

# *Concurrency, Time & Constraints*

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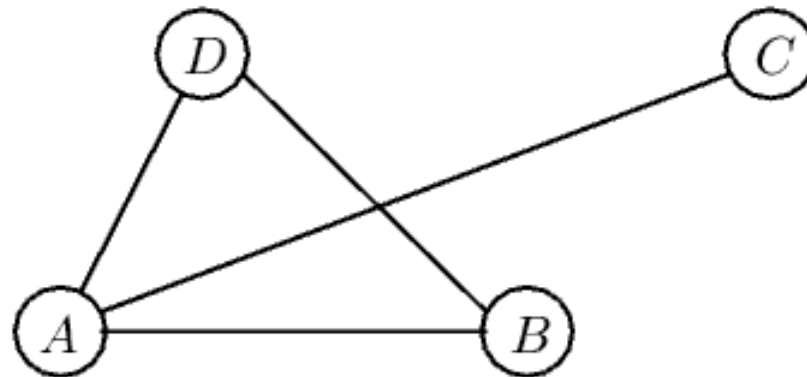
CLEI Cali, Oct 2005



# Concurrency

*Concurrent Systems*: **Agents** (or **processes**) that interact with each other.

- Systems as networks where **arcs represent agent interaction**.

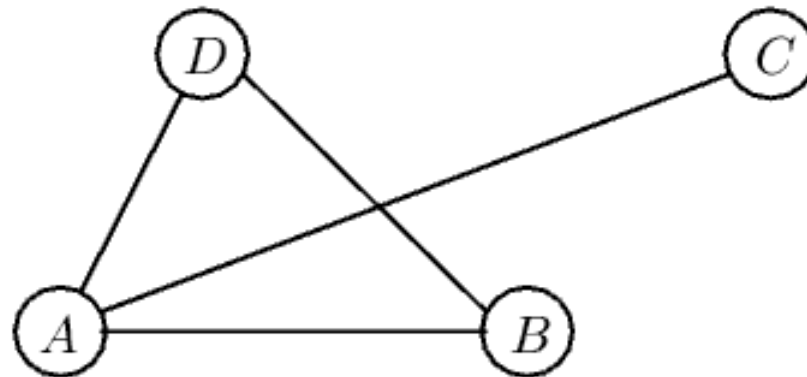


- Models for Concurrency: CCS, Pi-Calculus, CSP. Arcs denote **Links**.

## Concurrency, Constraints

In CCP [Saraswat, '89]: **Agents** interact via **constraints** over shared variables.

- Systems as networks where **arcs represent agent interaction**.



- **Arcs as constraints** on the (shared-variables of) agents.

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# Concurrency, Constraints, and Time

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As other models, CCP has extended for new and wider phenomena

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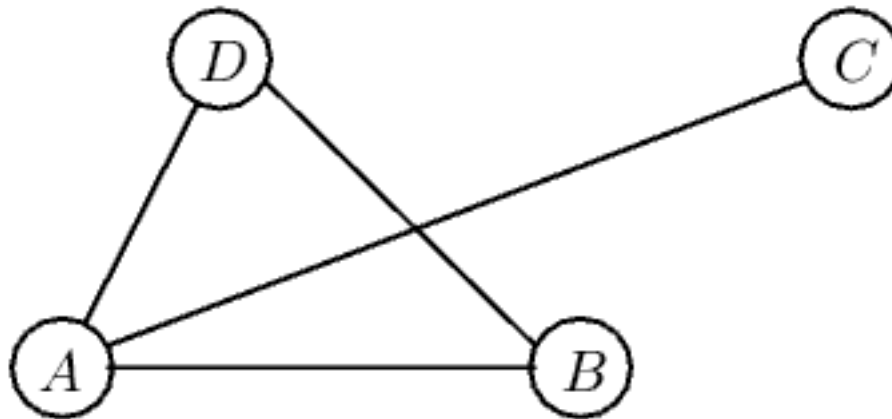
E.g:

- *Mobility* [Gilbert and Palamidessi '00, Réty '98, Rueda & Valencia '97].
  - *Stochastic Behavior* [Saraswat, Jagadeesan '98, Gupta-Panangaden-Jagadeesan '99]
  - *Timed Behavior*
    - (Basic) Timed CCP [Gupta-Jagadeesan-Saraswat '94]
    - Timed Default CCP [Gupta-Jagadeesan-Saraswat '95]
    - Hybrid CCP [Gupta-Jagadeesan-Saraswat '96]
    - Timed CCP: the tccp model [DeBoer-Gabbrielli-Meo '00]
    - Nondeterministic (Basic) Timed CCP [Nielsen-Palamidessi-Valencia '01]
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## Timed CCP: the tcc model

In the tcc model, as time passes, some constraints may *disappear* or be created.

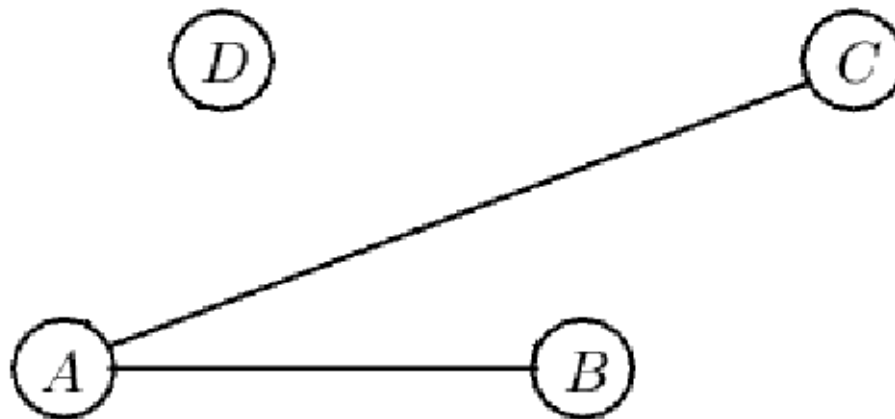
E.g.:  $t = 1$



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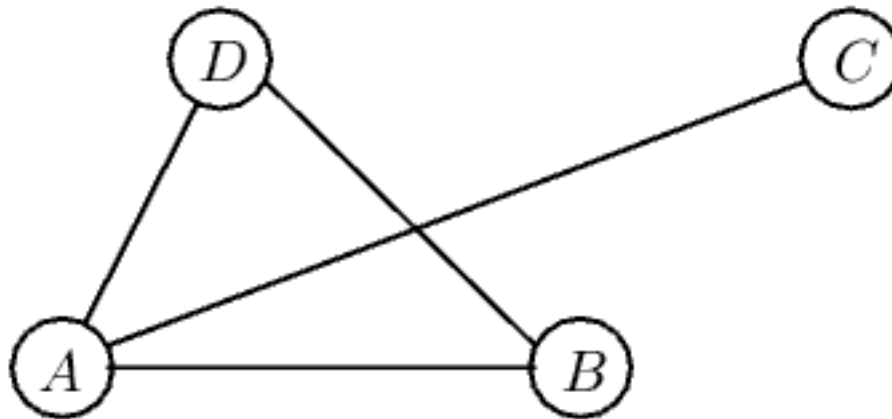
E.g.:  $t = 2$



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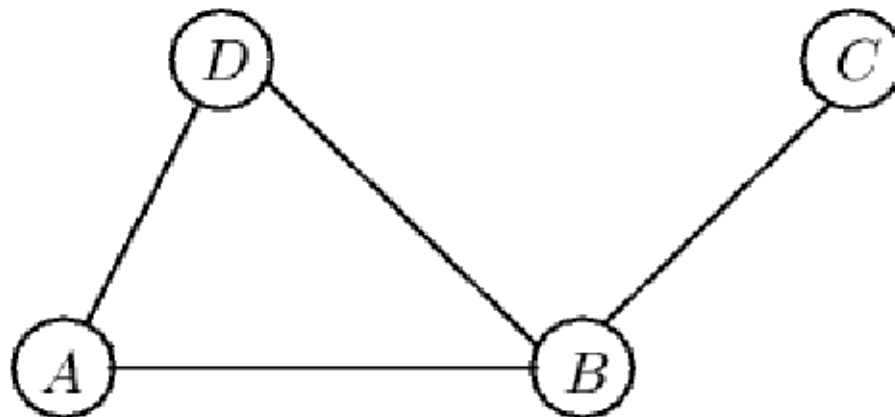
E.g.:  $t = 3$



## Timed CCP: the tcc model

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E.g.:  $t = 4$





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## The Goal of this Talk

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*“...One of the outstanding challenges in concurrency is to find the right marriage between **logic** and **behavioural** approaches”. R. Milner.*

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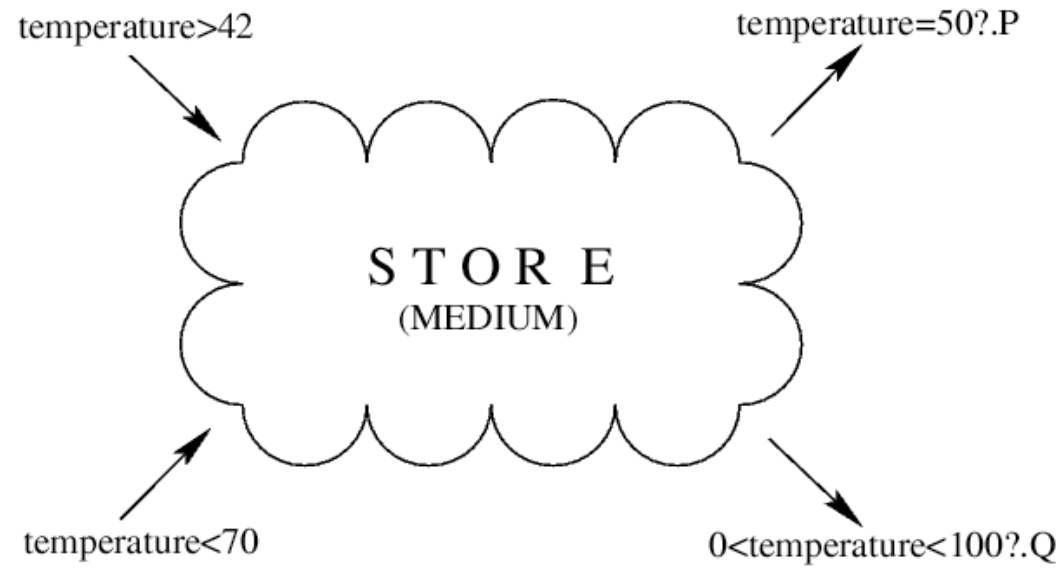
About Timed CCP (I shall argue that):

- It is **simple**.
  - It **expresses** interesting real-world temporal situations.
  - It is **rigorously formalized** upon process algebra and logic.
  - It offers **reasoning techniques** from denotational semantics and process logic.
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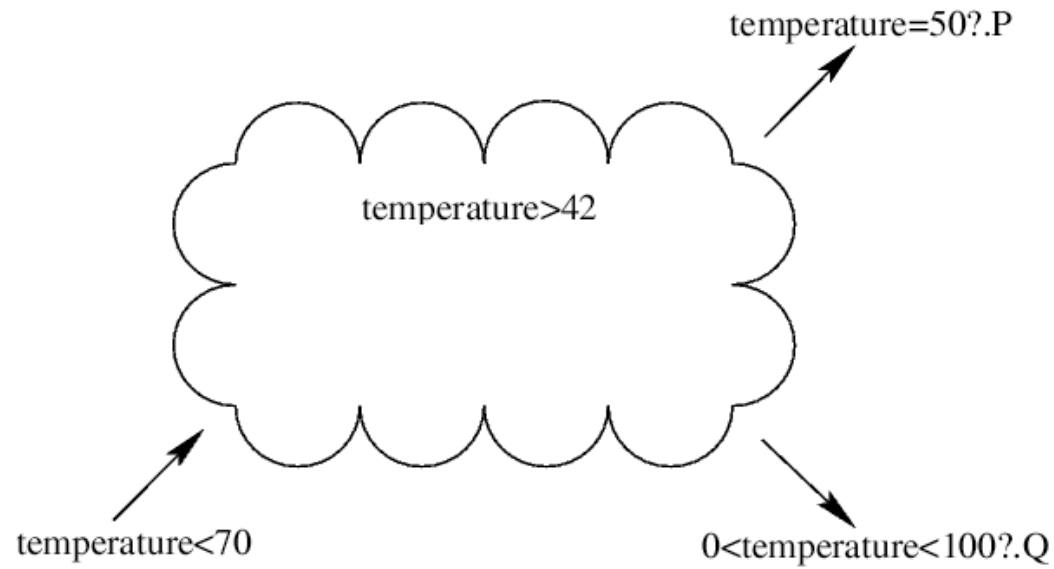
# Agenda

- Basic Timed CCP intuitions.
- Semantics.
- A Logic and Proof System.
- Applications.
- Behavior.
- Hierarchy of temporal CCP languages
- Future Work

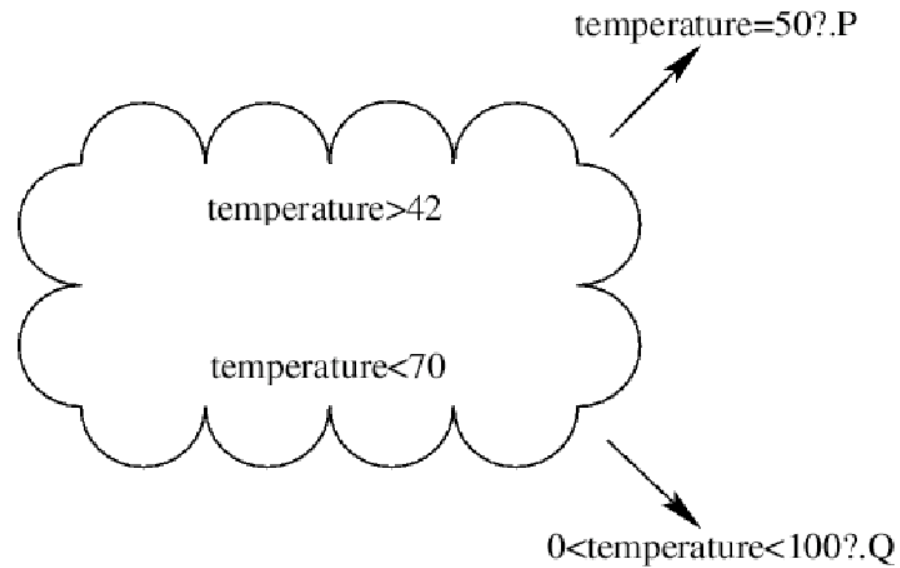
# CCP Intuitions: A Typical CCP Scenario



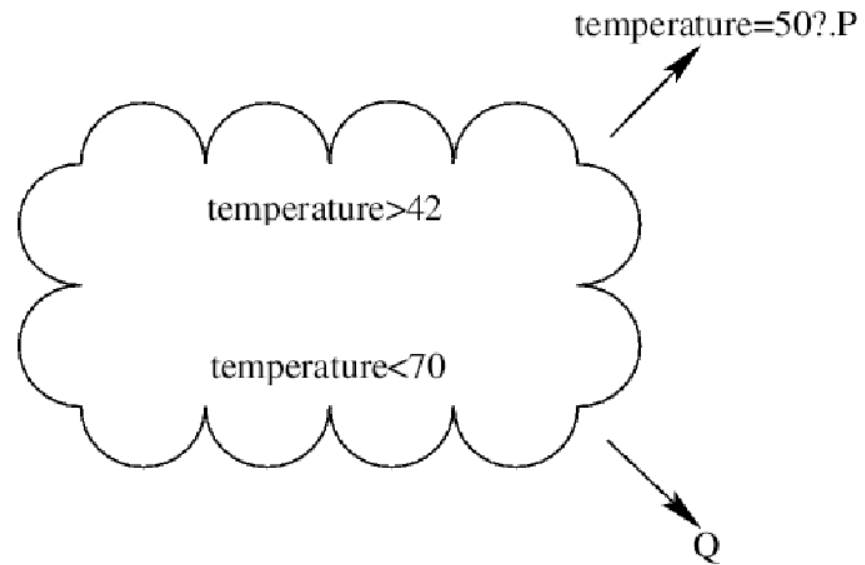
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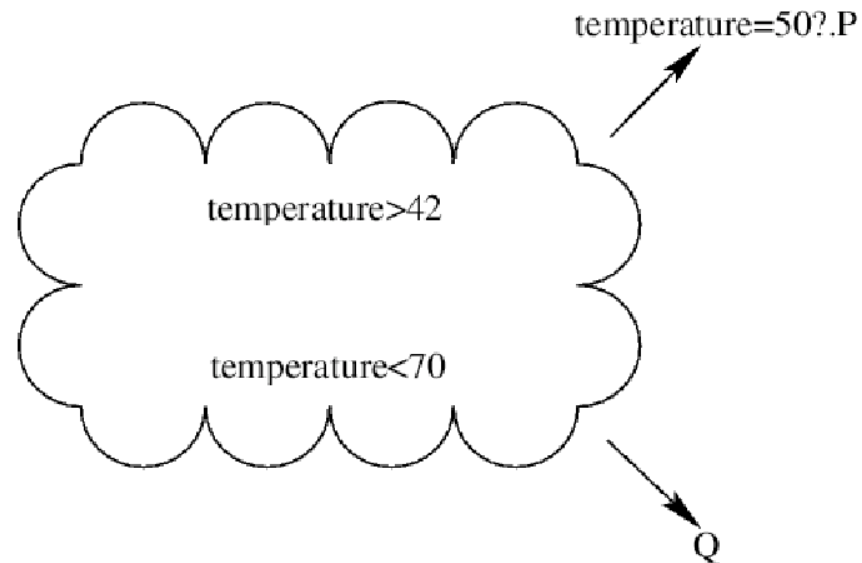
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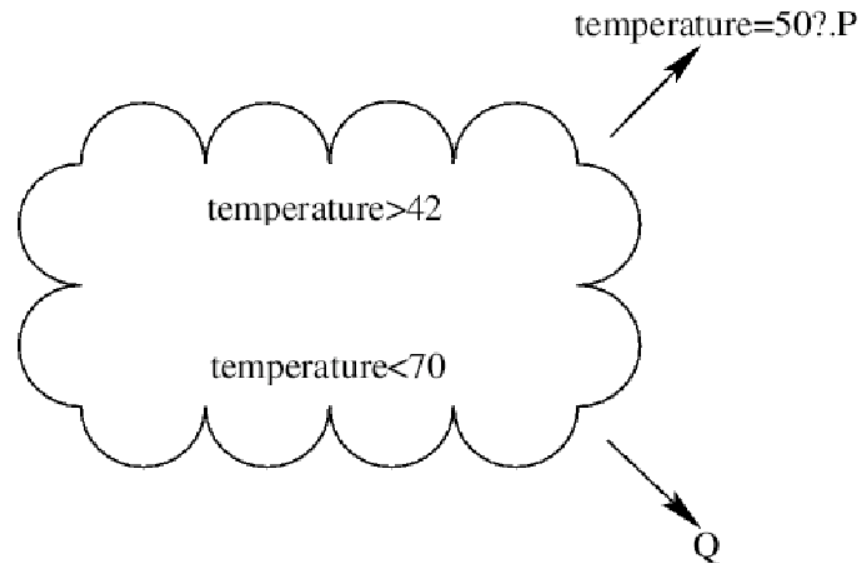


## CCP Intuitions: A Typical CCP Scenario



- **Partial Information** (e.g. temperature is some *unknown* value  $> 20$ ).

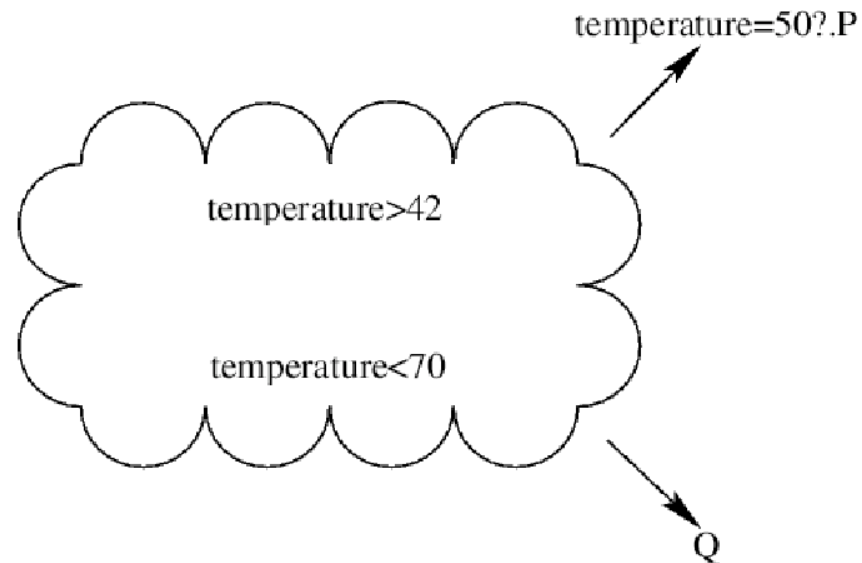
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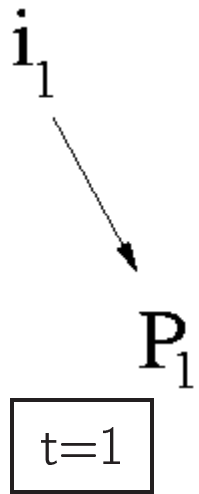
- **Partial Information** (e.g. temperature is some *unknown* value  $> 20$ ).
- **Concurrent Execution** of Processes.
- **Synchronization** via Blocking-Ask.

# CCP Intuitions: Representing Partial Information

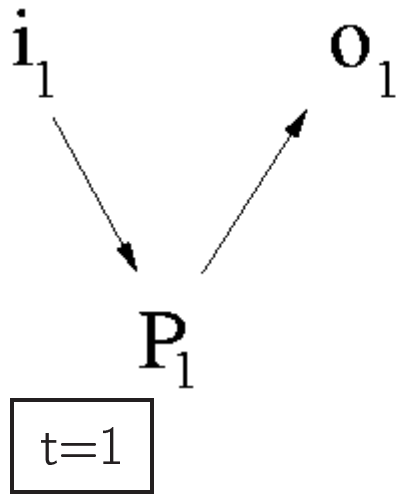
**Definition.** A **constraint system** consists of a signature  $\Sigma$  and first-order theory  $\Delta$  over  $\Sigma$ .

- **Constraints**  $a, b, c, \dots$ : formulae over  $\Sigma$ .
- **Relation**  $\vdash_{\Delta}$ : decidable entailment relation between constraints.
- $\mathcal{C}$ : set of constraints under consideration.

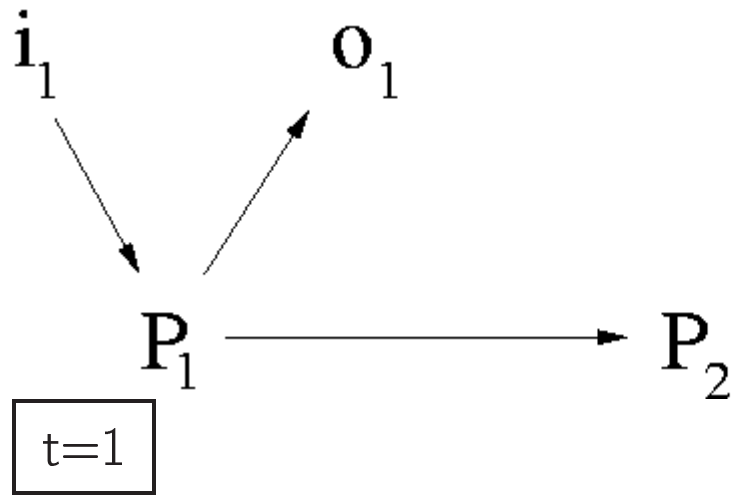
# Reactive Systems



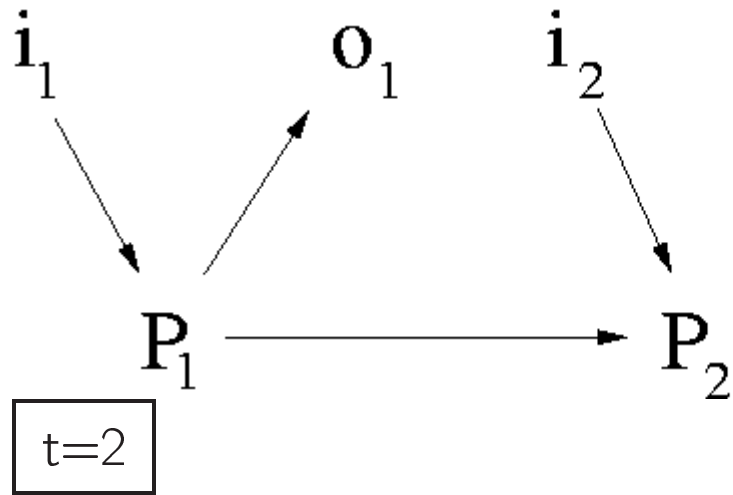
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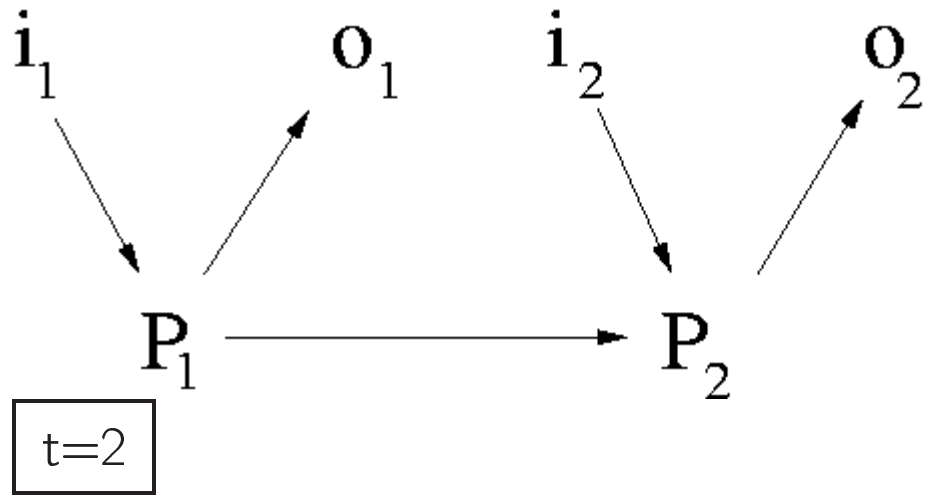
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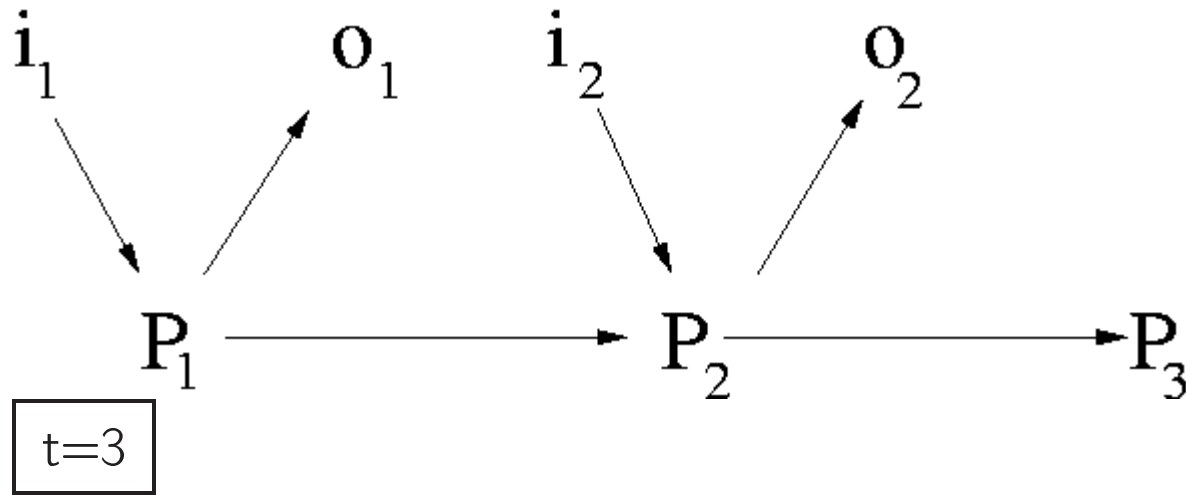
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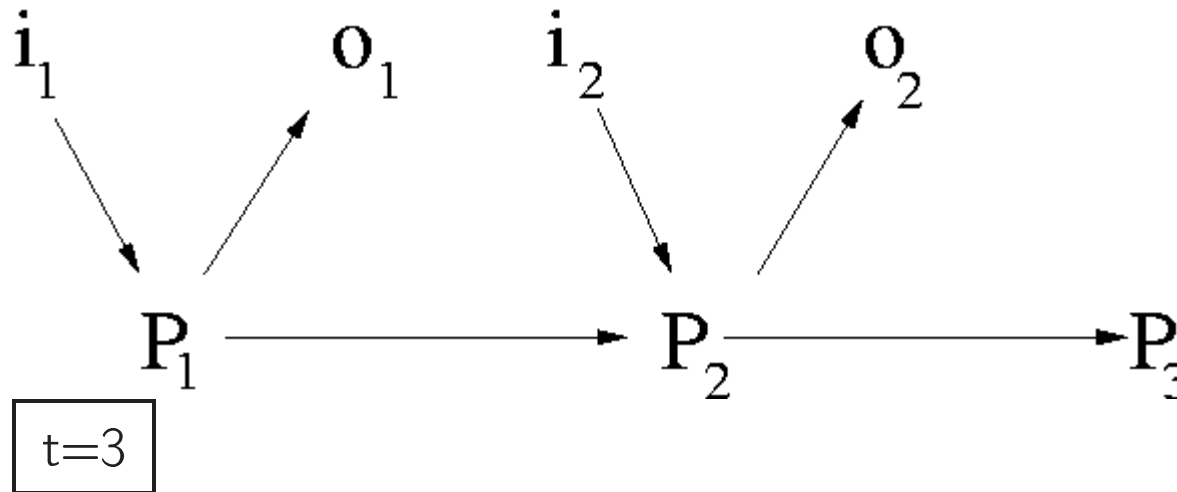


# Reactive Systems






## Reactive Systems



- **Stimulus**  $i_i$  : input information (as a constraint) for  $P_i$  .
- **Response**  $o_i$ : output information (as a constraint) of  $P_i$  .
- **Stimulus-Response** duration: **time interval** (or **time unit**).

**Examples:** PLC's, RCX Robots, Micro-Controllers, Synchronous Languages.

# Basic tcc Processes

Processes	Description	Action within the time interval
<p> <b>tell</b>(<i>c</i>)</p>	<p>telling information</p>	<p>add <i>c</i> to the store</p>

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• <b>unless</b> $c$ <b>next</b> $P$	time-out	unless $c$ now in the store do $P$ next
$P \parallel Q$	parallelism	execute $P$ and $Q$

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<ul style="list-style-type: none"> <li>• <math>!P</math></li> </ul>	<p>replication</p>	<p>execute <math>P</math> each time unit</p>

## Some Derived Constructs

- **Abortion**

$\text{abort} \stackrel{\text{def}}{=} !(\text{tell}(\text{false})).$

- **Asynchronous Parallel**

$P \mid Q \stackrel{\text{def}}{=} (\star P \parallel Q) + (P \parallel \star Q)$

- **Bounded Replication**

$!_{[t,t']} P \stackrel{\text{def}}{=} \prod_{t \leq i \leq t'} \text{next}^i P$

- **Bounded Delay**

$\star_{[t,t']} P \stackrel{\text{def}}{=} \sum_{t \leq i \leq t'} \text{next}^i P$

## Power Saver Example

• A power saver :

!(**unless** (lights = off) **next** ★ **tell**(lights = off))

## Power Saver Example

• A refined power saver :

```
!(unless (lights = off) next  $\star_{[0,60]}$  tell(lights = off))
```

## Power Saver Example

- A more refined one; deterministic power saver:

!(**unless** (lights = off) **next tell**(lights = off))

# Operational Semantics

## Internal Transitions:

$$\begin{array}{l}
 RT \frac{}{\langle \mathbf{tell}(c), a \rangle \longrightarrow \langle \mathbf{skip}, a \wedge c \rangle} \qquad RG \frac{a \vdash c_j}{\langle \sum_{i \in I} \mathbf{when} \ c_i \ \mathbf{do} \ P_i, a \rangle \longrightarrow \langle P_j, a \rangle} \\
 RB \frac{}{\langle ! P, a \rangle \longrightarrow \langle P \ \|\ \mathbf{next} \ ! P, a \rangle} \qquad RS \frac{}{\langle \star P, a \rangle \longrightarrow \langle \mathbf{next}^n P, a \rangle}^{(n \geq 0)}
 \end{array}$$

## Observable Transition

$$RO \frac{\langle P, a \rangle \longrightarrow^* \langle Q, a' \rangle \not\rightarrow}{P \xrightarrow{(a, a')} \mathbf{F}(Q)} = \begin{cases} Q' & \text{if } Q = \mathbf{next} \ Q' \\ Q' & \text{if } Q = \mathbf{unless} \ (c) \ \mathbf{next} \ Q' \\ \mathbf{F}(Q_1) \ \|\ \mathbf{F}(Q_2) & \text{if } Q = Q_1 \ \|\ Q_2 \\ \mathbf{local} \ x \ \mathbf{in} \ \mathbf{F}(Q') & \text{if } Q = \mathbf{local} \ x \ \mathbf{in} \ Q' \\ \mathbf{skip} & \text{otherwise} \end{cases}$$

# Operational Semantics

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## Observations to Make of Processes

### • Stimulus-response interaction

$$P = P_1 \xrightarrow{(c_1, c'_1)} P_2 \xrightarrow{(c_2, c'_2)} P_3 \xrightarrow{(c_3, c'_3)} \dots$$

denoted by  $P \xrightarrow{(\alpha, \alpha')} \omega$  with  $\alpha = c_1.c_2 \dots$  and  $\alpha' = c'_1.c'_2 \dots$

### Observable Behavior

• **Input-Output**  $io(P) = \{(\alpha, \alpha') \mid P \xrightarrow{(\alpha, \alpha')} \omega\}$

• **Output**  $o(P) = \{\alpha' \mid P \xrightarrow{(\text{true}^\omega, \alpha')} \omega\}$

• **Strongest Postcondition**  $sp(P) = \{\alpha' \mid P \xrightarrow{(-, \alpha')} \omega\}$

# Strongest-Postcondition Denotational Semantics

$$\llbracket \text{tell}(a) \rrbracket = \{c \cdot \alpha \in C^\omega : c \vdash a, \}$$

$$\llbracket P \parallel Q \rrbracket = \llbracket P \rrbracket \cap \llbracket Q \rrbracket$$

$$\llbracket !P \rrbracket = \{ \alpha : \text{for all } \beta \in C^*, \alpha' \in C^\omega : \alpha = \beta.\alpha' \text{ implies } \alpha' \in \llbracket P \rrbracket \}$$

$$\llbracket \star P \rrbracket = \{ \beta.\alpha : \beta \in C^*, \alpha \in \llbracket P \rrbracket \}$$

$$\llbracket \sum_{i \in I} \text{when } (a_i) \text{ do } P_i \rrbracket = \bigcup_{i \in I} \{c \cdot \alpha : c \vdash a_i \text{ and } c \cdot \alpha \in \llbracket P_i \rrbracket\} \cup \left( \bigcap_{i \in I} \{c \cdot \alpha : c \not\vdash a_i, \alpha \in C^\omega\} \right)$$

**Definition.**  $P$  is **locally-independent** iff its guards depend on no local variables.

**Theorem.**  $sp(P) \subseteq \llbracket P \rrbracket$  and, if  $P$  is a locally-independent,  $sp(P) = \llbracket P \rrbracket$

## Related Work & Road Map

- IO Denotation for Timed CCP: [ Gupta-Jagadeesan-Saraswat '94]

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### RoadMap:

Operational and Denotational Models for Timed CCP

🌐 Coming Next: Logic & Specification.

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**Syntax.**  $A ::= c \mid A \wedge A \mid \neg A \mid \exists_x A \mid \circ A \mid \diamond A \mid \square A$

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**Semantics.** Say  $\alpha = c_1.c_2.\dots \models A$  iff  $\langle \alpha, 1 \rangle \models A$  where

$\langle \alpha, i \rangle \models c$	iff	$c_i \vdash c$
$\langle \alpha, i \rangle \models \neg A$	iff	$\langle \alpha, i \rangle \not\models A$
$\langle \alpha, i \rangle \models A_1 \wedge A_2$	iff	$\langle \alpha, i \rangle \models A_1$ and $\langle \alpha, i \rangle \models A_2$
$\langle \alpha, i \rangle \models \circ A$	iff	$\langle \alpha, i + 1 \rangle \models A$
$\langle \alpha, i \rangle \models \square A$	iff	for all $j \geq i$ $\langle \alpha, j \rangle \models A$
$\langle \alpha, i \rangle \models \diamond A$	iff	there exists $j \geq i$ s.t. $\langle \alpha, j \rangle \models A$
$\langle \alpha, i \rangle \models \exists_x A$	iff	there is $\alpha'$ $x$ -variant of $\alpha$ s.t. $\langle \alpha', i \rangle \models A$ .

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For example

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- $\square \diamond A \Leftrightarrow \diamond \square A$  ??

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$\bullet$   $!(\mathbf{unless}(\text{LightsOff}) \mathbf{next} \star \mathbf{tell}(\text{LightsOff})) \models \diamond(\text{LightsOff})$

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E.g.,

$$\bullet \text{!(unless (LightsOff) next } \star \text{ tell(LightsOff))} \models \diamond(\text{LightsOff})$$

$$\bullet \text{!(when (AlarmGoesOff) do tell(CloseGate))} \models \square \text{AlarmGoesOff} \Rightarrow \square \text{CloseGate}$$

## Specification and Satisfaction

Specifications can be expressed as LTL formulae. We then say:

$P$  meets  $A$ , written  $P \models A$ , iff all sequences  $P$  outputs satisfy  $A$

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- $!(\text{when}(\text{AlarmGoesOff}) \text{ do } \text{tell}(\text{CloseGate})) \models \square \text{AlarmGoesOff} \Rightarrow \square \text{CloseGate}$
- **But how can we prove  $P \models A$  ?**

## Proof System for $P \models A$

$$\begin{array}{c}
 \text{tell}(c) \vdash c \text{ (tell)} \\
 \\
 \frac{P \vdash A \quad Q \vdash B}{P \parallel Q \vdash A \wedge B} \text{ (par)} \qquad \frac{P \vdash A}{\text{local } x \text{ in } P \vdash \exists x A} \text{ (hide)} \\
 \\
 \frac{P \vdash A}{\text{next } P \vdash \bigcirc A} \text{ (next)} \\
 \\
 \frac{P \vdash A}{!P \vdash \square A} \text{ (rep)} \qquad \frac{P \vdash A}{*P \vdash \diamond A} \text{ (star)} \\
 \\
 \frac{\forall i \in I \ P_i \vdash A_i}{\sum_{i \in I} \text{ when } c_i \text{ do } P_i \vdash \bigvee_{i \in I} (c_i \wedge A_i) \vee \bigwedge_{i \in I} \neg c_i} \text{ (sum)} \\
 \\
 \frac{P \vdash A \quad A \Rightarrow B}{P \vdash B} \text{ (rel)}
 \end{array}$$

Theorem. (*Completeness*) For every locally-independent process  $P$ ,

$$P \models A \quad \text{iff} \quad P \vdash A$$

## Verification $P \models A$

- Can we prove  $P \models A$  *automatically* ?

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**Theorem.** *Given a locally-independent  $P$  and a negation-free  $A$ , the problem of whether  $P \models A$  is **decidable**.*

*...and the proof uses the **denotational semantics** rather than the operational semantics !.*

# Theoretical Applications: Pnueli's First-Order LTL

Pnueli's First-Order LTL (FOLTL):

- Syntax like that of the Timed CCP Logic.

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- [Abadi '89] proved the full-language to be **undecidable**.
- Several work identifying decidable fragments of FOLTL.
- *Without rigid variables, FOLTL is **decidable**.* Proof by using the theory of **Timed CCP**.

## Programming Applications: Cells

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**Example.**  $x : (3) \parallel y : (5) \parallel exch_7(x, y) \Longrightarrow x : (7) \parallel y : (3)$ .

# Applications: Logic & Proof System at Work

Proposition.

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$$\frac{\frac{\frac{\frac{\frac{}{x : (g(w)) \vdash x = g(w)}{Pr.1}}{\frac{}{y : (w) \vdash y = w}}{Pr.1}}{LPAR}}{\frac{}{x : (g(w)) \parallel y : (w) \vdash x = g(w) \dot{\wedge} y = w}}{LNEXT}}{\frac{}{next(x : (g(w)) \parallel y : (w)) \vdash \bigcirc(x = g(w) \dot{\wedge} y = w)}}{Lem.(3)}}}{\frac{\forall w \in \mathcal{D} \text{ tell}(change(x)) \parallel \text{tell}(change(y)) \parallel \text{next}(x : f(w) \parallel y : (w)) \vdash \bigcirc(x = g(w) \dot{\wedge} y = w)}{LSUM}}{\frac{exch_f(x, y) \vdash \dot{\bigvee}_{w \in \mathcal{D}} (x = w \dot{\wedge} \bigcirc(x = g(w) \dot{\wedge} y = w)) \dot{\bigwedge}_{w \in \mathcal{D}} \dot{\vdash} x = w}{LCONS}}{\frac{exch_f(x, y) \vdash \dot{\bigwedge}_{w \in \mathcal{D}} (x = w \Rightarrow \bigcirc(x = g(w) \dot{\wedge} y = w))}{LCONS}}{\frac{exch_f(x, y) \vdash (x = v \Rightarrow \bigcirc(x = g(v) \dot{\wedge} y = v))}{LCONS}}$$

## Programming Applications: LEGO Zigzagging

**Specification.** Go *forward* (f), *right* (r) or *left* (l) but DO NOT go:

- f if preceding action was f,
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<i>GoForward</i>	<u>def</u>	$f_{exch}(act_1, act_2) \parallel \mathbf{tell}(\mathbf{forward})$
<i>GoRight</i>	<u>def</u>	$r_{exch}(act_1, act_2) \parallel \mathbf{tell}(\mathbf{right})$
<i>GoLeft</i>	<u>def</u>	$l_{exch}(act_1, act_2) \parallel \mathbf{tell}(\mathbf{left})$
<i>Zigzag</i>	<u>def</u>	( $\mathbf{when}(act_1 \neq \mathbf{f}) \mathbf{do} \mathit{GoForward}$ $\mathbf{when}(act_2 \neq \mathbf{r}) \mathbf{do} \mathit{GoRight}$ $\mathbf{when}(act_2 \neq \mathbf{l}) \mathbf{do} \mathit{GoLeft}$ ) $\parallel$ $\mathbf{next} \mathit{Zigzag}$
<i>StartZigzag</i>	<u>def</u>	$act_1:(0) \parallel act_2:(0) \parallel \mathit{Zigzag}$

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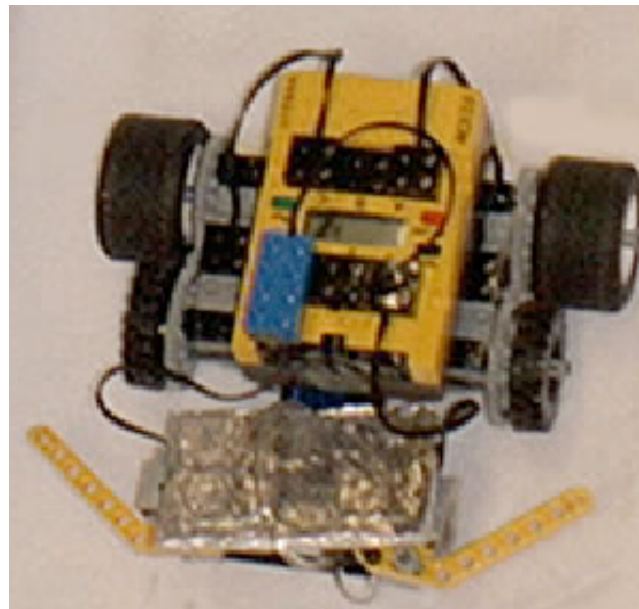
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<i>GoForward</i>	$\stackrel{\text{def}}{=}$	$f_{\text{exch}}(act_1, act_2) \parallel \text{tell}(\text{forward})$
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<i>GoLeft</i>	$\stackrel{\text{def}}{=}$	$l_{\text{exch}}(act_1, act_2) \parallel \text{tell}(\text{left})$
<i>Zigzag</i>	$\stackrel{\text{def}}{=}$	( <b>when</b> ( $act_1 \neq f$ ) <b>do</b> <i>GoForward</i> + <b>when</b> ( $act_2 \neq r$ ) <b>do</b> <i>GoRight</i> + <b>when</b> ( $act_2 \neq l$ ) <b>do</b> <i>GoLeft</i> )    <b>next</b> <i>Zigzag</i>
<i>StartZigzag</i>	$\stackrel{\text{def}}{=}$	$act_1:(0) \parallel act_2:(0) \parallel \text{Zigzag}$

**Proposition.**  $\text{StartZigzag} \vdash \Box(\Diamond\text{right} \wedge \Diamond\text{left})$

# A Timed CCP Programming Language for Robots

**LMAN** (Hurtado&Munoz 2003): A timed ccp reactive programming language for **LEGO RCX** Robots.



## Music Applications: Controlled Improvisation.

Music composition and performance is a complex task of defining and controlling concurrent activity. E.g:

- ▷ There are  $M_1, \dots, M_m$  musicians (or *Voices* if you wish). Each  $M_i$  is given a three-notes pattern  $p_i$  of delays between each note in the block.
- ▷ Once her block is played, the musician waits for the others to finish their respective blocks before start playing a new one.
- ▷ The exact time a new block will be started is not specified, but should not be later than  $p_{dur}$ ; the sum of the durations of all patterns.
- ▷ Musicians keep playing notes until all of them play a note simultaneously.

# Music Applications: Controlled Improvisation

$$M_i \stackrel{\text{def}}{=} \sum_{(j,k,l) \in \text{perm}(p_i)} ( \text{Play}_{(j,k,l)}^i \parallel \text{next}^{j+k+l} ( \text{flag}_i := 1 \parallel \text{whenever } (go = 1) \text{ do } \star_{[0, \text{pdur}]} M_i ) )$$

$$\text{Play}_{(j,k,l)}^i \stackrel{\text{def}}{=} \begin{aligned} & !_{[0, j-1]} \text{tell}(note_i = \text{sil}) \parallel \text{next}^j \text{tell}(c_i[note_i]) \\ & \parallel !_{[j+1, j+k-1]} \text{tell}(note_i = \text{sil}) \parallel \text{next}^{j+k} \text{tell}(c_i[note_i]) \\ & \parallel !_{[j+k+1, j+k+l-1]} \text{tell}(note_i = \text{sil}) \parallel \text{next}^{j+k+l} \text{tell}(c_i[note_i]) \end{aligned}$$

$$C \stackrel{\text{def}}{=} \begin{aligned} & !( \text{when } \bigwedge_{i \in [1, m]} (flag_i = 1) \wedge (stop = 0) \text{ do} \\ & \quad ( \text{tell}(go = 1) \parallel \prod_{i \in [1, m]} flag_i := 0 ) ) \\ & \parallel \text{next whenever } \bigwedge_{i \in [1, m]} (note_i \neq \text{sil}) \text{ do } stop := 1 \end{aligned}$$

## Music Applications: Controlled Improvisation

$$Init \stackrel{\text{def}}{=} \prod_{i \in [1, m]} (\text{tell}(c_i[note_i]) \parallel flag_i : 0) \parallel stop : 0$$

$$Sys \stackrel{\text{def}}{=} Init \parallel C \parallel \prod_{i \in [1, m]} M_i$$

- ▷ Notice that *regardless the musicians' choices* the system always terminates iff

$$Sys \vdash \diamond stop = 1.$$

- ▷ Notice that *there are some musicians' choices* on which the system terminates iff

$$Sys \not\vdash \square stop = 0.$$

The above statements can be effectively verified!

## More Timed CCP Applications and Languages

- **Music Composition and Performance** (Rueda&Valencia 2004).
- **Biological System** (Olarte&Rueda 2005, Gutierrez&Perez&Rueda 2005).
- **TimedGentzen** (Saraswat 1995): A tcc-based programming language for reactive-systems implemented in Prolog.
- **JCC** (Saraswat&Gupta 2003): An integration of timed ccp into **JAVA**. See <http://www.cse.psu.edu/~saraswat/jcc.html>

## Related Work & Road Map

- Logic & Proof System for Timed CCP: [ Gupta-Jagadeesan-Saraswat '94,'95]



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• Verification (Model Checking) for Timed CCP [Falaschi and Villanueva '03]

### RoadMap:

Operational and Denotational Models for Timed CCP

Timed CCP Logic and its Applications

• Coming Next: Behavioral Equivalences.

## Observations to Make of Processes

### • Stimulus-response interaction

$$P = P_1 \xrightarrow{(c_1, c'_1)} P_2 \xrightarrow{(c_2, c'_2)} P_3 \xrightarrow{(c_3, c'_3)} \dots$$

denoted by  $P \xrightarrow{(\alpha, \alpha')} \omega$  with  $\alpha = c_1.c_2 \dots$  and  $\alpha' = c'_1.c'_2 \dots$

### Observable Behavior

• **Input-Output**  $io(P) = \{(\alpha, \alpha') \mid P \xrightarrow{(\alpha, \alpha')} \omega\}$

• **Output**  $o(P) = \{\alpha' \mid P \xrightarrow{(\text{true}^\omega, \alpha')} \omega\}$

• **Strongest Postcondition**  $sp(P) = \{\alpha' \mid P \xrightarrow{(-, \alpha')} \omega\}$

## Behavioral Equivalences

**Definition.** Let  $l \in \{o, io, sp\}$ . Define  $P \sim_l Q$  iff  $l(P) = l(Q)$ .

Unfortunately, neither  $\sim_{io}$  nor  $\sim_o$  are congruences. Let  $\approx_{io}$  and  $\approx_o$  be the corresponding congruences.

**Theorem.**  $\approx_{io} = \approx_o \subset \sim_{io} \subset \sim_o$ .

## Distinguishing Context Characterizations

**Theorem.** Given  $P, Q$  and  $\sim \in \{\approx_o, \sim_{io}, \sim_{sp}\}$ , one can construct a context  $C_{\sim}^{(P,Q)}[.]$  such that:

$$P \sim Q \quad \text{if and only if} \quad C_{\sim}^{(P,Q)}[P] \sim_o C_{\sim}^{(P,Q)}[Q]$$

- Interesting consequence of the theorem:

**Decidability** of all  $\sim_{io}, \sim_{sp}, \approx_o$  and  $\approx_{io}$  reduce to that of  $\sim_o$ .

- Interesting result introduced for the proof:

Given  $P$  one can **construct a finite set** including all relevant inputs.

## Behavioral Equivalence: Decidability.

**Definition.** A star-free  $P$  is **locally-deterministic** iff all its summations occur outside of its local processes.

**Theorem.** Given a locally-deterministic  $P$  one can effectively construct a Büchi automaton  $B_P$  that recognizes  $o(P)$ .

As a corollary,

**Theorem.**  $\approx_o, \approx_{io}, \sim_{io}, \sim_{sp}$  **are all decidable** for locally-deterministic processes.



## Related Work & Road Map

- Decidability of Various Equivalences [Valencia '03]

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- Timed CCP Bisimilarity Equivalence and its Axiomatization [Tini '00]

### RoadMap:

Operational and Denotational Models for Timed CCP

Timed CCP Logic and its Applications

Behavioral Equivalences

- Coming Next: Timed CCP Language Hierarchy.

## Variants and their Expressive Power

Basic Timed CCP with the following alternatives for *infinite behavior*.

- **tcc[Rec]**

Recursive definitions  $A(x_1, \dots, x_n) \stackrel{\text{def}}{=} P$  with  $fv(P) \subseteq \{x_1, \dots, x_n\}$ .

- **tcc[Rec, Identical Parameters]**

As above but every call of  $A$  in  $P$  is of the form  $A(x_1, \dots, x_n)$ .

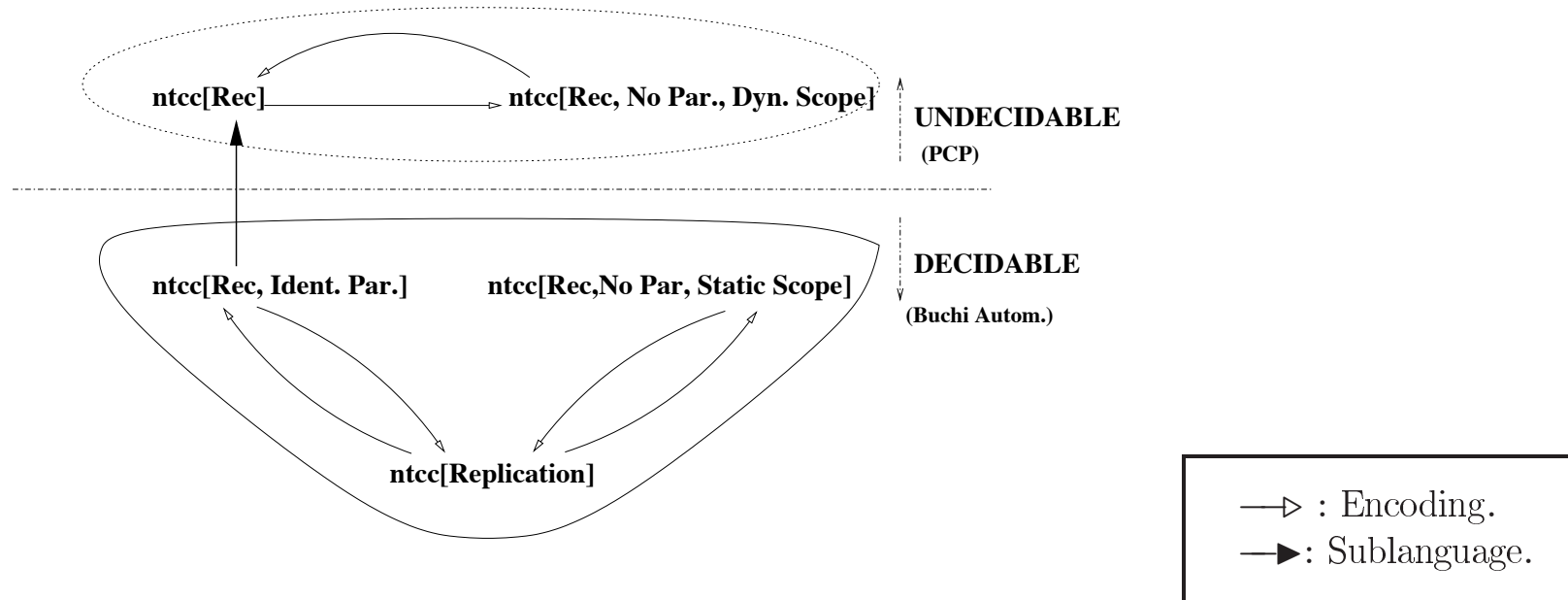
- **tcc[Rec, No Parameters, Dyn. Scoping]**

Recursive definitions  $A \stackrel{\text{def}}{=} P$  with Dynamic Scoping

- **tcc[Rec, No Parameters, Static Scoping]**

Recursive definitions  $A \stackrel{\text{def}}{=} P$  with Static Scoping.

# TCC Hierarchy and $\sim_{io}$ (un)decidability.



- Qualitative distinction between dynamic and static scope.
- The results have inspired similar results for CCS.

## Final Remarks

Timed CCP combines the [declarative](#) view of *LTL* with the [operational-behavioral](#) view from *process calculi* .

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- **Simple** ideas from concurrency and temporal logic.
- It **expresses** interesting real-world temporal situations.
- **Formalization** upon process algebra and logic.
- **Techniques** from a denotational semantics and process logic.

## Ongoing and Future Work

- Implementation of Automatic Tools for analyzing Timed CCP Processes.
- Probabilistic Timed CCP (Olarte&Rueda 2005, Perez 2005).
- *Secure CCP* (Ecole Polytechnique, IBM, Univ. Pisa, Javeriana, Univalle).
- *Timed CCP for reasoning about Biological Systems* (Olarte&Rueda 2005, Gutierrez&Perez 2005).

## Examples of Observables

$$\underbrace{\begin{array}{l} \text{when } a \text{ do next} \\ \text{when } b \text{ do next tell}(d) \\ + \\ \text{when } c \text{ do next tell}(e) \end{array}}_P, \quad \underbrace{\begin{array}{l} \text{when } a \text{ do next when } b \text{ do next tell}(d) \\ + \\ \text{when } a \text{ do next when } c \text{ do next tell}(e) \end{array}}_Q$$

Assuming  $a, b, c, d$  and  $e$  mutually exclusive:

- $o(P) = o(Q) = \{\text{true}^\omega\}$ .
- $io(P) \neq io(Q)$ : If  $\alpha = a.c.\text{true}^\omega$  then  $(\alpha, \alpha) \in io(Q)$  but  $(\alpha, \alpha) \notin io(P)$
- $sp(P) \neq sp(Q)$ : If  $\alpha = a.c.\text{true}^\omega$  then  $\alpha \in sp(Q)$  but  $\alpha \notin sp(P)$ .