Ordered Resolution and Paramodulation

October 25, 2005

Outline

Equality

Axiomes de l'égalité

$$x = x \qquad \text{(reflexivity)}$$
$$x = y \ \vee \ \neg y = x \qquad \text{(symmetry)}$$
$$x = z \ \vee \ \neg x = y \vee \neg y = z \qquad \text{(transitivity)}$$

$$f(\overline{x}) = f(\overline{y}) \ \vee \ \neg \overline{x} = \overline{y} \qquad \left(\begin{array}{c} \text{fonctional} \\ \text{monotonicity} \end{array} \right)$$

$$P(\overline{x}) = P(\overline{y}) \ \vee \ \neg \overline{x} = \overline{y} \qquad \left(\begin{array}{c} \text{predicative} \\ \text{monotonicity} \end{array} \right)$$

The are as many fonctional (resp. predicative) monotonicity axioms as the number of function (resp. predicate) symbols in the vocabulary.

- Adding these axioms for each equality predicate leads to a blow up in the number of clauses generated by ordered resolution, most of which are useless.
- Example (take the order rpo($f > a$))

$$\neg a = f(a) \quad f(x) = f^2(x)$$

- Robinson et Wos proposed to replace all equality axioms except reflexivity by one, specific, less prolific inference rule:

$$\frac{C \vee l = r \qquad \pm D \vee A[u]}{C\sigma \vee D\sigma \vee \pm A[r\sigma]} \quad \text{si} \begin{cases} u \notin \mathcal{X} \\ \sigma = mgu(l = u) \end{cases}$$

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Resolution

$$\frac{+A \vee C \qquad -A' \vee C'}{C\sigma \vee C'\sigma} \quad \begin{array}{l} \sigma = mgu(A = A') \\ A\sigma \not\succ B \ \forall B \in C\sigma \vee C'\sigma \end{array}$$

Factoring

$$\frac{+A \vee +A' \vee C}{+A\sigma \vee C\sigma} \quad \begin{cases} \sigma = mgu(A = A') \\ A\sigma \not\succ B \ \forall B \in C\sigma \end{cases}$$

Reflexivity

$$\frac{-u = v \vee C}{C\sigma} \quad \begin{cases} \sigma = mg(u = v) \\ u\sigma = v\sigma \not\succ B \ \forall B \in C\sigma \end{cases}$$

$\mathcal{ORPF}$ continued: Paramodulation

Ordered Paramodulation

$$\frac{C \vee l = r \quad D \vee \pm A[u]}{C\sigma \vee D\sigma \vee \pm A\sigma[r\sigma]} \quad \begin{cases} \sigma = mgu(l = u) \\ {\color{red} l\sigma \nprec r\sigma} \\ l\sigma = r\sigma \nprec B \ \forall B \in C\sigma \\ A\sigma \nprec B \ \forall B \in D\sigma \end{cases}$$

Monotonic Ordered Paramodulation

$$\frac{C \vee l = r \quad D \vee \pm A[u]}{C\sigma \vee D\sigma \vee \pm A\sigma[r\sigma]} \quad \begin{cases} \sigma = mgu(l = u) \\ {\color{red} A\sigma[l\sigma] \nprec A\sigma[r\sigma]} \\ l\sigma = r\sigma \nprec B \ \forall B \in C\sigma \\ A\sigma \npreceq B \ \forall B \in D\sigma \end{cases}$$

Completeness Theorem

$\mathcal{ORPF}$ is refutationally complete for any partial quasi-ordering $\succeq$ satisfying the following properties:

1. $\succeq$ is stable on terms,
2. $\succeq$ restricts on ground terms to a total well-founded monotonic ordering $\succ$
3. $\succeq$ is extended to atoms so as to satisfy: monotonicity: $s \succ t$ implies $A[s] \succ A[t]$ for any atom $A[s]$; minimality of $=$: $s \succ t$ implies $A[s] \succ s = t$ if A is not an equality atom.
4. $\succeq$ is extended to litterals and clauses in a natural way.
5. Both rules coincide under these conditons.

Subterm violated

Monotonic paramodulation is incomplete:

$$\{fb \neq fa,\ b = fb,\ a = fb\}$$
$$ffa \succ ffb \succ a \succ b \succ fa \succ fb$$

$$\frac{b = fb \qquad a = fb}{NO} \qquad \frac{ffb = fb \qquad a = fb}{NO}$$

$$\frac{fb \neq fa \qquad b = fb}{NO} \qquad \frac{fb \neq fa \qquad a = fb}{NO}$$

Ordered paramodulation generates:

$$\left\{ \begin{array}{l} f^m b \neq fa,\ f^{n+1} b \neq f^{m+1} b, \\ a = f^m b,\ f^n b = f^m b,\ a = b \end{array} \middle|\ n \geq 0,\ m > 0 \right\} \cup \{\Box\}$$

Is not a decision procedure when ground.

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$$fg^3b \succ fgb \succ fb \succ fg^2b \succ gb \succ b$$

The set of (ground unit) clauses is already closed.

Using ordered paramodulation:

$$\{gb = b, fg^2b \neq fb, fgb \neq fb, fb \neq fb, \square\}$$

Completeness of ordered paramodulation does not need monotonicity
[Bachmair, Ganzinger, Nieuwenhuis, Rubio, 2003]

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Completeness: the roadmap

- Compactness yields a finite unsatisfiable set of ground clauses;
- Build the tree of equality interpretations;
- Define the branch ending at an inference node;
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Herbrand equality interpretations

Problem of the construction

Let I a node in the tree interpreting the atoms $\{A_j\}_{0 \leq j < i}$ such that E_I is the set of equalities interpreted in T by I which we turn into a set of rules E_I.

We want exactly three cases now:

(i) A_i is $s = s$ that we interpret in T;

(ii) A_i is reducible in E_I to some atom B and both interpretations must be the same;

(iii) A_i is irreducible and has two successors.

These three cases correspond respectively to reflexivity, paramodulation and resolution.

Problems

- E_I must be confluent to ensure consistent decisions;
- The atom B must belong to $\{A_j\}_{0 \leq j < i}$.
- For an arbitrary finite unsatisfiable set of ground instances of the clauses, these assumptions are usually not met.

Completion of the set of atoms

$\mathcal{A}$: finite set of atoms
$\mathcal{E}$ be the set of equalities in $\mathcal{A}$

$$A \longrightarrow_{\mathcal{E}} B$$

if

$$A = A[s],\ B = A[t],\ s = t \in \mathcal{E},\ s \succ t$$

Ordered completion:

$$\frac{\mathcal{A} \cup \{A\} \qquad A \longrightarrow_{\mathcal{E}} B}{\mathcal{A} \cup \{B\}}$$

Observation: the tree of all possible completion sequences is finite.

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Key property of Ordered Completion

Lemma

Let $\mathcal{A}$ be a finite set of atoms, $\mathcal{E}$ be its subset of equality atoms, $\overline{\mathcal{A}}$ the closure of $\mathcal{A}$ under all possible sequences of ordered completion, and $\overline{\mathcal{E}}$ its subset of equality atoms. Then
(i) $\overline{\mathcal{A}}$ is finite;
(ii) $\overline{\mathcal{E}}$ is closed under critical pair computation, hence defines a convergent set of rules again written $\overline{\mathcal{E}}$;
(iii) $\overline{\mathcal{A}}$ is closed under rewriting with $\overline{\mathcal{E}}$.

Proof: easy.

Ordered Completion achieves its goals

We assume that $\overline{\mathcal{A}} = \mathcal{A}$, hence $\overline{\mathcal{E}} = \mathcal{E}$.
Let E_H be the subset of equalities (rules) in $\mathcal{E}$ interpreted by T in the Herbrand interpret. H.

Lemma

Let $\{A_i\}_{i<j\leq n}$ be an initial segment of $\mathcal{A}$, and H an equality interpretation of $\{A_i\}_{i<j}$. Then,
(i) $A_i \longrightarrow_{E_H} B$ implies that $B = A_k$ for some $k < i$,
(ii) E_H is a convergent subset of $\mathcal{E}$.

Proof: Since $\mathcal{A}$ is closed under ordered completion, $B \in \mathcal{A}$, and $A_i \succ B$ implies $k < i$.
A critical pair between two rules of $\{A_i\}_{i<j}$ belongs to $\{A_i\}_{i<j}$ by (i), hence to E_H by def. □

Herbrand equality interpretations

The *tree of Herbrand equality interpretations* over $\mathcal{A} = \{A_j\}_j$ is defined inductively. Each node I in the tree defines a partial equality interpretation of $\{A_j\}_{j<i}$ and a set E_I of equalities interpreted by true in I.

1. Assume that A_i has the form $s = s$. Then, I has one successor J s.t. $[A_i]_J = T$
2. Assume otherwise that $A_i \longrightarrow_{R_I} A_j$ with $i > j$. Then I has one successor J s.t. $[A_i]_J = [A_j]_I$.
3. Otherwise, I has a left successsor J and a right K s.t. $[A_i]J = T$ and $[A_i]_K = F$.

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Lemma

The set of leaves is in bijection with the set of equality Herbrand interpretations of $\mathcal{A}$.

Proof: We show that H is a partial Herbrand equality interpretation over an initial segment $\{A_i\}_{i<j\leq n}$ iff
(a) for any atom $s = s \in \{A_i\}_{i<j}$, $[s = s]_H = T$,
(b) for any two atoms A_k, A_l such that $j > k > l$ and $A_k \longrightarrow_{E_H} A_l$, then $[A_k]_H = [A_l]_H$.

Clearly, an Herbrand equality interpretation satisfies (i) and (ii). We show the converse.

Proof

Let $s = t \in E_H$ and $u[s] = u[t] \in \{A_i\}_{i<j}$ for some $u[] \neq []$. Then $u[s] = u[t] \longrightarrow_{E_H} u[t] = u[t]$ which belongs to $\{A_i\}_{i<j}$ by previous lemma (i), assuming $u[s] \succ u[t]$. By (a) and (b), $[u[s] = u[t]]_H = [u[t] = u[t]]_H = T$, hence $u[s] = u[t] \in E_H$.

Assuming now that $A_k \longleftrightarrow^*_{E_H} A_l$ with $k > l$, we show that $[A_k]_H = [A_l]_H$ by induction on k. By previous lemma (ii), there exist atoms B, C such that $A_k \longrightarrow_{E_H} B \longrightarrow^*_{E_H} C \longleftarrow^*_{E_H} A_l$. By previous lemma (i), $B = A_m$ for some $m < k$. By induction hypothesis, $[A_m]_H = [A_l]_H$. By assumption (b), $[A_k]_H = [A_m]_H$. We conclude by transitivity. □

Tree of equality interpretations for $fb \succ fa \succ a \succ b$

Semantic trees

Generating Interpretation

We assume an unsatisfiable set $\mathcal{G}$ of ground clauses built upon the atoms in $\mathcal{A}$, which is closed under the rules in $\mathcal{ORPF}$.

Definition

We call *failure node* a partial equality interpretation J for which there exists $C \in \mathcal{G}$ such that $[C]_J = F$ and $[C]_I$ is undefined for any $I < J$. We call semantic tree associated with $\mathcal{G}$ the tree obtained from the tree of equality interpretations by replacing each failure node J by a leaf labelled with a clause in $\mathcal{G}$ refuting J.

Inductive set of generating interpretations

1. If the partial interpretation I is a leaf, done.
2. If I has a unique successor I' in the semantic tree, choose I'.
3. If I has two successors J (the left one) and K (the right one) such that K is a failure node, choose J. In case K is labelled by the clause $s = t \vee D$ such that $s = t$ is maximal, we say that $s = t$ is *generated*.
4. If I has two successors J (the left one) and K (the right one) such that K is not a failure node and $A_{|I|}$ is an equality atom, choose K.
5. Otherwise, choose either J or K.

G will denote any generating interpretation.

Property of generating interpretations

Lemma

Assume that G is a generating interpretation of a semantic tree associated with the unsatisfiable set $\mathcal{G} = \{A_i\}_{i<n}$ of ground clauses closed under the rules in $\mathcal{ORPF}$.
Let us assume that A_i is reducible by E_G. Then, there exists a generating clause $s = t \vee C$ in $\mathcal{G}$ such that:
(i) $A_i \longrightarrow_{s=t \in E_G} A_j \in \mathcal{A}$, with $s \succ t$,
(ii) $s = t \succ A$ for every atom A of C,
(iii) $[C]_G = F$.

Proof A straightforward key property of G is that the generated equations are exactly the equations $s = t$ irreducible in $E_{G\setminus\{s=t\}}$.
Let I be the father of G.
(i). We need proving that each reducible atom A_i rewrites to atom $B \in \mathcal{A}$ with an irreducible equation.
Let $s = t$, $s \succ t$, be an equation reducing A_i, that is, $A_i = A_i[s]$ and $A_j = A_i[t]$ for some $j < i$, such that (s, t) is minimal with respect to $\succ$.
If t is reducible to t' by some equation $u = v$ interpreted in T by G, then $s = t' \in E_G$, hence A_i is reducible by a smaller equation. Contradiction.

proof

If s is reducible by some equation $u = v$ interpreted in T by G, then, by monotonicity, $w[s]$ is reducible by $u = v$, hence A_i is reducible by an equation smaller than $s = t$. Contradiction, or $s = u, t = v$ up to renaming. Therefore, $s = t$ is irreducible for $E_G \setminus \{s = t\}$.
(ii) and (iii). Since $s = t$ is the last atom enumerated by G, it is maximal in the clause. Since $\mathcal{G}$ is closed under positive factoring, we can assume that $s = t \notin C$, hence $[C]_G = [C]_I = F$ and $s = t$ is strictly bigger than any atom in C.

proof, continued

Since I has two successors, by definition of the tree of Herbrand equality interpretations, s and t must be irreducible by E_I. Let now $u = v \in E_G \setminus (E_I \cup \{s \to t\})$ and assume without loss of generality that $u \succ v$. By definition of the tree of Herbrand equality interpretations, $u = v \succ s = t$. By properties of $\succ$, $u \succ s$ and $u \succ t$, hence u is not a subterm of s or of t. It follows that $s = t$ cannot be reduced by $u \to v$. $\square$

Refutational Completeness of $\mathcal{ORPF}$

Theorem and Proof

Theorem

A set of clauses $\mathcal{C}$ is unsatisfiable iff the empty clause belongs to its closure under $\mathcal{ORPF}$.

Proof: By compactness, we chose a finite unsatisfiable set $\mathcal{G}$ of ground instances of $\mathcal{C}$, built over a finite set $\mathcal{A}$ of ground atoms. By ordered completion, we complete $\mathcal{A}$ into a new finite set $\overline{\mathcal{A}}$. We can now generate a new set of ground instances of $\mathcal{C}$

$$\overline{\mathcal{G}} = \{C\gamma \mid C \in \mathcal{C},\ C\gamma \text{ ground }, A \in \overline{\mathcal{A}}\ \forall A \in C\gamma\}$$

By construction, $\overline{\mathcal{G}}$ contains $\mathcal{G}$, hence is unsatisfiable, and is closed under $\mathcal{ORPF}$.

We construct a minimal semantic tree $\mathcal{W}$ for $\overline{\mathcal{G}}$, for the ordering comparing in $(>_{\mathbb{N}}, \succ_{mul})_{lex}$ the pair $(|\mathcal{W}|, \{\text{clauses refuting the leaves of } \mathcal{W}\})$. Assume $\mathcal{W}$ is non-empty: choose an arbitrary generating interpretation J with father node I. By definition of $\mathcal{W}$, J is refuted by a clause $\pm B \vee C$, in which $B = P(\overline{u}\gamma)$ is last enumerated atom, hence is maximal in C. By minimality assumption and closure of $\mathcal{G}$ under positive factoring, B does not occur in C when positive. By minimality assumption, definition of equality interpretations, closure of $\mathcal{A}$ under ordered paramodulation and construction of $\mathcal{G}$, γ can be assumed in normal form for E_J.

Proof continued

We now exhibit an inference between the clause refuting J and another clause in $\mathcal{W}$. The infered clause will belong to $\mathcal{G}$ and refute the interpretation I, therefore contradicting our minimality assumption.

This is done by cases upon the definition of G.

1. $P(\overline{u}\gamma)$ is of the form $s = s$, in which case I has J as single successor labelled by $\neg s = s \vee C$. By closure property of $\mathcal{G}$ under the rules in $\mathcal{ORPF}$, $\mathcal{G}$ contains C. There are two cases. If $s = s \in C$, then C refutes J, otherwise it refutes a node $N < J$, contradicting our minimality assumption in both cases.

Proof continued

2. $P(\overline{u}\gamma)$ is irreducible by a rule in E_I. Then, I has two successors, and by definition, J must be the left successor of I and the right successor must be itself a leaf. Hence I has two successors labelled by clauses in both of which the atom $P(\overline{u}\gamma)$ is maximal. Let these clauses be $+P(\overline{u}\gamma) \vee C$, in which $P(\overline{u}\gamma)$ is strictly bigger than any atom occuring in C, and $-P(\overline{u}\gamma) \vee D$. By construction, $\mathcal{G}$ contains the clause $C \vee D$ obtained by ordered resolution from both previous clauses. By definition of $\succ$, $C \vee D$ is strictly smaller than the clause $-P(\overline{u}\gamma) \vee D$, hence refuting a node $N \leq J$, which contradicts our minimality assumption.

Proof, end

3. $P(\overline{u}\gamma)$ is reducible by E_I. Since γ is irreducible, $P(\overline{u}\gamma)$ must be reducible at a non-variable position p of $P(\overline{u})$ by an equation $s = t \in E_I$, yielding the atom $B = P(\overline{u}\gamma)[t]_p \in \mathcal{A}$. By Lemma, $s = t$ is generated by a clause $s = t \vee D$ such that $s \succ t$ and $s = t$ is strictly bigger than any atom in D. Therefore, there is an ordered paramodulation between $l = r \vee D$ and the clause $\pm P(\overline{u}\gamma) \vee C$, yielding $B \vee C \vee D$, which belong to $\mathcal{G}$ by construction. Then, the infered clause refutes an ancestor node, the obtained semantic tree is smaller than the starting one, a contradiction again.

Conclusion

1. Apply compactness before anything else;
2. Inductive construction of the tree of interpretations requires the subterm property;
3. Interpretation of Bachmair-Ganzinger model generation technique as a maximal branch of the semantic tree;
4. No lifting needed !