{F}(t)\right) \quad (17)$$

and

$$\dot{\Phi}_L(t) = \frac{df_8}{dt}(H(t)) = f_{11}\left(H(t), \dot{H}(t)\right) \quad (18)$$

$$\Rightarrow \dot{\Phi}_L(t - \Delta T(t)) = f_{12}\left(H(t), \dot{H}(t), D_B(t), F(t)\right)$$

Substituting this results in equ.(16) leads to

$$\Omega_R(t) = f_{13}\left(\begin{matrix} D_B(t), \dot{D}_B(t), F(t), \dot{F}(t), \\ H(t), \dot{H}(t), \Omega_R(t - \Delta T(t)) \end{matrix}\right) \quad (19)$$

Since we suppose the system to be in a known stationary state for $t < 0$, it is possible to show

that $\Omega_R(t)$ is determined by the output of the system and its derivatives with respect to time (Fleck, 2004).

$$\Omega_R(t) = f_{14}\left(D_B(t), \dot{D}_B(t), F(t), \dot{F}(t), H(t), \dot{H}(t)\right) \tag{20}$$

It is difficult to find an explicit expression for $\Omega_R(t)$, because of the occuring delayed values $\Omega_R(t-\Delta T)$. But if we assume that the past of the system is known, the delayed values of $\Omega_R(t-\Delta T)$ are known and $\Omega_R(t)$ can be calculated using the flat outputs and its past values.
Finally the input variable $M_{SP}(t)$ can be expressed: rearranging the equations (1) and (20) yields:

$$\begin{aligned}\dot{M}_{SP}(t) &= \dot{H}(t)\cdot\rho\cdot B \\ &\quad\cdot\left(D_B(t)+2\cdot\left(R-\sqrt{R^2-H^2(t)}\right)\right) \\ &\quad+\rho\cdot B\cdot R\cdot D_B(t)\cdot\Omega_R(t) \\ &= f_{15}\left(H(t),\dot{H}(t),F(t),\dot{F}(t),D_B(t),\dot{D}_B(t)\right)\end{aligned} \tag{21}$$

Now all appearing system variables have been expressed by $H(t), F(t), D_B(t)$ and their time derivatives. Therefore $\mathbf{y}_f = (H(t), F(t), D_B(t))$ holds the second part of the definition (1) and can be considered as a flat output of the system. It should be noted that instead of choosing the $F(t)$ and $D_B(t)$ as a component of the flat output, it is also possible to choose two arbitrarily variables among $z_h(t), z_s(t), \delta_h(t), \delta_s(t), \Delta T(t), \Phi_K(t), F(t), D_B(t), D_E(t)$ (in this case a component of a flat output is an input of the system!) and that $H(t)$ could be replaced by $\Phi_L(t)$. This means that depending on the given control problem, one can choose the most suitable flat output.

4. SIMULATION RESULTS

Because for lack of place and without loss of generality, only the subsystem consisting of the inputs $\Omega_R(t)$ and $M_{SP}(t)$ and the contact time $\Delta T(t)$ and the pool height $H(t)$ as the flat output is considered for the simulations. The model of the forming zone is not taken into account and, for simplification, D_B is set to a constant value.

As above alluded, an offline calculation of the trajectories is very difficult since the determination of an explicit expression for $\Omega_R(t)$ as a function of $\Delta T(t)$ and $H(t)$ is highly complex.
Therefore the calculation has to be done online. To avoid discontinuities in the time shape of the manipulated variables, the function of the trajectories for $H(t)$ and $\Delta T(t)$ have to be chosen to be smooth enough between the two operating points. Since both components of the flat output appear with at most the first derivative in the parametrization of $\Omega_R(t)$ and $\dot{M}_{SP}(t)$, a continuously derivable polynomial joining the two stationary points would be sufficient. To calculate the corresponding time shape of the input variables, the trajectories of $\Delta T(t)$ and $H(t)$ are injected in the system equations in the following order: eq.(2), eq.(6), eq.(7) solved for $\Phi_K(t)$, eq.(16), eq.(21). The calculation is done recursively and repeated for each time step. In the following sections simulation results for the transitions of the pool height and the contact time are shown.

4.1 Constant Pool Height, Variable Contact Time

The system is supposed to be at its operating point. The contact time $\Delta T(t)$ should be changed from $\Delta T(t_0) = \Delta T_0$ to a new steady state $\Delta T(t_1) = \Delta T_1$ within the transition time T_Δ while the pool height remains constant:

$$H(t) = H(t_0),\ t > t_0 \tag{22}$$

For the transition of $\Delta T(t)$ the following polynomial is chosen which is sufficiently smooth:

$$g(t) = \begin{cases} 0 & \text{for } t < 0 \\ 6\cdot\left(\frac{t}{T_\Delta}\right)^5 - 15\cdot\left(\frac{t}{T_\Delta}\right)^4 + 10\cdot\left(\frac{t}{T_\Delta}\right)^3 & \\ 1 & \text{for } t > T_\Delta \end{cases}$$

$$\Delta T(t) = \Delta T(t_0) + (\Delta T(t_1) - \Delta T(t_0))\cdot g(t), \forall t \tag{23}$$

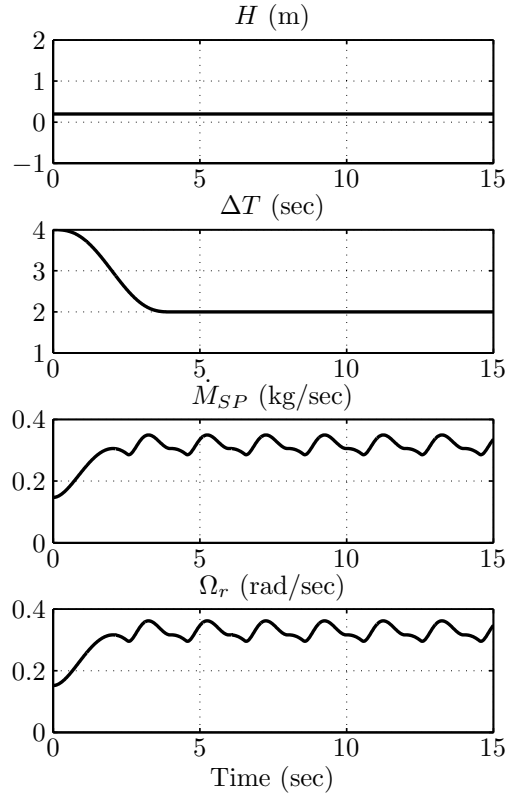

Fig. 6. $H(t) = const.$, $T_\Delta = 4sec$

Simulation results are shown in figure 6.

4.2 Variable Pool Height, Constant Contact Time

The system is supposed to be at its operating point. The pool height $H(t)$ should be changed from $H(t_0) = H_0$ to the new steady state $H(t_1) = H_1$ within the transition time T_Δ while the contact time remains constant:

$$\Delta T(t) = \Delta T(t_0) \quad , t > t_0. \tag{24}$$

For the transition of $H(t)$ a polynomial similar to function (23) is chosen and the trajectories for the manipulated variables are calculated as mentioned before. Figure 7 shows the simulation results. In both cases, it is interesting to note that

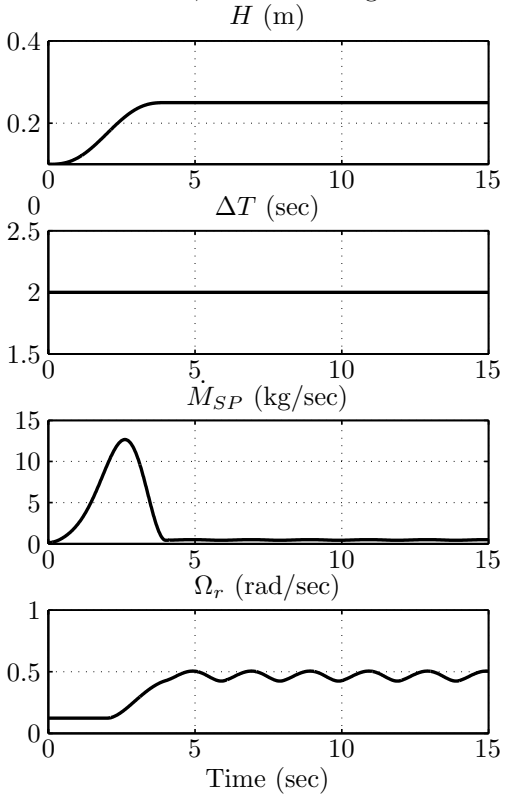

Fig. 7. $\Delta T(t) = const.$, $T_\Delta = 4sec$

the manipulated variables $\dot{M}_{sp}(t)$ and $\Omega_R(t)$ are periodic functions of the time with a period of $\Delta T(T_\Delta) \approx 2sec$. One explanation can be given by considering the state variables of the system to be stationary (i.e. $\dot{H}(t), \Delta\dot{T}(t), \dot{\Phi}_K(t) = 0$). In this case eq. (16) results in

$$\Omega_R(t) = \Omega_R(t - \Delta T(T_\Delta)) \tag{25}$$

which obviously owns a periodical solution and eq. (21) becomes

$$\dot{M}_{sp}(t) = \rho \cdot B \cdot R \cdot D_B \cdot \Omega_R(t) \tag{26}$$

and states that $M_{sp}(t)$ is proportional to $\Omega_R(t)$ and therefore shares the same properties as $\Omega_R(t)$.

5. CONCLUSION

In this paper it has been shown that the continuous strip casting process - a nonlinear system with variable dead time - is flat. Since the natural output is identical to the flat output, the controllability and observability of the system have been shown, properties which are quite difficult to prove with classical approaches in the presences of variable dead time. By using flatness, trajectories for the manipulated variables have been calculated which ensure that the pool height $H(t)$ and contact time $\Delta T(t)$ can be changed without influencing each other. The flatness of the system simplifies the analysis of the system behaviour and it can be easily determined which transitions between operating points are physical possible and which are not. At the same time constraints on system variables can be easily considered for the trajectory calculation. For furhter details about the twin roll caster, specially another interersting decoupling problem involving the output mass flow and the thickness of the strip, readers are referred to (Fleck, 2004).

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