On aircraft deconfliction by bilevel programming

Martina Cerulli[1], Claudia D'Ambrosio[1], and Leo Liberti[1]

[1]CNRS - LIX, Ecole Polytechnique, 91128, Palaiseau, France

Abstract

We present a bilevel programming formulation for the aircraft deconfliction problem with multiple lower-level subproblems. We propose two reformulation based on the KKT conditions and the dual of the lower-level subproblems. Finally, we compare the results obtained implementing these formulations using global optimization solvers.

1 Introduction

We consider the problem of *aircraft deconfliction*, or, in other words, detection and resolution of aircraft conflicts, which is one of the main tasks of Air Traffic Management. Aircraft are said to be potentially *in conflict* if their relative distance is smaller than a given safety threshold. Despite the importance of this kind of control, it is still widely performed manually on the ground by air traffic controllers, who essentially monitor the air traffic of a certain period of time on a radar screen, giving instructions to the pilots. Since the level of automation reached on aircraft is very high, the need for automatic tools to integrate human work on the ground is evident.
There are several ways in which aircraft conflicts can be avoided. The most common is based on the change of the trajectory or the flight level of the involved aircraft. This is the way air traffic controllers usually solve potential conflicts. Another strategy consists in slightly changing the speeds while keeping the trajectories unchanged. The latter is the variant on which we will focus in this paper. We present a Mathematical Programming (MP) formulation for aircraft separation based on speed regulation. For a wider introduction to this problem, see [1].
We will assume that aircraft fly within a fixed altitude layer: they can thus be modeled as points in $\mathbb{R}^2$ (see Figure 1 as an example).

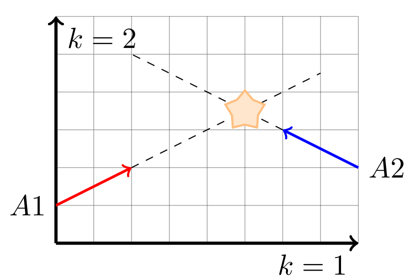

Figure 1: Two conflicting aircraft

Deconfliction is expected to involve minimal deviations from the original aircraft flight plan. Therefore, the objective function to minimize will take into account the percentage of speed change of each aircraft. Of course, for each aircraft pair, we must assure that they do not get closer to each other than a given safety distance at every time t of a fixed interval $[0, T]$ (note that this generates an uncountably infinite set of constraints).

2 Mathematical Formulations

We propose a MP formulation of the speed-change problem variant. The terminology and symbols are taken from [1].

1. **Sets:**
   - $A = \{1, .., n\}$ is the set of aircraft (n aircraft move in the shared airspace)
   - $K = \{1, 2\}$ is the set of directions (the aircraft move in a Euclidean plane)
2. **Parameters:**
   - T is the length of the time horizon taken into account [hours]
   - d is the minimum required safety distance between a pair of aircraft [NauticalMile NM]
   - x^0_{ik} is the k-th component of the initial position of aircraft i
   - v_i is the initial speed of aircraft i [NM/h]
   - u_{ik} is the k-th component of the direction of aircraft i
   - q_i^{min} and q_i^{max} are the bounds on the ratio of the speed for each aircraft
3. **Variables:**
   - q_i is the is the possible increase or decrease of the original speed of aircraft i: $= 1$ if the speed is unchanged, $q_i > 1$ if it is increased, $q_i < 1$ if it is decreased
   - t_{ij} is the instant of time defined for the aircraft pair i and j; these variables help us compute the relative distance between i and j in time interval $[0, T]$

2.1 Bilevel formulation of the problem

In order to address the issue of uncountably many constraints for each value in $[0, T]$, we propose to formulate the problem as a bilevel MP with multiple second level problems. Each of these subproblems ensures that the minimum distance between each aircraft pair exceeds the safety distance threshold. Thus, each lower-level subproblem involves the lower-level variable t, and is parameterized by the upper-level variables q.

$$\min_{q,t} \quad \sum_{i \in A} (q_i - 1)^2 \tag{1}$$

$$\forall i \in A \quad q_i^{\mathsf{min}} \leq q_i \leq q_i^{\mathsf{max}} \tag{2}$$

$$\forall i < j \in A \quad d^2 \leq \min_{t_{ij} \in [0,T]} \sum_{k \in \{1,2\}} ((x^0_{ik} - x^0_{jk}) + t_{ij}(q_i v_i u_{ik} - q_j v_j u_{jk}))^2 \tag{3}$$

The upper-level (convex) objective function is the sum of squared aircraft speed changes. This corresponds to finding the feasible solution with the minimum speed change, as mentioned before. It must be minimized w.r.t the time t and the variable q, with each q_i within the given range $[q_i^{\mathsf{min}}, q_i^{\mathsf{max}}]$. The objective of each lower-level subproblem is to minimize over $t_{ij} \in [0, T]$ the relative Euclidean distance between the two aircraft it describes; note that this is also a convex function. This minimum distance, reached at t^*_{ij}, must be at least d^2. This corresponds to imposing the minimum safety distance d between aircraft i and j within $[0, T]$.

2.2 KKT reformulation

We follow standard practice and replace each convex lower-level subproblem by its Karush-Kuhn-Tucker (KKT) conditions. Assuming some regularity condition (e.g. the Slater's condition) holds, this yields a single-level MP with complementarity constraints. Given the KKT multipliers μ_{ij} and λ_{ij} defined for each lower-level problem, we have:

$$
\begin{aligned}
\min_{q,t} \quad & \sum_{i\in A}(q_i-1)^2 && (4)\\
\text{s.t.} \quad & \forall i \in A \quad q_i^{min} \le q_i \le q_i^{max} && (5)\\
& \forall i<j \in A \quad \sum_{k\in\{1,2\}} (2t_{ij}(q_i v_i u_{ik} - q_j v_j u_{jk})^2 + \\
& \qquad + 2(x^0_{ik} - x^0_{jk})(q_i v_i u_{ik} - q_j v_j u_{jk}) - \mu_{ij} + \lambda_{ij}) = 0 && (6)\\
& \forall i<j \in A \quad \mu_{ij},\ \lambda_{ij} \ge 0 && (7)\\
& \forall i<j \in A \quad \mu_{ij}\, t_{ij} = 0 && (8)\\
& \forall i<j \in A \quad \lambda_{ij}\, t_{ij} - \lambda_{ij}\, T = 0 && (9)\\
& \forall i<j \in A \quad -t_{ij} \le 0,\ t_{ij} \le T && (10)\\
& \forall i<j \in A \quad \sum_{k\in\{1,2\}} ((x^0_{ik} - x^0_{jk}) + t_{ij}(q_i v_i u_{ik} - q_j v_j u_{jk}))^2 \ge d^2 && (11)
\end{aligned}
$$

The last constraint Eq. (11) is necessary to ensure that each KKT solution t^*_{ij} respects the safety distance.

2.3 Dual reformulation

We propose another closely related reformulation of the bilevel problem (1)-(3), which arises because the lower-level subproblems are convex Quadratic Programs (QP). Specifically, their duals are also QPs which only involve dual variables [2, 3]. In particular, an upper-level constraint such as Eq. (3) has the form $\mathsf{const} \le \min\{\frac{1}{2}\, x^\top Q x + p^\top x \mid Ax \ge b \wedge x \ge 0\}$ with Q positive semidefinite. By strong duality it can be written as follows:

$$\mathsf{const} \le \max\{-\frac{1}{2}\, y^\top Q y + b^\top z \mid A^\top z - Qy \le p \wedge z \ge 0\}, \tag{12}$$

where the maximization QP on right hand side is the dual of the previous minimization one [3].

Proposition 1. *Eq.* (12) *can be replaced by* $\mathsf{const} \le -\frac{1}{2}\, y^\top Q y + b^\top z \wedge A^\top z - Qy \le p \wedge z \ge 0$ $(*)$ *in Eq.* (1)-(3).

Proof. If Eq. (12) is active, then the maximum objective function value of the QP is const. Because of the max operator, the objective function of the QP cannot attain any larger value. This means that $(*)$ can only be feasible when $-\frac{1}{2}\, y^\top Q y + b^\top z$ attains its maximum over $A^\top z - Qy \le p$ and $z \ge 0$. If Eq. (12) is inactive, it has no effect on the optimum. Since $(*)$ is a relaxation of Eq. (12), the same holds. □

Prop. 1 yields the following reformulation of 1-(3).

$$\min_{\substack{q\in[q^{\mathsf{min}},q^{\mathsf{max}}]\\ z\ge 0, y}} \quad \sum_{i\in A}(q_i-1)^2 \tag{13}$$

$$\forall i<j \in A \quad -\sum_{k=1}^{2}(q_i v_i u_{ik} - q_j v_j u_{jk})^2\, y_{ij}^2 + (-T) z_{ij} \ge d^2 - \sum_{k=1}^{2}(x^0_{ik} - x^0_{jk})^2 \tag{14}$$

$$\forall i<j \in A \quad -\frac{z_{ij}}{2} - \sum_{k=1}^{2}(q_i v_i u_{ik} - q_j v_j u_{jk})^2\, y_{ij} \le \sum_{k=1}^{2}(x^0_{ik} - x^0_{jk})(q_i v_i u_{ik} - q_j v_j u_{jk}) \tag{15}$$

3 Computational results

We considered the set of instances proposed in [1], where n aircraft are placed on a circle of given radius r, with initial speed v_i and a trajectory defined by a heading angle such that aircraft fly toward the center of the circle (or slightly deviating with respect to such direction). Then we also considered instances in which aircraft move along straight trajectories intersecting in n_c conflict points. We set: $T = 2$ hours, $d = 5\,\mathrm{NM}$, $v_i = 400\,\mathrm{NM/h}$ for each $i \in A$. For the "*circle instances*" the heading angles cap_i are randomly generated and parameters x^0_{ik} and u_{ik} are given by $u_{i1} = \cos(\mathsf{cap}_i)$, $u_{i2} = \sin(\mathsf{cap}_i)$, $x^0_{ik} = -r\,u_{ik}$. The bounds q^{min}_i and q^{max}_i are set to 0.94 and 1.03 respectively. We implemented the proposed formulations using the AMPL modeling language [4] and solved them with the global optimization solver Baron [5] (B in the Table 1) or, when Baron was not successful, with a Multistart algorithm (MS in the Table 1) with 1000 iterations in total. The Multistart method for the KKT reformulation uses SNOPT [6] at each iteration, while the one for the Dual reformulation uses IPOPT [7]. Our results are reported in Table 1, and compared with those that are the best among the ones obtained with different methods in [1] and [8].

Instance			Cafieri	KKT reformulation			Dual reformulation		
n	n_c	r	*obj (Best solution)*	*obj*	*time(s)*	solver	*obj*	*time(s)*	solver
Circle instances									
2	-	100	0.002531	0.002524	0.28	B	0.002526	0.41	B
3	-	200	0.001667	0.001664	1.49	B	0.001663	3.70	B
4	-	200	0.004009	0.004025	65.42	B	0.004017	184.4	B
5	-	300	0.003033	0.003052	12511	B	0.003050	13978	B
6	-	300	0.006033	0.006088	31.99	MS	0.006096	7.84	MS
Non-circle instances									
6	5		0.001295	0.001254	53.31	MS	0.001254	14.88	MS
7	4		0.001617	0.001591	238.82	MS	0.001591	31.17	MS
7	6		0.001579	0.001566	86.95	MS	0.001566	33.18	MS
8	4		0.002384	0.002384	1163	MS	0.002384	39.54	MS
10	10		0.001470	0.001469	835.24	MS	0.001397	78.90	MS

Table 1

The value of the objective function is always very low, given the nature of the problem (q must be in $[0.94, 1.03]$). Comparing the solutions obtained on the instances considered, it appears that they are comparable.

References

[1] N. Durand S. Cafieri. Aircraft deconfliction with speed regulation: New models from mixed-integer optimization. *J. of Global Optimization*, 58:613–629, 2014.

[2] W. S. Dorn. Self-dual quadratic programs. *Journal of the Society for Industrial and Applied Mathematics*, 9(1):51–54, 1961.

[3] W. S. Dorn. Duality in quadratic programming. *Quarterly of Applied Mathematics*, 18(2):155–162, 1960.

[4] Robert Fourer, David M. Gay, and Brian W. Kernighan. *AMPL: A Modeling Language for Mathematical Programming.* 2002.

[5] N.V. Sahinidis and M. Tawarmalani. *BARON 7.2.5: Global Optimization of Mixed-Integer Nonlinear Programs,* User's Manual, 2005.

[6] P.E. Gill. *User's guide for SNOPT version 7.2.* Systems Optimization Laboratory, Stanford University, California, 2006.

[7] A. Wächterm and L. T. Biegler. On the implementation of a primal-dual interior point filter line search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 2006.

[8] C. D'Ambrosio S. Cafieri. Feasibility pump for aircraft deconfliction with speed regulation. *Journal of Global Optimization, Springer Verlag*, 71(3):501–515, 2018.