► DALE MILLER, *Focused sequent calculus proof systems.*
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Gentzen invented the sequent calculus as a setting to develop his cut-elimination theorem for classical and intuitionistic logics [4]. The sequent calculus has also been used to motivate and describe proofs in linear logic [5]. Since the sequent calculus capture proofs in these three logics in a modular fashion (i.e., intuitionistic proofs have no structural rules on the right; linear logic proofs have no structural rules on the left and right), they make an appealing proof system to study and apply. Anyone attempting to uncover structure within sequent calculus proofs is forced, however, to recognize that such proofs are chaotic and formless: tedious arguments about the permutation of inference rules usually dominate such an effort. For example, early attempts by computer scientists to use the sequent calculus to provide a proof theory for logic programming encountered such arguments [8].

Focused sequent calculus proof systems impose useful structure on sequent calculus proofs. Andreoli developed the first such proof system for linear logic [1] and several others researchers subsequently developed focused proof systems for classical and intuitionistic logics. Liang and Miller [6, 7] have given comprehensive presentations of focused proof systems for linear, intuitionistic, and classical logics. Focused proofs systems provide a flexible setting to define *synthetic* inference rules with the automatic guarantee that cut-elimination holds for them as well. The full picture of focusing also allows for *multifocusing* in which synthetic inferences rules can be applied in parallel. Although parallelism in sequent calculus proofs seems an oxymoron, multifocused sequent proofs have been used to describe such parallel proof structures as proof nets [3] and expansion trees [2].

In this talk, I will introduce focused proof systems and illustrate their applications to the theory of proofs more generally.

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