Smooth Non-Rigid Shape Matching via Effective Dirichlet Energy Optimization

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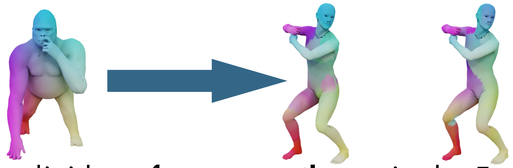
Code : https://github.com/RobinMagnet/smoothFM

1. Motivation

- Spectral algorithms **fail** in the presence of non-isometry.

Source ZoomOut DiscreteOpt



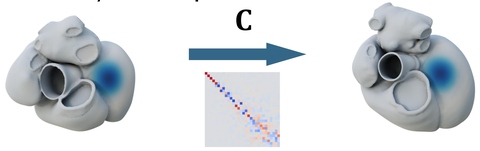
- Can we explicitly **enforce smoothness** in the Functional Map pipeline ?

2. Contributions

- Extension of the Discrete Optimization framework to **pointwise energies**
- A new efficient method to **promote smoothness** for non-isometric shape matching
- Uniformization of **several formulations** for smoothness using coherent notations
- New **non-isometric dataset** with dense ground truth correspondences

3. Background

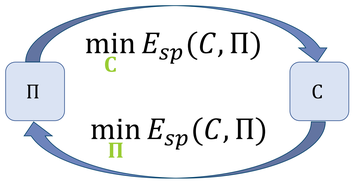
- Functional map $\boldsymbol{C}$ transfer **functions** between shapes. In the spectral basis, they are represented as $K \times K$ matrices



- **Smoothness** is measured using the **Dirichlet Energy** of the pointwise map Π : $E_D(\Pi) = \frac{1}{4}\|\Pi X\|_W^2$

3. Spectral Algorithms

- **ZoomOut**[1] and **Discrete Optimization**[2] compute correspondences using **iterative conversions** between functional and pointwise maps. They however only handle **spectral** energies.



3. Our Algorithm

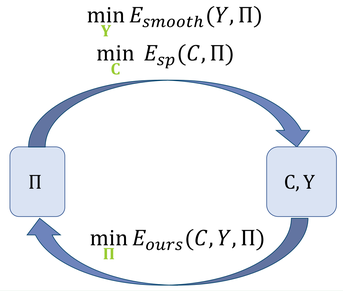
- **Extends** the DiscreteOpt energy with the Dirichlet Energy
- Using half quadratic splitting, we create an additional variable $\boldsymbol{Y}$, and use an **iterative solver** similar to DiscreteOpt.

$$E_{ours}(C,\mathrm{Y},\Pi) = E_{smooth}(\Pi,Y) + \alpha E_{sp}(C,\Pi)$$

$$E_{smooth}(\Pi,Y) = \frac{1}{4}\|Y\|_W^2 + \beta\|\Pi X - Y\|_2^2$$

ALGORITHM 1: Meta-algorithm

Input : Initial maps $\Pi_{12}^{\mathrm{in}}, \Pi_{21}^{\mathrm{in}}$ and vertex positions X_1, X_2
Output: Refined pointwise maps Π_{12}, Π_{21}
Initialization : $\Pi_{ij}^{(0)} = \Pi_{ij}^{\mathrm{in}}$, $Y_{ij}^{(0)} = \Pi_{ij}^{(0)} X_j$ for $i,j \in \{1,2\}$
while *Not converged* **do**
  $C^{(k+1)} = \arg\min_C \; E_{\mathrm{bij}}(\Pi^{(k)}, C)$
  $Y^{(k+1)} = \arg\min_Y \; E_{\mathrm{sm}}^{c}(\Pi^{(k)}, Y)$
  $\Pi^{(k+1)} = \arg\min_\Pi \; E_{\mathrm{ours}}(\Pi, C^{(k+1)}, Y^{(k+1)})$
end



4. Extension of Smoothness Energy

- The algorithm supports **multiple** smoothness energies : ARAP, nICP, Smooth Shells and RHM
- This allows to **compare** these energies in the same framework, without extra regularization

5. DeformingThings4D-Matching

- **New Dataset** with 433 non-isometric pairs
- Dense ground truth correspondences

6. Results

- Qualitative results on TOSCA Non-Isometric

Source ZoomOut DiscreteOpt Ours w/ RHM

- Quantitative results on DEFORMINGTHINGS4D-MATCHING, comparing **multiple energies**.

methods	*accuracy*	*bijectivity*	*smoothness*	*coverage*	*runtime (s)*
Init	12.71	11.70	3.60	24.57%	-
Ours w/ ARAP	12.16	11.70	0.71	31.0%	25.3
Ours w/ nICP	9.56	3.89	1.72	40.4%	100.8
Ours w/ Shells	8.41	2.59	2.18	51.7%	48.2
ZO	8.57	7.14	4.02	67.0%	17.5
DO	9.01	1.78	3.21	62.4%	40.9
Ours w/ D	8.19	2.63	1.56	50.4%	21.4
Ours w/ RHM	8.10	2.18	1.47	56.0%	42.1

- Significant improvement in smoothness and accuracy, with some loss of coverage.

References:
[1] Simone Melzi et al. Zoomout: Spectral upsampling for efficient shape correspondence, 2019
[2] Jing Ren et al. Discrete optimization for shape matching, 2021

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