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Localization on smart grids[*]

Pierre-Louis Poirion[1], Sonia Toubaline[1], Claudia D'Ambrosio[1], Leo Liberti[1,2]

[1]*CNRS LIX Ecole Polytechnique, Palaiseau 91128, France* {poirion,toubaline,dambrosio,liberti}@lix.polytechnique.fr

[2]*IBM "T.J. Watson" Research Center, Yorktown Heights, 10598 NY, USA* leoliberti@us.ibm.com

Abstract We formalize the problem of localizing monitoring equipment on electrical networks. Each monitoring device can be installed on any link of the network. Various constraints must be taken into account, including topological constraints and Euclidean distance constraints. This yields a Mixed-Integer Nonlinear Program (MINLP) with combinatorial as well as Euclidean distance constraints.

Keywords: MINLP, electrical networks, sensor networks

1. Introduction

An electrical network is a distribution network for the electricity commodity. Some nodes are production nodes, some nodes are demand nodes, there may be intermediate nodes, and the links are usually cables. The technical constraints which regulate the electrical flow involve the physics of alternating currents, and include frequency and phase terms [1–3]. Although the definition of a *smart grid* is somewhat fuzzy, there is a general agreement that a smart grid should be:

- accountable as regards cost, capacity and resilience down to a very precise detail (e.g. at each second, at each node, and so on);
- robust to failures;
- make use of very different energy sources (hopefully environmentally friendlier than burning coal and gas).

Of course these properties are not independent: the network can be robust *if* it is continuously and precisely monitored, and, in the case of failures, alternative sources of energy are readily available. In this work, we focus on the first of the above properties, i.e. accountability.

For an electrical network to be fully accountable, many monitoring devices have to be installed on its nodes and links (more precisely, the device could be localized anywhere along any link). Also, the information collected by these devices has to be sent to centralization servers which are supposed to store and/or perform computation on these data. Data communication can be achieved by using the power lines or wirelessly. The main objective is to install as few devices as possible subject to the network being satisfactorily monitored.

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2. Formulation

2.1 Parameters, variables, objective

Let $G = (V, E)$ be the graph representation of the power network, with node set V and link set E. Each link is a pair $\{i, j\}$ of nodes. The network G is embedded in $\mathbb{R}^2$: for each $i \in V$ let $\nu_i = (\nu_{i1}, \nu_{i2}) \in \mathbb{R}^2$ be the position of node i, and for each $\{i, j\} \in E$ let $\gamma_{ij} : [0, 1] \to \mathbb{R}^2$ be the closed-form description of the embedding of the link $\{i, j\}$ in the plane, such that $\gamma_{ij}(0) = \nu_i$ and $\gamma_{ij}(1) = \nu_j$, and $\gamma_{ij}(t) = (\gamma_{ij1}(t), \gamma_{ij2}(t))$ for each $t \in [0, 1]$.

For each $i \in V$ let $z_i = 1$ iff a node device is installed at i, and 0 otherwise. For each $\{i, j\} \in E$ let $y_{ij} = 1$ iff a link device is installed on $\{i, j\}$, and 0 otherwise. Let $x_{ij} \in \mathbb{R}^2$ be the position of the link device on $\{i, j\}$, and $t_{ij} \in [0, 1]$ be such that $\gamma_{ij}(t_{ij}) = x_{ij}$ if the corresponding link is active:

$$\forall \{i, j\} \in E \quad x_{ij} = y_{ij}\gamma_{ij}(t_{ij}) \tag{1}$$

Cost minimization yields:

$$\min_{x,y,z} \sum_{i \in V} z_i + \sum_{\{i,j\} \in E} y_{ij}. \tag{2}$$

Extending this function to different unit costs for different equipment is very easy.

2.2 Covering constraints

Next, there are covering constraints on nodes and links:

$$\begin{aligned} \forall i \in V \quad z_i + \sum_{\substack{j \in V \\ \{i,j\} \in E}} z_j &\geq 1 && (3) \\ \forall \{i, j\} \in E \quad \sum_{\substack{\{h,k\} \in E \\ h=i \vee h=j}} y_{hk} &\geq 1. && (4) \end{aligned}$$

These constraints ensure that for each node/link neighbourhood at least one monitoring device is installed.

2.3 Communication extent constraints

The communication extent constraints concern the ability of the communication devices to perform their function. If the communication occurs on the power lines, then the constraints are technical (being on either side of a voltage drop barrier, making sure that frequencies and phase overlap nondestructively) and largely depend on the specific properties of the network and the device, so they are difficult to generalize. If the communication is wireless, then it is either anchor-based or point-to-point.

In the first case, it means that every communication device sends its data to a wired hub, commonly located at each node, which has to be within a distance threshold ρ of the device:

$$\forall \{i, j\} \in E \quad y_{ij}(z_i + z_j - z_i z_j)\|x_{ij} - z_i\nu_i - z_j(1 - z_i)\nu_j\|_2 \leq \rho. \tag{5}$$

Eq. (5) makes sure that every communication device on a link is close enough to a hub on a nearby node.

In the second case, we need to ensure connectivity with additional flow variables on the completion of the line graph G, i.e. the complete graph $\bar{G}$ having E as vertex set: for each $e, f, g, h \in E$ let $w_{ef}^{gh} = 1$ if the communication devices on links g and h use the communication devices on links e, f as intermediate hops because they are within the distance threshold ρ, and

0 otherwise. We use these variables in a multicommodity flow setting, where the sources and targets are the links which have a communication device installed.

$$\begin{aligned}
\forall g \neq h, e \neq f \in E \quad y_e y_f \|x_e - x_f\|_2 \quad &\leq \quad w_{ef}^{gh} \rho && (6)\\
\forall g \neq h \in E \quad \sum_{\substack{e \in E \\ e \neq g}} y_e (w_{ge}^{gh} - w_{eg}^{gh}) \quad &= \quad y_g y_h && (7)\\
\forall g \neq h \in E, e \in E \smallsetminus \{g,h\} \quad \sum_{\substack{f \in E \\ f \neq e}} y_f (w_{ef}^{gh} - w_{fe}^{gh}) \quad &= \quad 0. && (8)
\end{aligned}$$

Eq. (6) enforces $w = 1$ on those link pairs having installed communication devices, Eq. (7) are the multicommodity flow constraints at the source nodes, and Eq. (8) are the flow conservation equations.

2.4 Reformulation

The above formulation is a nonconvex MINLP. By assuming box bounds on all continuous variables, however, it can be reformulated exactly to a MINLP where the only nonconvexity is given by Eq. (1) in case the γ functions are nonlinear. First, reformulate Eq. (5) and Eq. (6) by means of the standard "big M" technique. Second, all of the remaining products only involve binary variables, and can therefore be linearized using Fortet's reformulation [4, 5].

2.5 Validation

The hub model (with Eq. (5)) was validated with a few randomly generated neworks with at most 100 nodes and 1500 arcs, using straight line and quadratic curve segments as arcs.

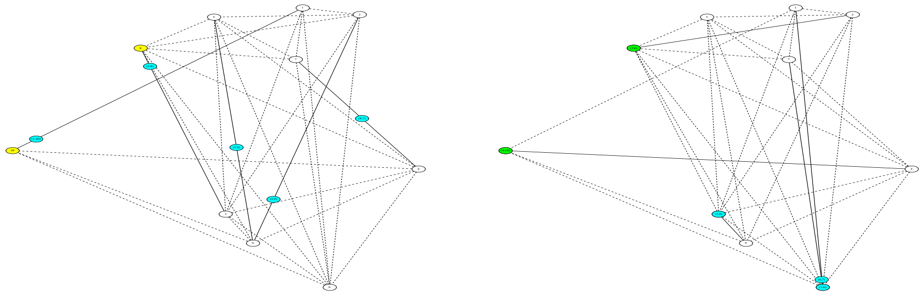

Figure 1. Two 2D hub instances with 10 vertices and edge probability 0.5: linear segments (left) and quadratic curve segments (right). Blue nodes are link devices, and yellow nodes are node devices (hubs). Obviously, green = blue + yellow.

References

[1] D. Bienstock and A. Verma. The n-k problem in power grids: new models, formulations and computation. *SIAM Journal on Optimization*, to appear.

[2] J.F. Bonnans. Mathematical study of very high voltage power networks i: The optimal dc power flow problem. *SIAM Journal on Optimization*, 7:979–990, 1997.

[3] J.F. Bonnans. Mathematical study of very high voltage power networks ii: The ac power flow problem. *SIAM Journal on Applied Mathematics*, 58:1547–1567, 1998.

[4] R. Fortet. Applications de l'algèbre de Boole en recherche opérationelle. *Revue Française de Recherche Opérationelle*, 4:17–26, 1960.

[5] L. Liberti, S. Cafieri, and F. Tarissan. Reformulations in mathematical programming: A computational approach. In A. Abraham, A.-E. Hassanien, P. Siarry, and A. Engelbrecht, editors, *Foundations of Computational Intelligence Vol. 3*, number 203 in Studies in Computational Intelligence, pages 153–234. Springer, Berlin, 2009.