Reduced Representation of Deformation Fields for Effective Non-rigid Shape Matching

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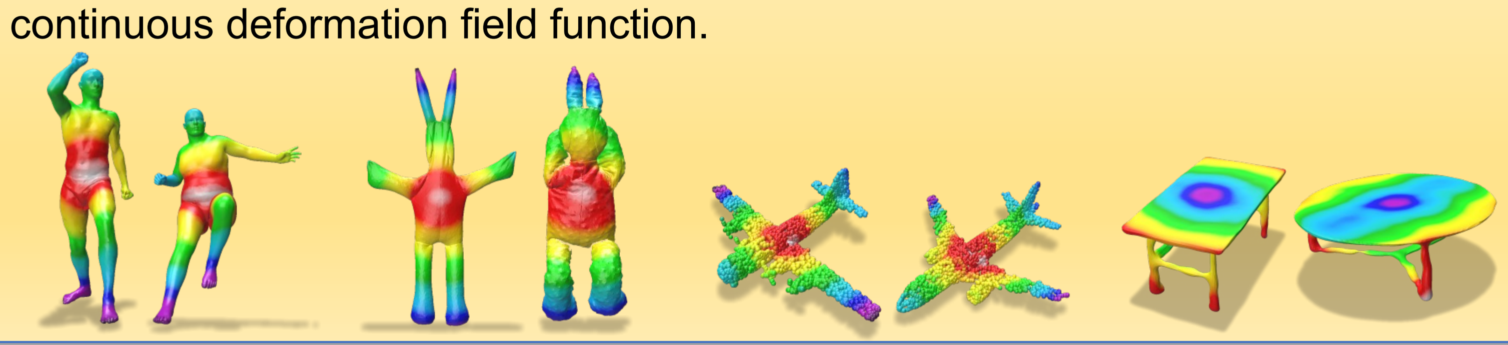
https://github.com/Sentient07/DeformationBasis

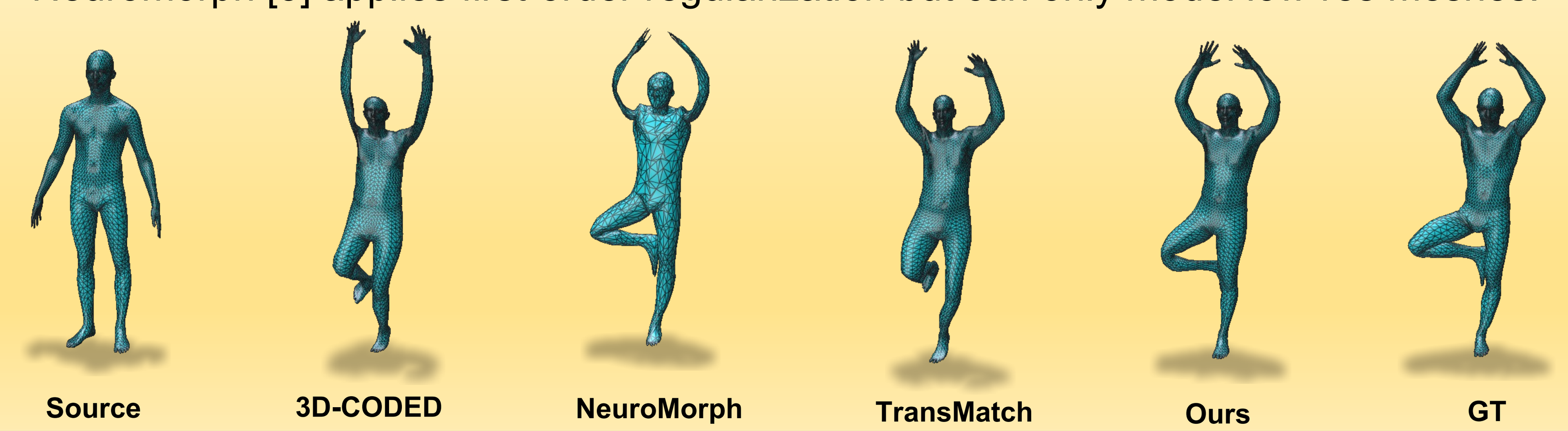
Introduction

- **Objective:** Build an **efficient** and **versatile** non-rigid shape matching technique.
- **Motivation:** Existing methods are poor at modelling finer-details and require abundant training data.
- **Solution:** Learn a coarser representation with limited supervision while approximating a continuous deformation field function.



Related Works

- 3D-CODED [1] learns point-wise deformation field and cannot faithfully model finer-details.
- TransMatch [2] uses transformer to pairwise deformation and needs abundant data to train.
- Neuromorph [3] applies first-order regularization but can only model low-res meshes.



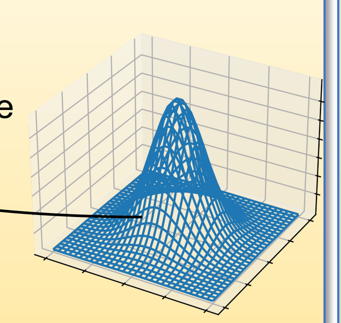
Background: Meshfree Methods

- Construct a continuous approximation of a function $u(\mathbf{x})$ based on observation at points in space (nodes)

$$u(\mathbf{x}) = \sum_{i=1}^{K} \Phi_i(\mathbf{x}) u_i$$

- Where $\Phi_i(\cdot)$ is the shape function which produces smooth approximation near the node

$$\Phi_i(\mathbf{x}) = p^T(\mathbf{x})[M(\mathbf{x})]^{-1} w_i(\mathbf{x}) p(\mathbf{x}_i)$$



- $p(\cdot)$ is a polynomial basis, *M* is the moment matrix and the compact weighting function $w_i(\cdot)$ is given by

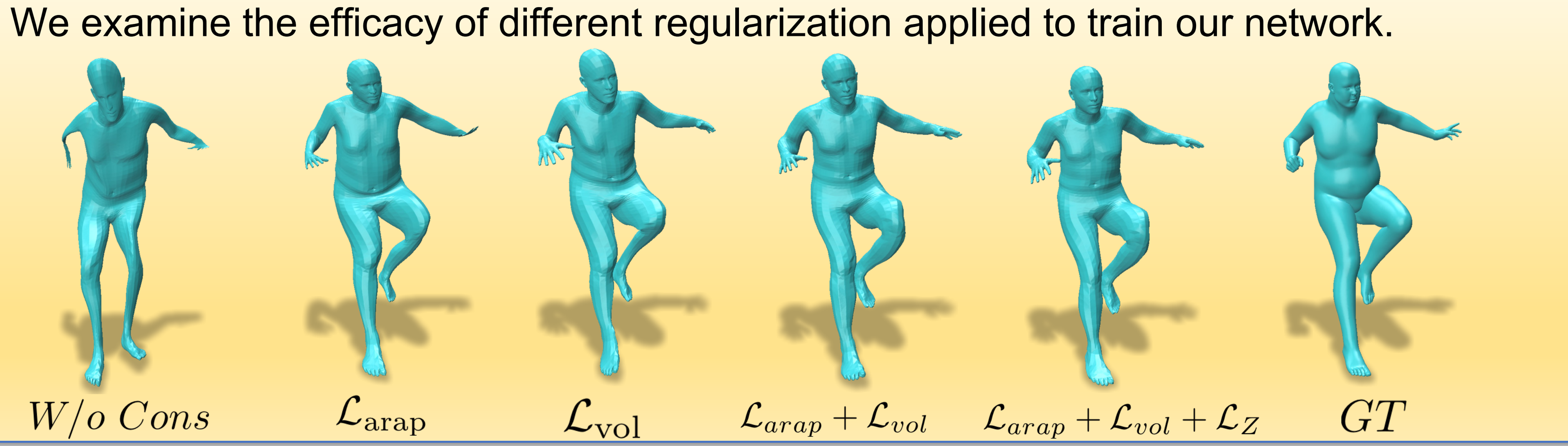
$$w_i(\mathbf{x}) = \begin{cases} \left(1 - \frac{\|\mathbf{x}-\mathbf{x}_i\|_2^2}{r_i}\right)^3, & \text{if } \|\mathbf{x}-\mathbf{x}_i\|_2^2 \leq r_i \\ 0, & \text{otherwise} \end{cases}$$

- Gradient of deformation field is computed w.r.t the shape function and not u_i

$$\mathbb{J} = \nabla_{x,y,z} u(\mathbf{x}) = \sum_{i=1}^{K} \left[\frac{\partial \Phi_i(\mathbf{x})}{\partial x}, \frac{\partial \Phi_i(\mathbf{x})}{\partial y}, \frac{\partial \Phi_i(\mathbf{x})}{\partial z}\right]^T u_i$$

Ablation Study

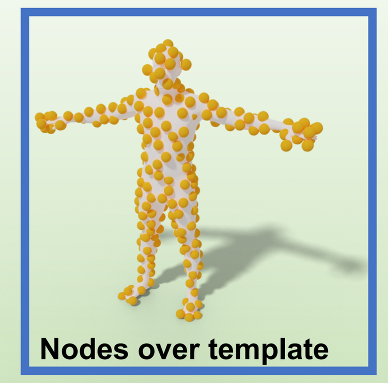
We examine the efficacy of different regularization applied to train our network.

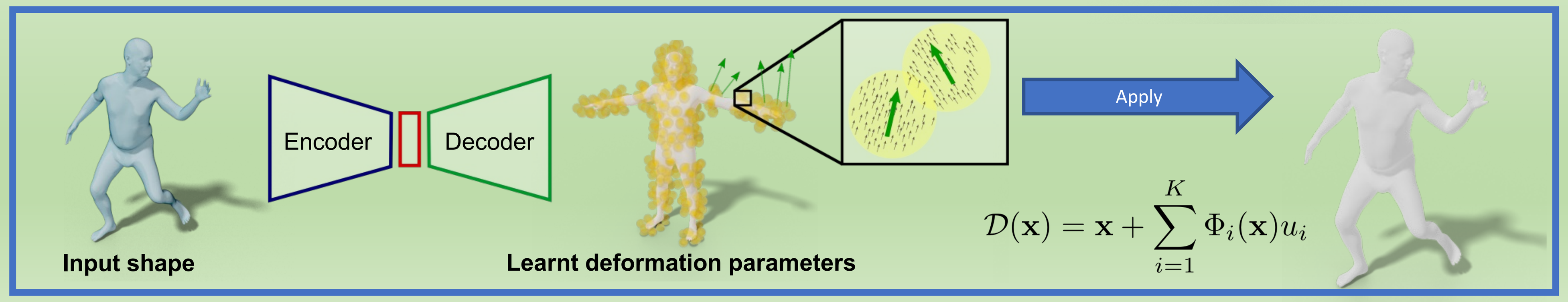


Approach

Three main steps:

- Fix a template and sample nodes near the shape volume.
- Learn deformation parameters at nodes using an auto-encoder.
- Extend and apply the deformation field over entire shape.





Training and Inference

- **Training:** Our objective is a combination of supervision and regularization.

$$\mathcal{L}_{\mathrm{net}} = \underbrace{\boxed{\lambda_1 \mathcal{L}_{\mathrm{cor}}}}_{\text{Supervision}} + \underbrace{\boxed{\lambda_2 \mathcal{L}_{\mathrm{vol}} + \lambda_3 \mathcal{L}_{\mathrm{arap}} + \lambda_4 \mathcal{L}_Z}}_{\text{Regularization}}$$

- $\mathcal{L}_{\mathrm{cor}}$ is the correspondence loss dense or key-point based.

$$\mathcal{L}_{cor} = \sum_{(\mathbf{x}_k, \mathbf{x}_l)}^{|\mathcal{C}|} \|\mathcal{D}(\mathbf{x}_l) - \mathbf{x}_k\|_2^2$$

- $\mathcal{L}_{\mathrm{vol}}$ is the regularization to promote volume preserving deformation fields

$$\mathcal{L}_{vol} = \sum_{j=1}^{N} \sum_{i=1}^{K} |\det(\mathbb{J}_j(\mathbf{q}_i)) - 1|_2^2$$

- $\mathcal{L}_{\mathrm{arap}}$ is to promote As-Rigid-As-Possible (ARAP) deformation fields

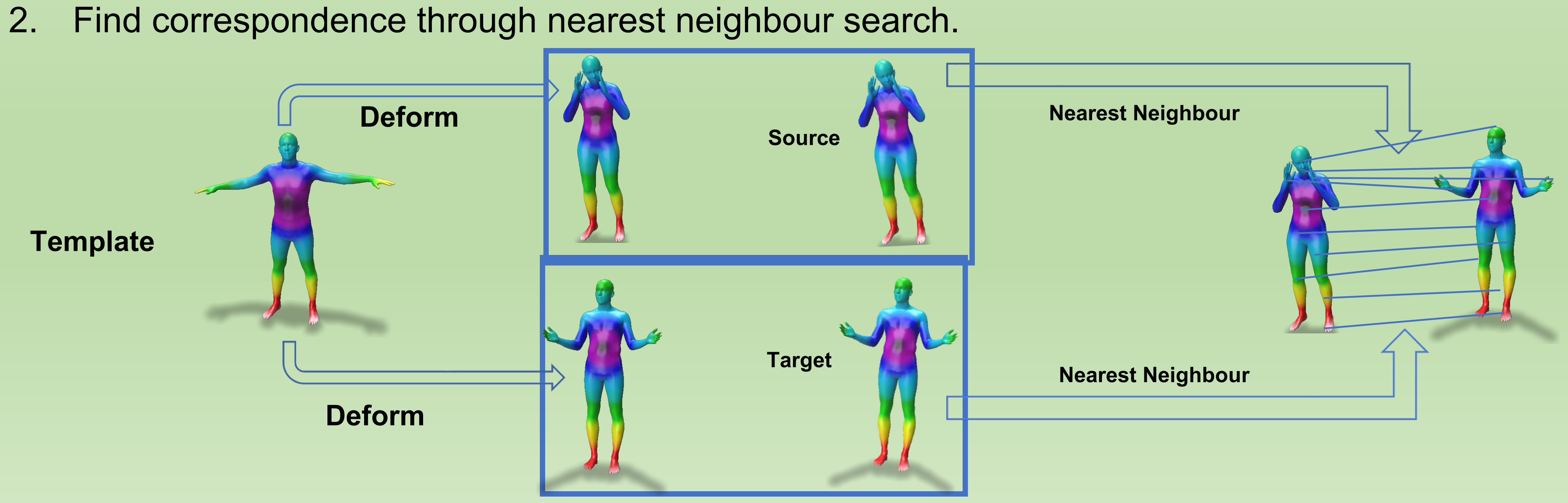
$$\mathcal{L}_{\mathrm{arap}} = \sum_{j=1}^{N} \sum_{i=1}^{K} \|\mathbb{J}_j^T(\mathbf{q}_i)\mathbb{J}_j(\mathbf{q}_i) - \mathbf{I}\|_F^2$$

- $\mathcal{L}_Z$ is the constraint applied to intermediate shapes to promote plausible latent deformation space

$$\mathcal{L}_Z = \sum_{l \neq j}^{|S|} \mathcal{L}_{arap}(\mathrm{dec}((1-\alpha)\mathbf{z}_j + \alpha \mathbf{z}_l)) + \sum_{l \neq j}^{|S|} \mathcal{L}_{vol}(\mathrm{dec}((1-\alpha)\mathbf{z}_j + \alpha \mathbf{z}_l))$$

- **Inference:**

1. Deform the given pair of shapes independently and optionally enhance the reconstruction.
2. Find correspondence through nearest neighbour search.

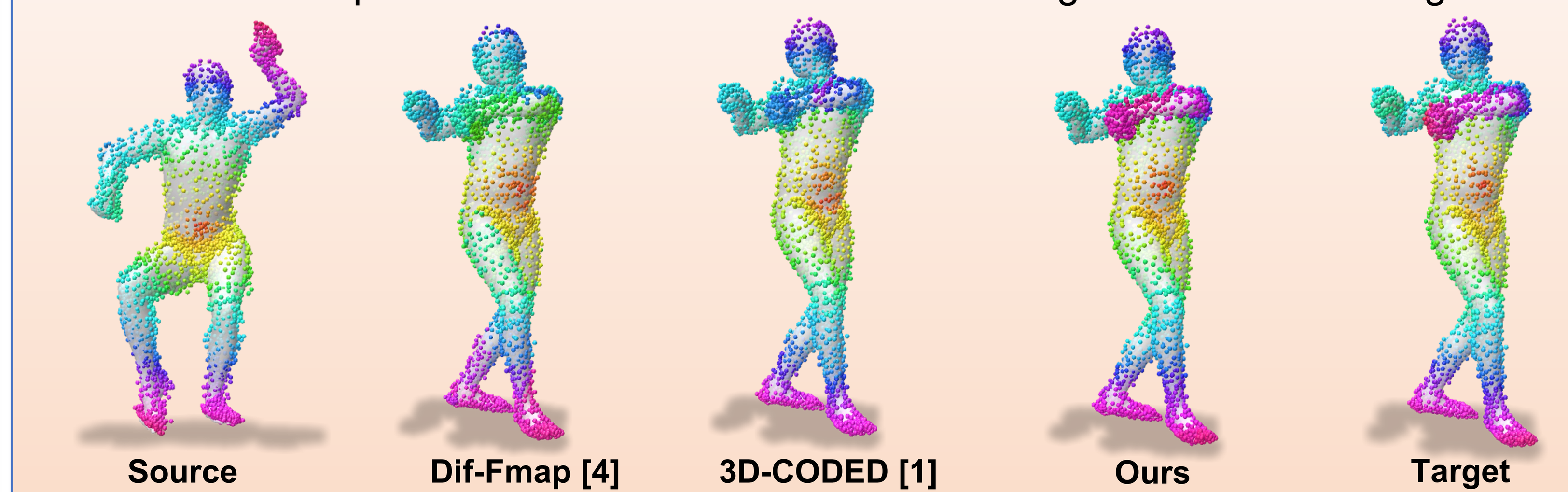


References

[1] 3D-CODED : 3D Correspondences by Deep Deformation, T. Groueix et.al, ECCV 2018
[2] Shape registration in the time of transformers, Trappolini et al NeurIPS 2021
[3] NeuroMorph: Unsupervised Shape Interpolation and Correspondence in One Go, Eisenberger et.al, CVPR 2021
[4]Correspondence Learning via Linearly-invariant Embedding, Marin et.al, NeurIPS 2020,
[5] Deformed Implicit Field: Modeling 3D Shapes with Learned Dense Correspondence, Deng et.al CVPR 2021
[6] Deep Implicit Templates for 3D Shape Representation, Zheng et.al CVPR 2021

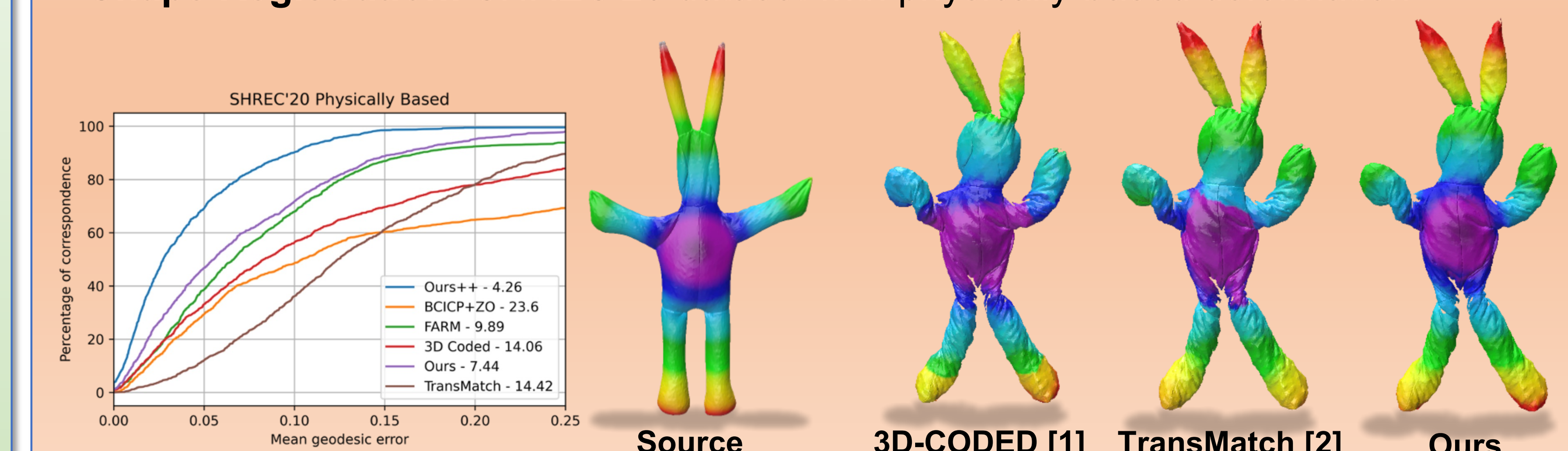
Results: Non-Rigid Shape Matching and Registration

- **Matching:** Qualitative and quantitative results on SCAPE-N and SHREC'19 dataset.
- Our method outperforms baselines with an order of magnitude fewer training data.



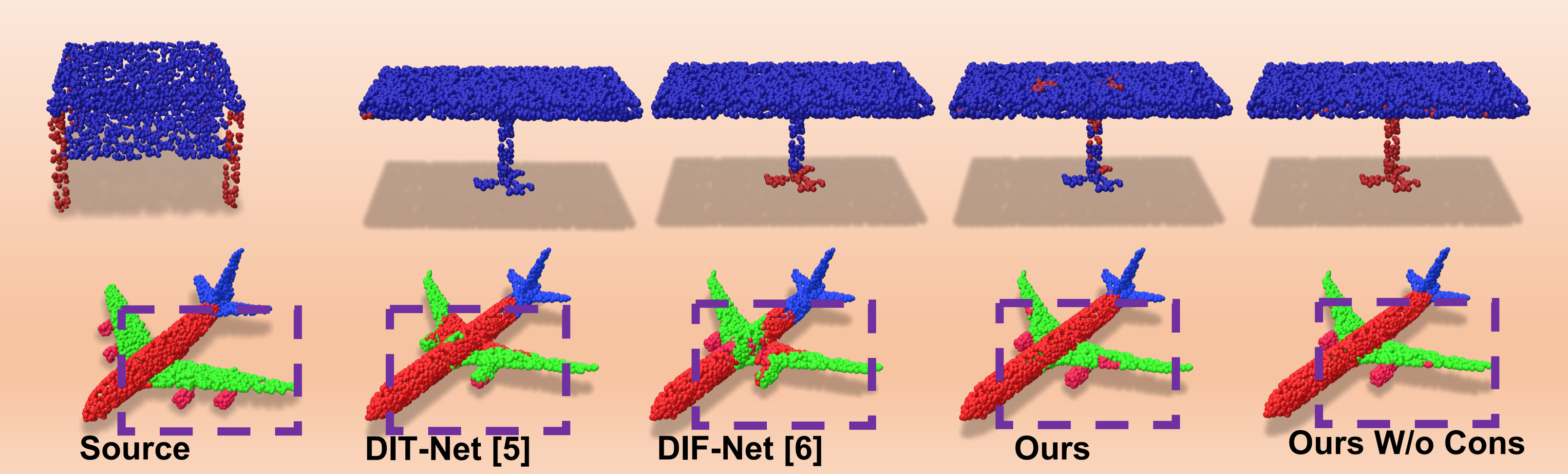
Type	Method	#Tr Data	SHREC'19	SCAPE
Spectral	GeoFMap	1.7	11.2	27.7
Pair-wise	Diff-Fmap	1.0	15.1	26.0
	CorrNet3D	15.0	9.6	38.0
Template Based	3D-CODED	23.0	10.3	18.7
	TransMatch	1.0	6.1	17.1
	Ours	**0.1**	**4.8**	**6.6**

- **Shape Registration:** SHREC'20 dataset with physically-based deformation



Unsupervised Shape Segmentation

Qualitative results on shapes from the ShapeNet dataset



Conclusion

1. Introduced a powerful reduced representation of deformation fields based on mesh-free approximation.
2. Direct analytical expression for the derivatives opens the door to apply first order regularizations effectively.
3. Versatile and shows state-of-the-art performance in down-stream tasks

Acknowledgements

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