

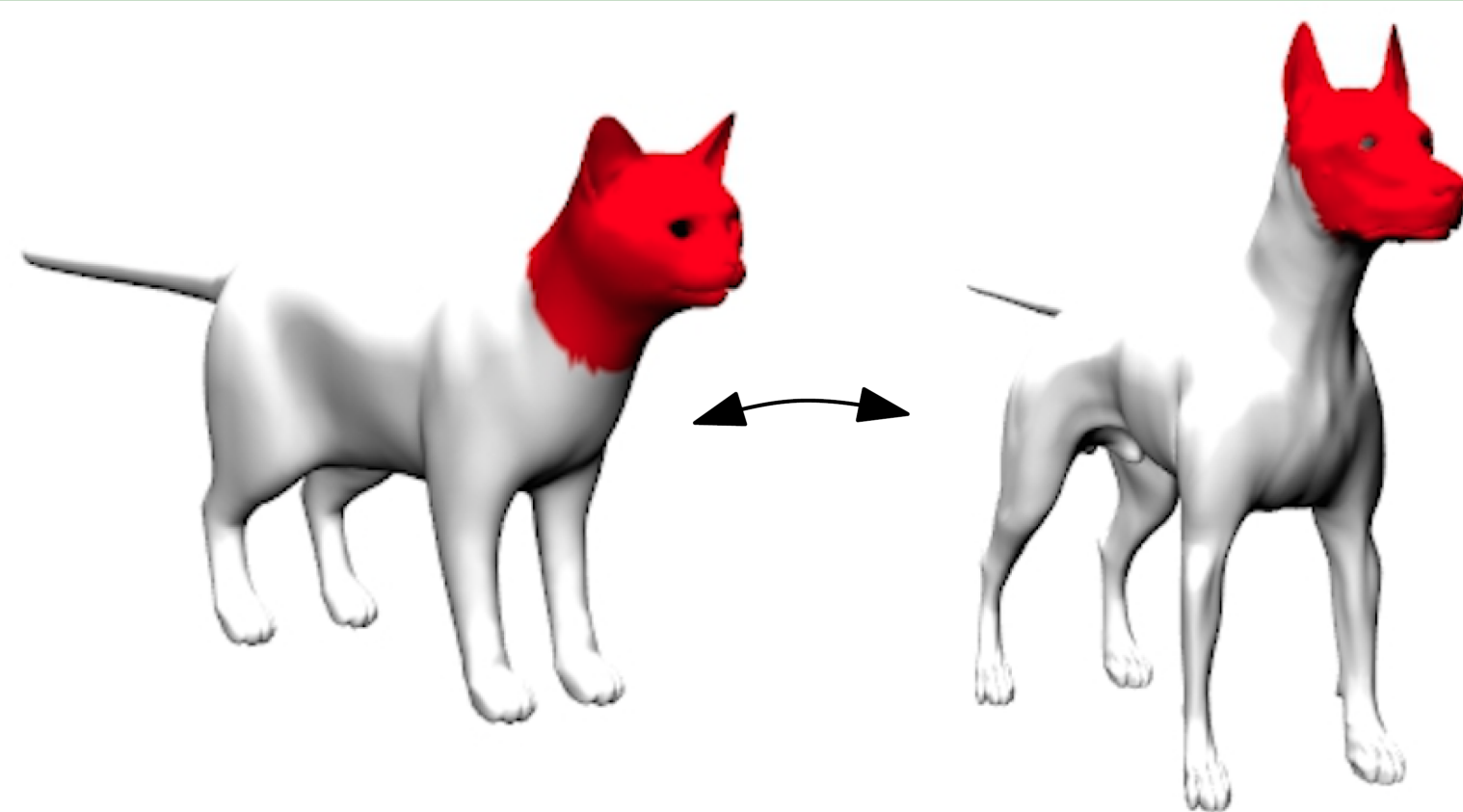
# REGION-BASED CORRESPONDENCE BETWEEN 3D SHAPES VIA SPATIALLY SMOOTH BICLUSTERING

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## Background

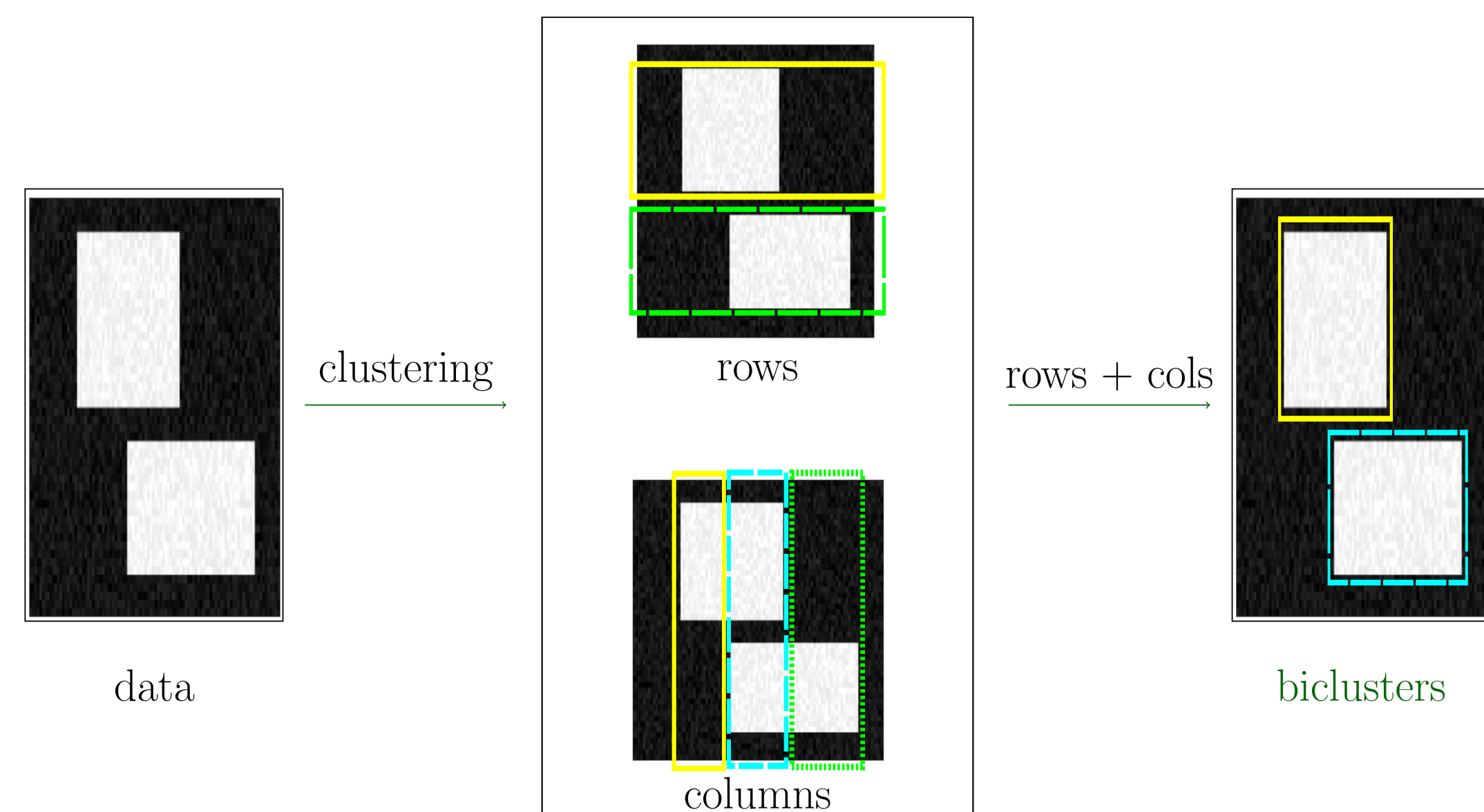
### Region-based Correspondence

Given a couple of shapes, RBC aims at finding regions on the shapes that behave similarly and can thus be easily put in correspondence



### Biclustering

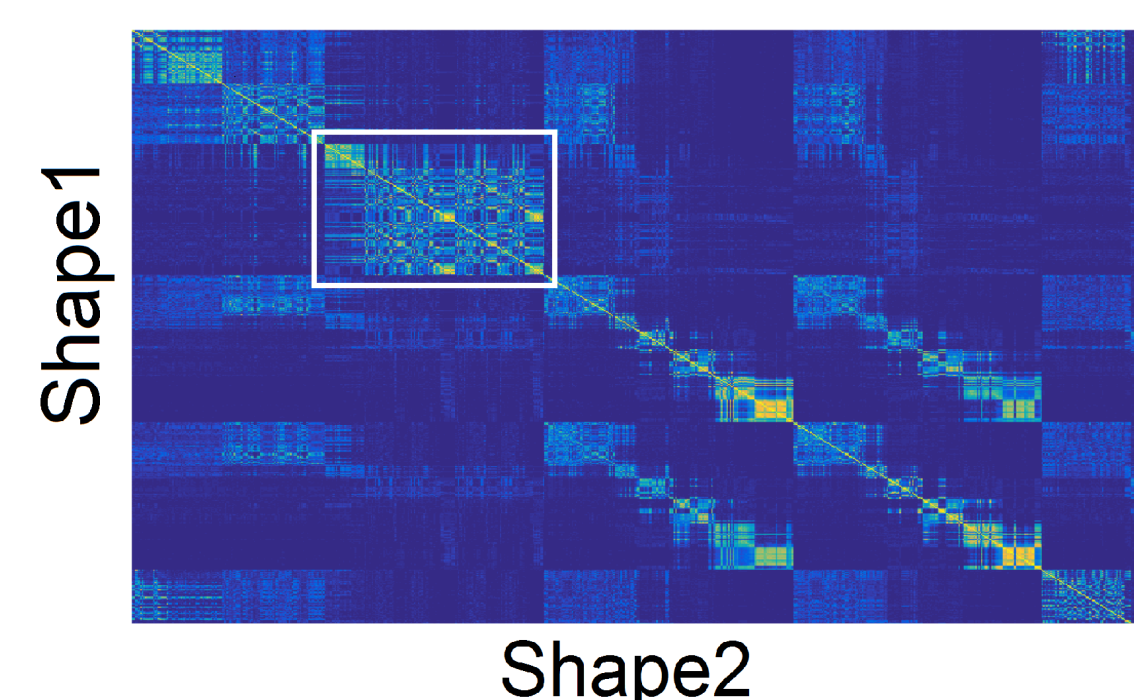
Simultaneous clustering of both rows and columns of a given data matrix



Bicluster: sub-matrix where rows present a coherent behaviour with columns (and viceversa)

### Starting viewpoint

1. StableRegion [SGP '16]
2. Similarity criterion between two vertices of different shapes
3. Affinity Matrix

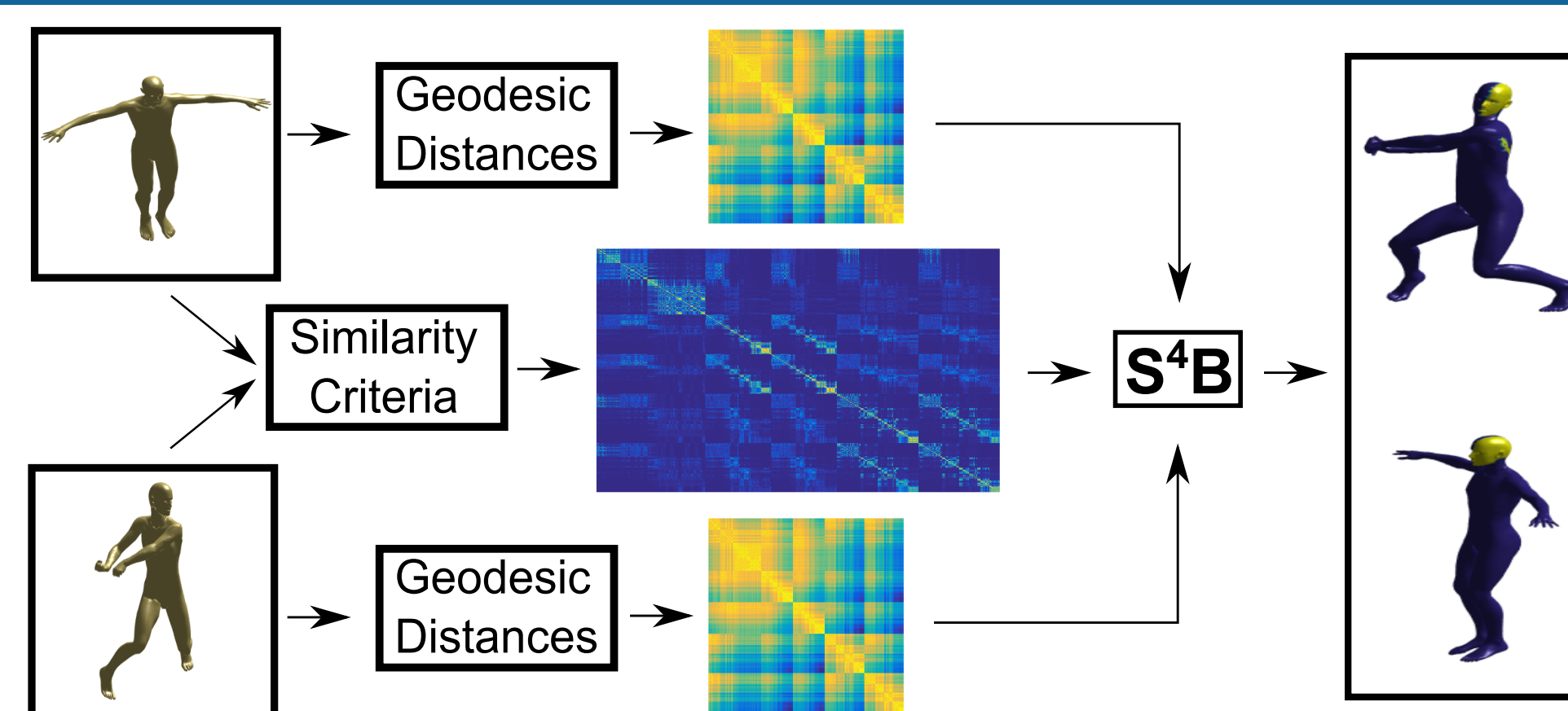


RBC is a biclustering problem. Bicluster: vertices of the first shape presenting a coherent behaviour with vertices of the second shape

## The method

IDEA:  
Near vertices should be grouped together!

### Overview



### S<sup>4</sup>B Ingredients

#### 1. low-rank matrix factorization

$$\begin{bmatrix} 10 & 20 & 0 & 0 \\ 20 & 40 & 0 & 0 \\ 30 & 60 & 0 & 0 \\ 40 & 80 & 0 & 0 \\ 0 & 0 & 5 & 10 \\ 0 & 0 & 5 & 10 \end{bmatrix} \xrightarrow{\text{rk}=2} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \end{bmatrix}_{v_1} + \begin{bmatrix} 10 & 20 & 0 & 0 \\ 20 & 40 & 0 & 0 \\ 30 & 60 & 0 & 0 \\ 40 & 80 & 0 & 0 \\ 0 & 0 & 5 & 10 \\ 0 & 0 & 5 & 10 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}_{z_2} \rightarrow \begin{bmatrix} 10 & 20 & 0 & 0 \\ 20 & 40 & 0 & 0 \\ 30 & 60 & 0 & 0 \\ 40 & 80 & 0 & 0 \\ 0 & 0 & 5 & 10 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

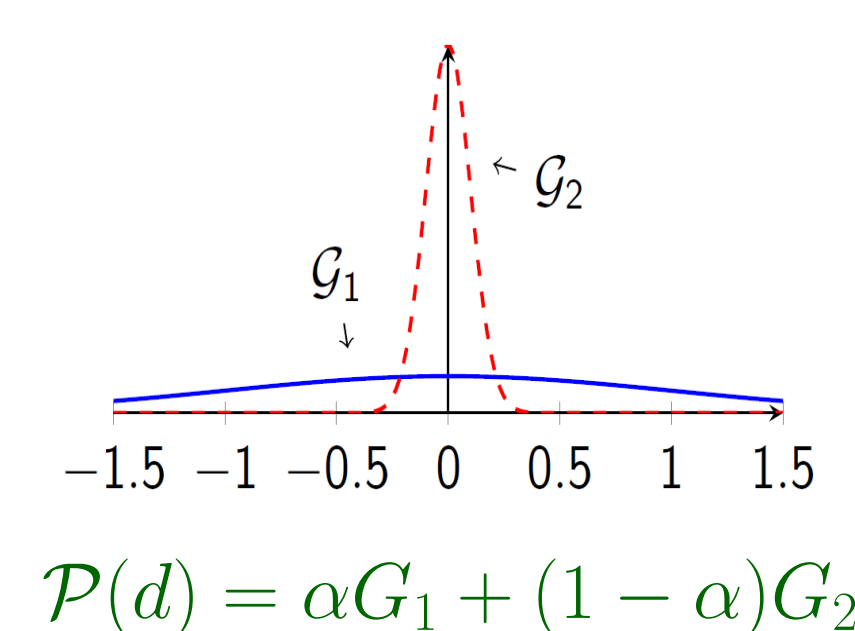
$$D = \sum_{p=1}^k v_p * z_p^T = VZ$$

with  $V = [v_1, \dots, v_k]$  and  $Z = [z_1, \dots, z_k]^T$

#### 2. Spike and Slab prior

Two Normal Distributions

- $G_2 = \mathcal{N}(0, \tau_2) \rightarrow$  generate zero points
- $G_1 = \mathcal{N}(0, \tau_1) \rightarrow \sim$  uniform function
- $\tau_1 \gg \tau_2$

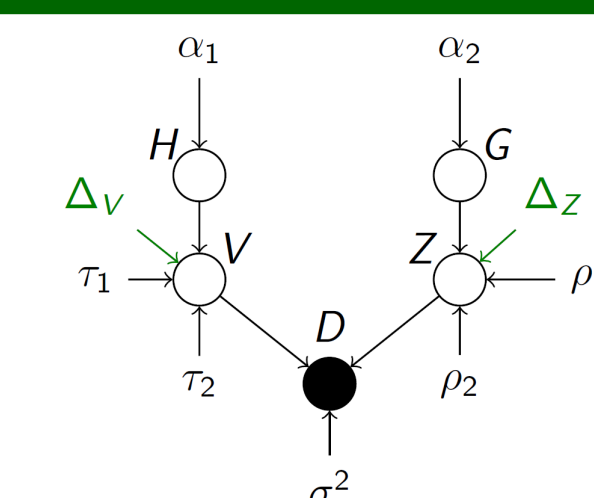


#### 3. Spatial Smoothness

- Encourage similar rows (cols) to be in the same bicluster
- $\Delta_V, \Delta_Z$  similarity matrices
- $s_{ij} \geq 0$ : preference for  $i^{th}$  and  $j^{th}$  columns (or rows) to be in the same bicluster

#### The model

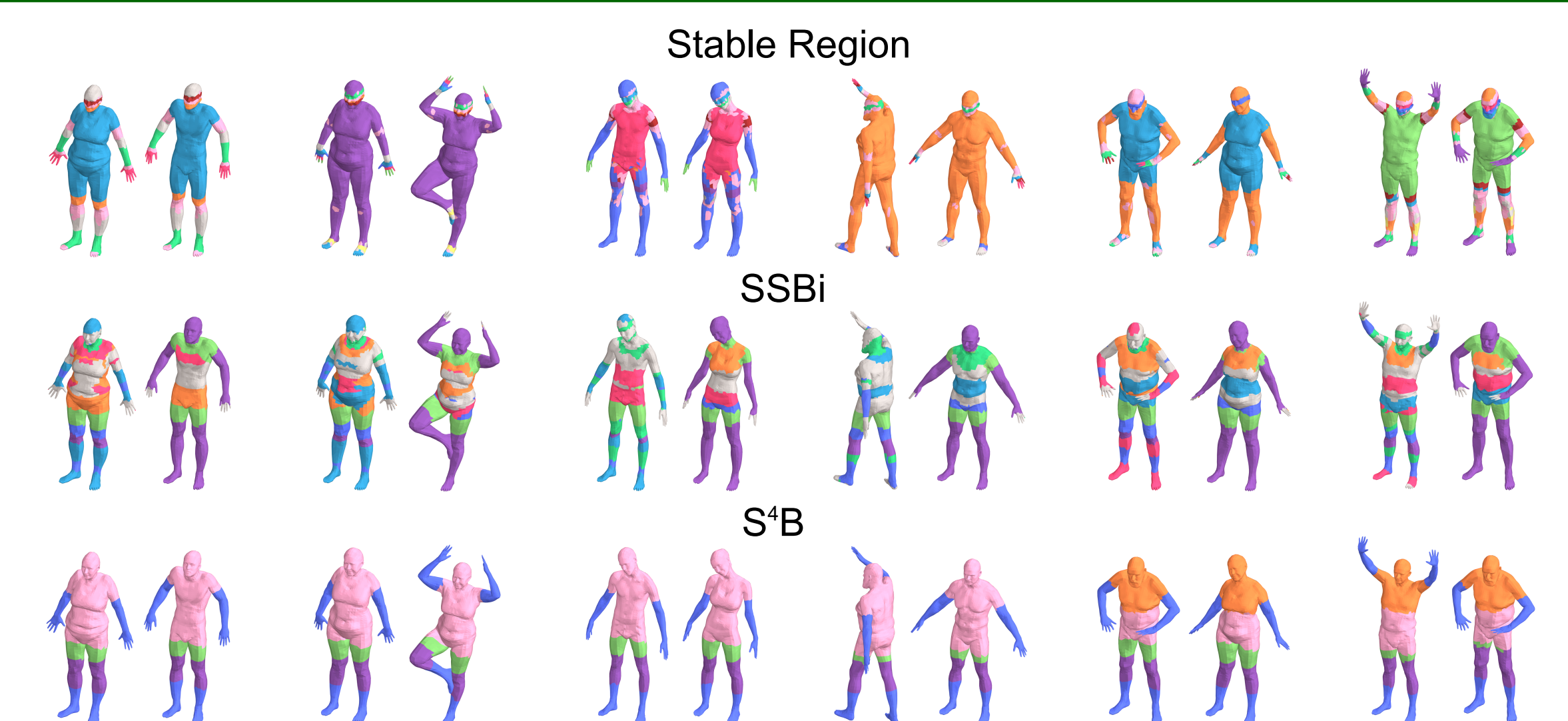
- Bayesian Network
- EM algorithm
- closed form update rules (GEM)



## Results

### FAUST Dataset

#### Qualitative results



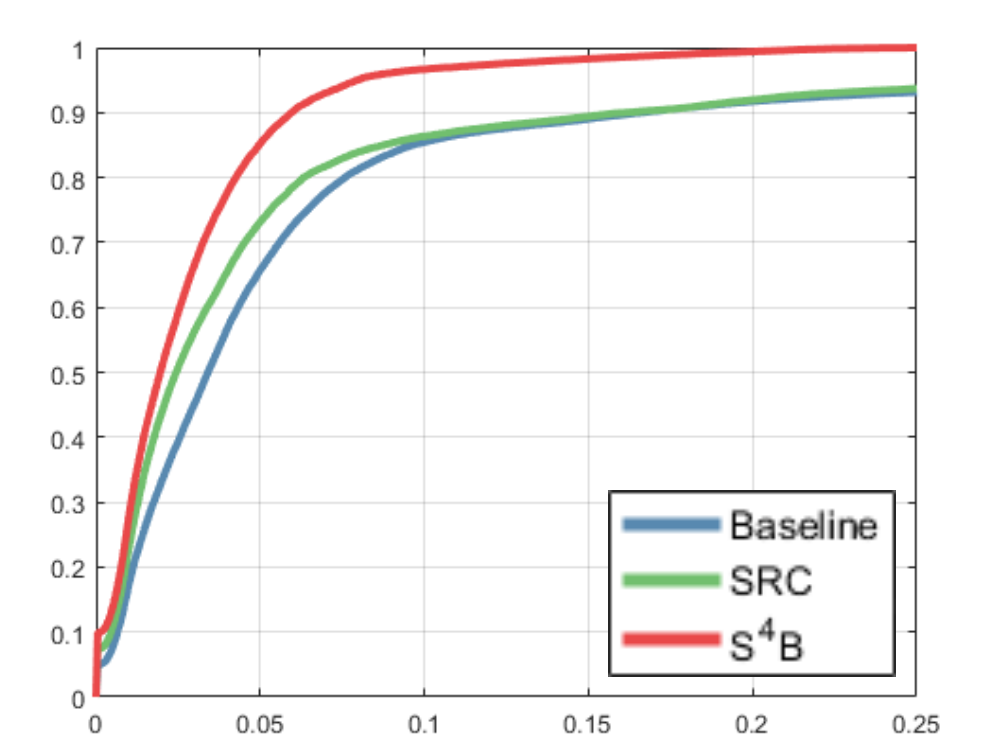
#### Quantitative results

	Stable Region	BIM Voronoi	BIM S <sup>4</sup> B	SSBi	S <sup>4</sup> B	Stable Region	BIM Voronoi	BIM S <sup>4</sup> B	SSBi	S <sup>4</sup> B
scenario1	95.37	95.87	<b>97.98</b>	29.91	97.36	94.95	96.76	97.84	30.07	<b>97.98</b>
scenario2	85.34	95.35	94.21	30.39	<b>95.73</b>	87.42	96.17	93.82	30.95	<b>96.63</b>
scenario3	85.39	92.51	92.5	32.32	<b>94.25</b>	87.63	92.82	92.55	33.77	<b>94.96</b>
global	86.58	93.26	93.36	31.8	<b>94.8</b>	89.33	93.1	93.15	31.26	<b>95.52</b>

Table 1: Scenario1: same subject in different poses. Scenario2: different subjects in the same pose. Scenario3: different subjects in different poses. Mean/median scores for each scenario, and the global mean/median score. SSBi is the S<sup>4</sup>B algorithm without the spatial smoothness prior. BIM details at [SIG '11].

### Point-wise map estimation

- Functional maps [TOG '12]: framework for p2p-map recovery
- such map is obtained combining information of vertices descriptors and known regions
- we compare the standard baseline, with SRC regions and with S<sup>4</sup>B regions
- **S<sup>4</sup>B regions clearly improve the performance!**



## References

- [SGP '16]: Stable Region Correspondences Between Non-Isometric Shapes, *V. Ganapathi-Subramanian et al*
- [TOG '12]: Functional maps: a flexible representation of maps between shapes. *M. Ovsjanikov et al*
- [SIG '11]: Blended intrinsic maps. *V. G. Kim*