A branch-and-price algorithm for the risk-equity constrained routing problem

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Abstract We study a multi-criteria variant of the problem of routing hazardous material on a geographical area subdivided in regions. The two objective functions are given by a generally defined routing cost and a *risk equity* equal to the maximum, over each region, of the risk *perceived* within a region. This is a multicommodity flow problem where integer variables are used to define the number of trucks used for the routing. This problem admits a straightforward path formulation, for which a branch-and-price problem where, for each node of the branch-and-bound tree, column generation is used to obtain a lower bound.

1 Introduction

The transportation of hazardous materials (*hazmat* from now on) has received a large interest in recent years, this results from the increase in public awareness of the dangers of hazmats and the enormous amount of hazmats being transported [3]. The main target of this problem is to select routes from a given origin-destination pair of nodes such that the risk for the surrounding population and the environment minimum, without producing excessive economic costs. When solving such a prob-

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lem by minimizing both cost and the total risk, typically several vehicles share the same (short) routes which results in high risks associated to regions surrounding these paths whereas other regions are less affected. In this case, one may wish to distribute the risk in an equitable way over the population and the environment. The computation of routes with a fairly distributed risk consists in generating dissimilar origin-destination paths, i.e paths which relatively don't impact the same zones. We classify solution approaches in two sets, *resolution-equity-based methods* and *model-equity-based methods*.

In *resolution-equity-based methods*, equity constraints are taken into account in the resolution process. These methods are based on a dissimilarity index which permits to indicate when two paths are considered as dissimilar. The iterative penalty method [10] consists of computing iteratively a shortest path and penalize its arcs by increasing their weights for discouraging the selection of the same arc set in the generated paths set in the next iteration. The Gateway shortest-paths method [13] consists of generating dissimilar paths by forcing at each time a new path to go through a different node (called the gateway node), the dissimilarity index is defined as the absolute difference between areas under the paths (areas between paths and the abscissa axis). The minimax method [12] consists of selecting k origin-destination shortest-paths and select among them iteratively a subset of dissimilar paths by means of a dissimilar index defined as the length of common parts between the paths. The p-dispersion method [1] generates an initial set U of paths and determines a maximal dissimilar subset S, i.e., the one with the maximum minimum dissimilarity among its paths, the dissimilarity index is the length of common parts or the common impact zones between the paths. The efficiency of these methods is based on the dissimilarity index and the initial set of paths [3].

Model-equity-based methods consist of taking into account equity constraints in the model formulation. In [8, 9], the authors propose an equity shortest path model that minimizes the total risk of travel, while the difference between the risks imposed on any two arbitrary zones does not exceed a given threshold, the authors solve the lagrangean relaxation of the problem and a gap-closing procedure is presented. In [3] is proposed a multi-commodity flow model for routing of hazmat, where each commodity is considered as one hazmat type. The objective function is formulated as the sum of the economical cost and the cost related to the consequences of an incident for each material (commodity). To deal with risk equity, the costs are defined as functions of the flow traversing the arcs, this imposes an increase of the arc's cost and risk when the number of vehicles transporting a given material increases on the arc.

Our problem is similar to that proposed in [3]. We consider the problem where a set of given quantities of hazmats has to be routed over a transportation network from specific origin points to specific destination points. Our goal is the minimization of the total routing cost *and* the maximization of the risk equity, the latter broadly defined as the risk shared by a set of regions that compose the geographical area under consideration. Thus our focus is a multi-criteria optimization problem which we describe more in detail below. The originality of our work is the integration of the objective of minimization of the maximum of risk imposed on all regions during the transportation activity into the multi-commodity flow model which can be solved using a Branch-and-Price algorithm.

This paper is organized as follows. In section 2, the problem is described and an optimization model is given. In section 3 a path formulation of the problem is given and a column generation procedure is described. We present in section 5 a branch-and-price procedure, in section 5 we present some preliminary experiments and we close the paper in section 6.

2 Description of the problem

Let the transportation network be represented as a directed graph $G = (N, A)$, with N being the set of n nodes and A the set of m arcs. Let C be the set of commodities, given as a set of point-to-point demands to transport a certain amount of hazmats. For any commodity $c \in C$, let s^c and t^c be respectively the source node and the destination node, and let D^c be the amount of hazmats to be shipped, by means of a set of trucks of given capacity F_c, from s^c to t^c. We for now assume that each commodity is associated with a unique type of hazmat.

We assume that the risk is computed on each arc of the network and is proportional to the flow traversing such an arc. We consider a set Q of regions, each given as subsets N_q of nodes for each $q \in Q$ of the transportation network, and we define r_{ij}^{cq} as the risk imposed on region $q \in Q$ when the arc $(i, j) \in A$ is used for the transportation of one unit of hazmats of type c. We remark that we employ a notion of *spread risk*, in that an accidental event on arc (i, j) within region $q \in Q$ may strongly affect another region $q' \in Q$.

2.1 Multiple objective functions

The problem of transporting hazmat is multi-objective in nature: one usually wants to minimize two (or more) objectives, namely the total cost of transportation, computed as a function of the amount of hazmat transported throughout the network and the trucks used for the transportation, and the *distributed risk*, which can be defined as a measure of risk that is shared among different regions. More specifically, for a given solution each region $q \in Q$ will be affected by a risk which is dependent on the transportation patterns in all other regions, and which can be summarized by a quantity ω_q. The second objective will then be $\max_{q \in Q} \omega_q$, and has to be minimized.

2.2 An optimization model

We introduce a flow variable f_{ij}^c defining the portion of commodity c being transported on arc (i,j). These variables are subject to flow conservation constraints

$$\sum_{j\in\delta^+(i)} f_{ij}^c - \sum_{j\in\delta^-(i)} f_{ji}^c = b_i^c \qquad \forall i\in N, c\in C$$

where $\delta^-(i)$ and $\delta^+(i)$ are the forward and backward star of i, i.e.,

$$\delta^-(i) = \{j\in N : (j,i)\in A\}, \qquad\qquad \delta^+(i) = \{j\in N : (i,j)\in A\},$$

and

$$b_i^c = \begin{cases} 1 & \text{if } i = s^c \\ -1 & \text{if } i = t^c \\ 0 & \text{otherwise.} \end{cases}$$

Also, y_{ij}^c defines the number of trucks to be used on arc (i,j) for commodity c. The link between variables f and y is given by the constraint

$$D_c f_{ij}^c \le F_c y_{ij}^c \qquad \forall (i,j)\in A, c\in C.$$

The first objective is a function of both f and y variables and is to be minimized: $\sum_{c\in C}\sum_{(i,j)\in A}(\alpha_{ij}^c f_{ij}^c + \beta_{ij}^c y_{ij}^c)$, with α and β suitable cost coefficients which we assume nonnegative. We define the risk ω_q imposed on a region $q\in Q$ as a linear combination of the flow variables:

$$\omega_q := \sum_{c\in C}\sum_{(i,j)\in A} r_{ij}^{cq} f_{ij}^c$$

and add a new variable $z := \max_{q\in Q}\omega_q$, which therefore is subject to the constraints

$$z \ge \sum_{c\in C}\sum_{(i,j)\in A} r_{ij}^{cq} f_{ij}^c \qquad \forall q\in Q.$$

The y variables represent trucks that transport hazmat from each source to each destination, and are therefore subject to flow conservation constraints. We write such constraints here for each commodity and for all intermediate nodes of each commodity, as the source and destination flow balance is redundant here (i.e., it is strictly dependent on the flow variables f):

$$\sum_{j\in\delta^+(i)} y_{ij}^c - \sum_{j\in\delta^-(i)} y_{ji}^c = 0 \qquad \forall i\in N\setminus\{s^c,t^c\}, \forall c\in C$$

The optimization model is therefore as follows:

$$\min \quad \sum_{c\in C}\sum_{(i,j)\in A}(\alpha_{ij}^c f_{ij}^c + \beta_{ij}^c y_{ij}^c) \tag{1}$$

$$\min \quad z \tag{2}$$

$$
\begin{array}{lll}
s.t.\ \sum_{j\in\delta^+(i)} f_{ij}^c - \sum_{j\in\delta^-(i)} f_{ji}^c = b_i^c, & \forall i \in N, c \in C & (3)\\
\sum_{j\in\delta^+(i)} y_{ij}^c - \sum_{j\in\delta^-(i)} y_{ji}^c = 0 & \forall i \in N \setminus \{s^c, t^c\}, \forall c \in C & (4)\\
D_c f_{ij}^c \leq F_c y_{ij}^c & \forall (i,j) \in A, c \in C & (5)\\
z \geq \sum_{c\in C} \sum_{(i,j)\in A} r_{ij}^{cq} f_{ij}^c & \forall q \in Q & (6)\\
f_{ij}^c \in [0,1] & \forall (i,j) \in A, c \in C & (7)\\
y_{ij}^c \in \mathbb{Z} & \forall (i,j) \in A, c \in C. & (8)
\end{array}
$$

Notice that constraints (4) and (5) guarantee that a sufficient number of trucks is allocated for each commodity regardless of the flow of hazmat. The path formulation described below is unable to provide such a guarantee and will therefore have to be modified.

3 Column generation formulation

The above *arc-flow* formulation is polynomial in $|N|$, $|A|$, $|Q|$, and $|C|$, but its size can make it impractical to solve real-world instances of our problem. A common approach is to use a *path-flow* formulation [7]. In these formulations, for each commodity c a variable is associated with every *path* from s^c to t^c. We denote by $\mathscr{P}^c$ the set of paths from s^c to t^c for a commodity $c \in C$ and by $\mathscr{P}_{ij}^c$ the set of paths in $\mathscr{P}^c$ containing arc $(i,j) \in A$. A new path variable f_p, $\forall p \in \mathscr{P}^c, \forall c \in C$, represents the portion of commodity transported on path p.

As for the flow of hazmat, in this formulation the number of trucks, previously denoted by variables y_{ij}^c, might be dependent on path variables f_p. They are by definition the number of trucks to be used on arc $(i,j) \in A$ for commodity $c \in C$. In practice, each truck drives on the whole path p, hence there should be a variable y_p that counts the number of trucks and that is related to variable f_p as follows:

$$F_c y_p \geq D_c f_p \qquad \forall p \in \mathscr{P}^c, c \in C. \tag{9}$$

This constraint substitutes the flow conservation constraint (4) and the "capacity" constraint (5). Let us write this path formulation for completeness:

$$
\begin{array}{llll}
\min & \sum_{c\in C}\left(\sum_{p\in\mathscr{P}^c} \alpha_p f_p + \sum_{(i,j)\in A} \beta_{ij} y_{ij}^c\right) & & (10)\\
\min & z & & (11)\\
s.t. & \sum_{p\in\mathscr{P}^c} f_p \geq 1 & \forall c \in C & (12)\\
& F_c y_p - D_c f_p \geq 0 & \forall p \in \mathscr{P}^c, c \in C & (13)\\
& z - \sum_{c\in C} \sum_{p\in\mathscr{P}^c} r_p^q f_p \geq 0 & \forall q \in Q & (14)\\
& f_p \geq 0 & \forall p \in \mathscr{P}^c, c \in C & (15)\\
& y_p^c \in \mathbb{Z} & \forall p \in \mathscr{P}^c, c \in C, & (16)
\end{array}
$$

where $\alpha_p = \sum_{(i,j)\in p} \alpha^c_{ij}$ are cost coefficients on the path $p \in \mathscr{P}^c$ and $r^q_p = \sum_{(i,j)\in p} r^{cq}_{ij}$ is the risk imposed on region $q \in Q$ when the path $p \in \mathscr{P}^c$ is used for the transportation of one unit of hazmats of type c. Constraint (12) is the path-flow counterpart of the flow conservation constraint (3) and requires that, regardless of the set of paths used, each commodity is fully routed. Constraints (13) and (14) are straightforward extensions of (5) and (6) respectively, given that the flow of commodity $c \in C$ on arc $(i, j) \in A$ is equal to $\sum_{p\in\mathscr{P}^c_{ij}} f_p$.

When restricting to a single-objective optimization problem, this model is an integer multicommodity flow problem. Regardless of considering only one objective, problem (11)-(16) contains $V = \sum_{c\in C} |\mathscr{P}^c|$ variables, which can be exponential in $|N|$. Therefore, solving it by introducing all path variables is in general impractical using the usual combinatorial optimization methods.

Column generation algorithms are very well suited for solving this kind of problems [4]. They use a relatively small initial set of columns to solve a problem, and iteratively introduce a new column when necessary to improve the objective function. Specifically, given a set of columns with negative reduced cost (among those that haven't been considered yet), one can introduce one or more such variables and apply a primal simplex method to resolve the amended problem. The problem with an initially small subset of columns is called the *restricted master problem*, while the problem of finding a variable (column) with negative reduced cost is called the *pricing problem*.

Constraint (13) introduces a major issue in the problem. In principle, introducing y variables indexed on paths rather than arcs and commodities allows to further reduce the number of columns, as we only need $1 + 2\sum_{c\in C} |\mathscr{P}^c|$ variables. However, analogously to columns, we do not want to have exponentially many rows (there are exponentially many paths). The above constraint could be *dynamically* generated, hence instead of column generation we would need *row-column* generation. One huge problem here is that to dynamically generate paths one needs to know all dual variables σ_p, for each $p \in \mathscr{P}^c$ and for all $c \in C$, to solve a pricing problem, and most of these dual variables are *not* available since we didn't generate all of them.

One possible way to deal with this is to use *surrogate constraints*: rather than impose all such constraints or generate them dynamically, we consider a *cover* of such set of constraints and impose conic combinations thereof (see for instance [14]). More specifically, for each $(i, j) \in A$, consider all constraints (9) summed up for all paths containing (i, j). We obtain

$$F_c \sum_{p\in\mathscr{P}^c_{ij}} y_p \geq D_c \sum_{p\in\mathscr{P}^c_{ij}} f_p \qquad \forall (i, j) \in A, c \in C. \tag{17}$$

The problem has now $|C|(1+m)+|Q|$ rows and $1+2\sum_{c\in C} |\mathscr{P}^c|$ variables. Since we relax all of the path constraints (9), the model (10)-(16) constitutes a relaxation of

(1)-(8). Column generation can be applied safely now, although it has to be applied to both f and y variables, and it converges to a *dual feasible* solution which gives a lower bound but not necessarily an optimal solution of the continuous relaxation of (1)-(8).

Suppose an integer solution is found as the optimal solution of the LP relaxation (solved with column generation). If at least one of the constraints (13) is violated, we are stuck with a solution that has no physical value but that cannot be proven primal infeasible unless a constraint is added. What we can do is therefore to create a second branching rule which discriminates between integer feasible solutions and eliminates the integer (but infeasible) solution just found. We will detail this procedure later in this paper, and instead provide insight on how to generate variables.

3.1 Handling one objective only

We consider from now on a continuous relaxation of (11)-(16) amended by the surrogate constraints:

$$\begin{aligned}
&\min && z && && (18)\\
&s.t. && \textstyle\sum_{p\in\mathscr{P}^c} f_p \geq 1 && \forall c\in C && (19)\\
& && F_c \textstyle\sum_{p\in\mathscr{P}^c_{ij}} y_p \geq D_c \sum_{p\in\mathscr{P}^c_{ij}} f_p && \forall (i,j)\in A, c\in C && (20)\\
& && z - \textstyle\sum_{c\in C}\sum_{p\in\mathscr{P}^c} r^q_p f_p \geq 0 && \forall q\in Q && (21)\\
& && f_p \geq 0 && \forall p\in\mathscr{P}^c, c\in C && (22)
\end{aligned}$$

We associate the dual variables vector $\mu\in\mathbb{R}^{|C|}_+$ with constraints (19), $\sigma\in\mathbb{R}^{m|C|}_+$ with constraints (20), and $\lambda\in\mathbb{R}^{|Q|}_+$ with constraints (21). We first analyze this problem considering the single objective (11). Let us define the subset of paths $\bar{\mathscr{P}}^c\subset\mathscr{P}^c, \forall c\in C$. The restricted master problem (RMP from now on) of (11)-(16), generated on a restricted subset of variables $f_p, p\in\bar{\mathscr{P}}^c, c\in C$, is as follows:

$$\begin{aligned}
&\min && z && && (23)\\
&s.t. && \textstyle\sum_{p\in\bar{\mathscr{P}}^c} f_p \geq 1 && \forall c\in C && (24)\\
& && F_c \textstyle\sum_{p\in\mathscr{P}^c_{ij}} y_p - D_c \sum_{p\in\bar{\mathscr{P}}^c_{ij}} f_p \geq 0 && \forall c\in C, (i,j)\in A && (25)\\
& && z - \textstyle\sum_{c\in C}\sum_{p\in\bar{\mathscr{P}}^c} r^q_p f_p \geq 0 && \forall q\in Q && (26)\\
& && f_p \geq 0 && \forall p\in\mathscr{P}^c, c\in C. && (27)
\end{aligned}$$

It is barely worth noting here that (23)-(27) is a restriction of the continuous relaxation of (11)-(16), which therefore provides neither a lower nor an upper bound. Only by applying column generation to (23)-(27), i.e., by iteratively amending columns with negative reduced cost, can we find a lower bound of (11)-(16).
The reduced cost of variables f_p, for each $c\in C, p\in\mathscr{P}^c$, is as follows:

$$\begin{aligned} h(f_p) &= -\mu_c + D_c \sum_{(i,j)\in p} \sigma_{ij}^c + \sum_{q\in Q} r_p^q \lambda_q \\ &= -\mu_c + D_c \sum_{(i,j)\in p} \sigma_{ij}^c + \sum_{q\in Q} \sum_{(i,j)\in p} r_{ij}^{cq} \lambda_q. \end{aligned} \tag{28}$$

Suppose an optimal primal solution $(\bar{f},\bar{y},\bar{z})$ and an optimal dual solution $(\bar{\mu},\bar{\sigma},\bar{\lambda})$ is given. At each iteration of the column generation algorithm, we look for a negative reduced cost variable by solving the problem:

$$\min_{c\in C, p\in\mathscr{P}^c} h(f_p),$$

which provides the column with most negative reduced cost. The *pricing problem* consists of finding the path p that minimizes (28), and is equivalent to solving a *shortest path* problem on a graph G where each arc $(i,j)\in A$ has weight $w_{ij} = D_c\bar{\sigma}_{ij}^c + \sum_{q\in Q} r_{ij}^{cq}\bar{\lambda}_q$. The path must have an origin-destination pair among those defined by the commodities in C. Suppose that, for the shortest path obtained, $-\bar{\mu}_c + D_c L_c + \sum_{q\in Q} r_p^q \bar{\lambda}_q < 0$. Then variable f_p has a negative reduced cost and can be introduced in the model.

One may also look for a negative reduced cost variable for *each* commodity, and add at most $|C|$ such variables. Although this usually speeds up convergence in terms of number of iterations, adding many column every time slows the primal simplex used to obtain a new solution. We obtain $|C|$ origin-destination shortest path problems, therefore the pricing problem becomes $|C|$ times slower — this is negligible given that most of the CPU time is usually spent on the primal simplex.

Notice that y variables do not need to be generated for the risk-objective problem: they only appear in the surrogate constraint, which makes them completely useless given that their value can be decided from an optimal f. This only happens if we consider the second objective function, while the first does contain those variables and would force us to generate them as well. Actually, no f variable is needed either as long as the y variables are only contained in the capacity constraint. The next subsection should shed light on this and introduce another use for y variables.

3.2 Risk on trucks

Another consideration is on risk equity associated to trucks: is the risk (especially the perceived one) only related to the real quantity, or portion, of hazmat transported, or is it also related on the trucks? If both quantity of hazmat and number of trucks should be considered, then the risk equity constraint would change. In this case, we could probably use a parameter s_{ij}^{cq} with an analogous meaning to that of parameter r, i.e., the influence of one truck driving through (i,j), transporting commodity $c\in C$, on region q, and modify (26) as follows:

$$z \geq \sum_{c \in C} \sum_{p \in \mathscr{P}^c} (r_p^q f_p + s_p^q y_p) \qquad \forall q \in Q.$$

where, similarly to r, we define $s_p^q := \sum_{(i,j) \in p} s_{ij}^{cq}$. This provides a motivation for the generation of both f and y variables. In fact, now the procedure to generate y variable can be defined as one that aims at finding a path p such that the reduced cost of the corresponding y_p is minimum:

$$\min_{c \in C, p \in \mathscr{P}^c} h(y_p) = \min\{-D_c \sum_{(i,j) \in p} \sigma_{ij}^c + \sum_{q \in Q} s_p^q \lambda_q : p \in \mathscr{P}\}$$

which provides a more difficult problem given that now the shortest path has to be found on a network with possibly both positive and negative weights.

4 Branch-and-price for single objective problems

In order to find an optimal integer solution to problem (10)-(16), the column generation approach outlined above must be coupled with a branch-and-bound algorithm. This class of algorithms, better known as *branch-and-price*, solve each branch-and-bound node by applying column generation on each lower bounding (continuous) subproblem [2]. For the single objective routing problem, we outline below an implementation of a branch-and-price, which we have implemented in ABACUS.

If only integer variables y_{ij}^c are not dynamically generated (but this no longer seems to be the case), the branching rule is rather simple: consider an optimal solution $(\overline{f}, \overline{y}, \overline{z})$ obtained after column generation at a branch-and-bound node. If, for all $c \in C$ and $(i, j) \in A$, we have $\overline{y}_{ij}^c \in \mathbb{Z}$, then the node can be fathomed as the solution is integer feasible. Otherwise, we select an arc $(i, j) \in A$ and a commodity $c \in C$ such that $\overline{y}_{ij}^c \notin \mathbb{Z}$ and generate two new branch-and-bound nodes with the amended constraints $y_{ij}^c \leq \lfloor \overline{y}_{ij}^c \rfloor$ and $y_{ij}^c \geq \lceil \overline{y}_{ij}^c \rceil$, respectively.

If we use y_p variables instead, we need to take special care in branching rules: given that these variables are generated, the branching rules have dual variables that need to be taken into account in the pricing problem. Furthermore, simple branching rules would not work and the branch-and-bound algorithm would not converge: the branching rule $y_p \leq k$, with $k \in \mathbb{Z}$, does not impose anything on the pricing problem, which might generate another variable that uses the same path as p with reduced cost. Another issue is making sure that the pricing problem remains a shortest path problem. One common branching rule for these cases is that used by Barnhart et al. [2].

5 Preliminary experiments and perspectives

In a first time, we test the efficiency of our model. We implement the formulation (11)-(16) in AMPL (A Modeling Language for Mathematical Programming) [6]. We report a sampling of our computational experiences with the model. We consider an instance with $N = 31$, $|C| = 3$ and $|Q| = 16$ (figure 1). We focus on risk equity objective function (11). Figure 2 present the solution obtained.

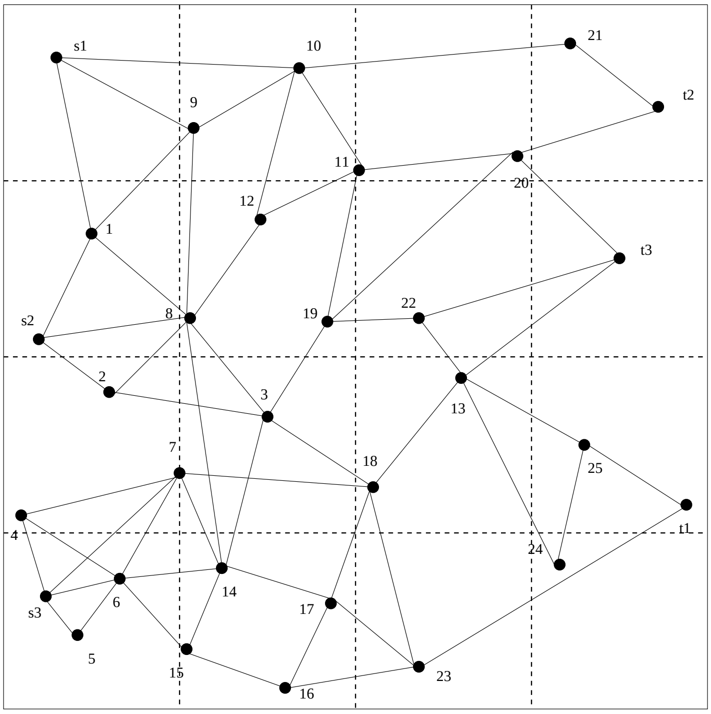

Fig. 1 Transportation risk model: network used in sample problem.

Throw our experimentations, we remarked that improving the equity of the risks imposed results in increased in the total risks imposed. When distributing the risk in an equitable way, routes can be longer, this increases both the total risk and the economic costs. We present on Table 1 the solution values generated by the weights (γ, δ), where γ is the weight on the equity objectives and δ is weight on the total risk objectives (the objective function became: $\gamma z + \delta(\sum_{c\in C,(i,j)\in A,q\in Q} r_{ij}^{cq} f_{ij}^{c})$).

The tradeoffs among risk and the equity of the risk imposed are complex and the number of options are extremely large. Distributing the risk in an equitable way can result in an increase in the total risk and economic cost. In this case, it seems to be not realistic to consider the model with only one objective function. In a branch-and-price algorithm, we can consider two possibilities for considering more than

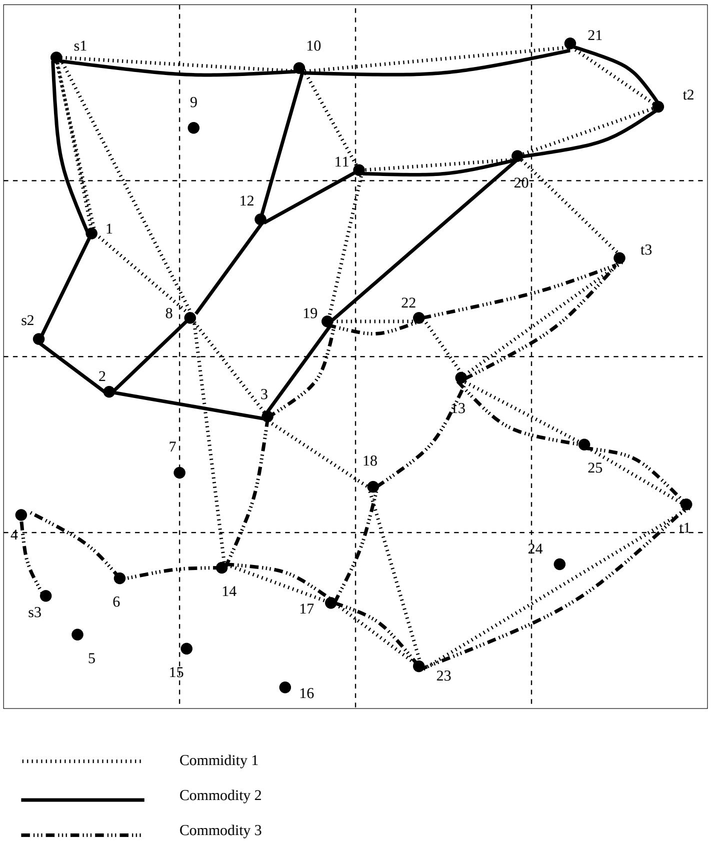

Fig. 2 Transportation risk model: solution of a sample problem.

(γ,δ)	$(0,1)$	$(0.3,0.7)$	$(0.5,0.5)$	$(0.7,0.8)$	$(1,0)$
z	18	16.0	13.8	11.7	10.3
total risk	124	124.7	126.5	129.3	152.6

Table 1 Weighting approach

one objective function, (1) the weighting method can be applied, and (2) the total risk objective function can be taken into account during the column generation algorithm, where the pricing problem will compute Pareto optimal paths considering both the reduced cost and the total risk generated by the path.

6 Conclusion

The transportation of hazmats is an important optimization problem in the field of sustainable development and in particular the equitable distribution of risks is of high interest. Within this study, we formalize this transportation problem as the minimization of two objectives (risk equity and economic cost) and show that a third objective function (total risk) has to be taken into account. Note that, for the moment an actual implementation has to prove in the future what is the effectiveness of the algorithm, which additional accelerating techniques of column generation can be used for solving large instances and how can we take into account many objective functions.

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